SEISMIC ANALYSIS OF ASYMMETRIC IN PLAN BUILDINGS

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ABSTRACT :

Buildings with an asymmetric distribution of stiffness and strength in plan undergo coupled lateral and torsional motions during earthquakes. In many buildings the centre of resistance does not coincide with the centre of mass. By reducing the distance between the centre of mass and the centre of stiffness, torsional effects should be minimized. The stiffness characteristics control the dynamic response of the building structure. The choice of the stiffness characteristics of structures is an important step in the conceptual design phase. The good behaviour of the structure can be provided with a well distributed lateral load resisting system. The inelastic seismic behaviour of asymmetric-plan buildings is considered by using the histories of base shear and torque (BST). The procedure to construct the BST surface of the system with an arbitrary number of resisting elements in the direction of asymmetry and of ground motion is proposed. The BST surface describes the inelastic properties of a system, however, the inelastic deformation cannot be computed unless a non-linear static or dynamic analysis is performed. The factors that determine the seismic response are the strength eccentricity, lateral and torsional capacity of the system, planwise distribution of stiffness and excitation.

KEYWORDS: Asymmetric buildings, torsion, base shear-torque surface, inelastic analysis, time history

1. INTRODUCTION

A lack of symmetry produces torsional effects that are sometimes difficult to assess, and can be very adverse. The preferred method of minimizing torsional effects is to select floor plans that are regular and reasonably compact. Complex plan buildings should be divided by seismic separation joints introduced between rectangular blocks. The behaviour of buildings during earthquakes will be satisfactory only if all measures are taken to provide a favourable failure mechanism. A special account must be taken so that torsional effects do not endanger or preclude the global ductile behaviour of the structure. Buildings with an asymmetric distribution of stiffness and strength in plan undergo coupled lateral and torsional motions during earthquakes. Because of torsion, the seismic demands of asymmetric buildings increase above those required by just translational deformation. It is well-known that the larger the eccentricity between the centre of stiffness and the centre of mass, the larger the torsional effects. An important aspect of the inelastic behaviour of asymmetric structures is the considerations of the degree of control over inelastic twist. One of the design aims should be to restrain the system against unrestricted inelastic twist. In the structures, which remain elastic during an earthquake, torsional vibrations may cause significant additional displacements and forces in the lateral load resisting elements. However, the design of the majority of buildings relies on inelastic response. In that case torsional motion leads to additional displacement and ductility demands. Hence, the relevance of current code recommendations, based on elastic torsional response, is open to questions.

2. BASE SHEAR AND TORQUE ULTIMATE SURFACE

An asymmetric-plan building consisting of rigid diaphragms with lumped storey masses is analysed. Lateral resistance of the building is provided by elasto-plastic structural elements (Fig. 1) located along resisting planes in the *x*- and *y*-directions. The resisting elements in the *y*-direction may have different stiffness and strengths, and may be arbitrarily located about the y-axis, creating an eccentricity e_{rx} between the centre of mass and the centre of stiffness of the building plan. On the other hand, the system is symmetric about the x-axis.

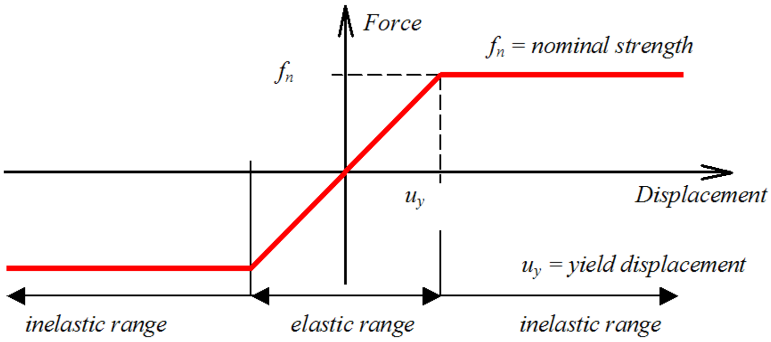

Figure 1. Elasto-plastic force displacement relationship of resisting elements

The BST ultimate surface is the locus of the storey shear and torque combinations that applied statically onto the story produce a plastic mechanism. Therefore, no combination of shear and torque can go beyond this surface. The inelastic behaviour of the system is represented in this force space as motions along the surface. Notice that plastic deformations can occur even when there is no motion along the BST surface. Thus, the inelastic deformation cannot be computed from the BST surface unless a step-by-step static or dynamic analysis is performed.

De la Llera and Chopra have been demonstrated that the BST surface is convex and it is point-symmetric with respect to the origin if the element yield displacements are the same under load reversals. In their paper (De la Llera and Chopra, 1994) the expressions for co-ordinates (V, M_t) of the BST surface vertices for a building with only three resisting planes along the y-axis (with the central plane passing through the centre of mass), and two resisting planes in the orthogonal direction are given. Here, the procedure to predict accurately the response of the system with an arbitrary number of resisting planes in y-direction (the direction of asymmetry and of ground motion) is proposed. The BST surface is composed of linear branches and can be constructed knowing a finite number of points (Fig. 2). Point A corresponds to a purely translational mechanism of the system and implies that all resisting planes in the y-direction must yield. Therefore, the equilibrium in the system gives:

$$V_{nA} = V_{ny} = \sum_{k=1}^{n} f_{nyk}, \qquad M_{tA} = V_{ny} \cdot e_{vx}, \qquad e_{vx} = \frac{1}{V_{ny}} \sum_{k=1}^{n} f_{nyk} \cdot x_k \tag{2.1}$$

where V_{ny} is the lateral capacity of the structure in the y-direction, f_{nyk} is the lateral capacity (nominal strength) of the k-th resisting element in the y-direction, n is the number of resisting planes in the y-direction, e_{vx} is the strength eccentricity, and x_k is the distance of k-th resisting plane from the centre of mass.

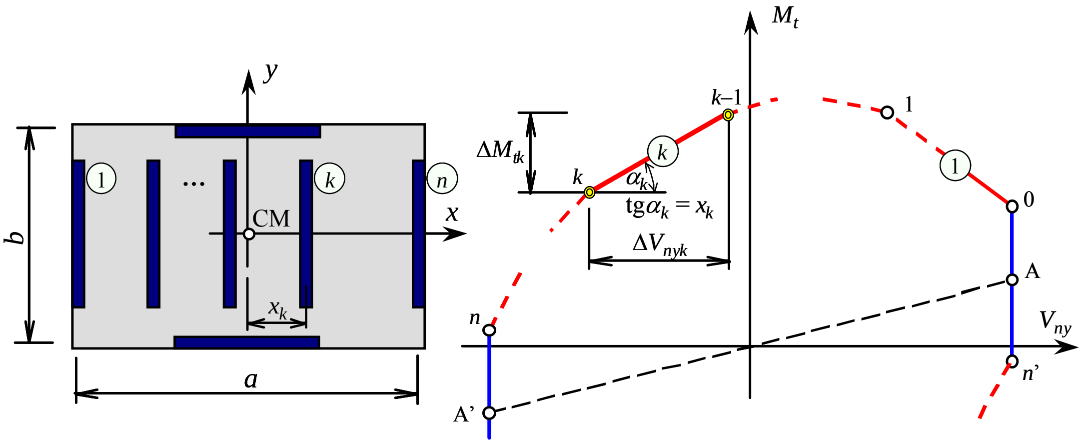

Figure 2. The BST surface of the system with an arbitrary number of resisting planes

The plastic mechanisms associated with branch A – 0 are generated by the rotation of the system about resisting plane 1, leaving the deformation of this element equal to the yield displacement u_{y1}. For this case, the equilibrium in the system gives:

$$V_n = V_{ny} = \sum_{k=1}^{n} f_{nyk}, \qquad M_t = V_{ny} \cdot e_{vx} + \lambda \cdot V_{nx} \cdot b, \qquad 0 \le \lambda \le 1 \tag{2.2}$$

where V_{nx} is the lateral capacity of the structure in the *x*-direction, *b* is the distance between resisting planes in the *x*-direction (Fig. 2), and λ is the parameter that includes the torque caused by the deformation of resisting planes in the *x*-direction ($\lambda = 0$ for equilibrium of the system at point A, and $\lambda = 1$ for equilibrium at point 0). For the *x*-direction, the equal nominal strength of the both orthogonal elements is assumed ($f_{nx1} = f_{nx2} = V_{nx} / 2$). All the plastic mechanisms associated with branch 0 – 1 have the same rotation, i.e. these collapse mechanisms are generated by the translation of the system, keeping the deformations of resisting plane 1 always in the elastic range (from $+u_{y1}$ to $-u_{y1}$). The equilibrium of the system at point 0 gives:

$$V_{n0} = V_{ny}, \qquad M_{t0} = V_{ny} \cdot e_{vx} + V_{nx} \cdot b \tag{2.3}$$

Because of the translation (from $+u_{y1}$ at point 0 to $-u_{y1}$ at point 1), the forces in resisting plane 1 are reduced, which in turn reduces the base shear V_{ny} and increase the base torque M_t resisted by the system:

$$V_{ny1} = V_{ny0} - 2 f_{ny1}, \qquad M_{t1} = M_{t0} + 2 f_{ny1} \cdot |x_1| \tag{2.4}$$

The plastic mechanisms associated with next branch are a sequel of the previous mechanisms, keeping the deformations of one resisting plane always in the elastic range. Each branch of the BST ultimate surface is defined with two points: "*k*–1" and "*k*" (Fig. 2). The co-ordinates (V_{nk}, M_{tk}) of the point "*k*" are given by the following expressions:

$$V_{nyk} = V_{nyk-1} - 2 f_{nyk}, \qquad M_{tk} = M_{tk-1} - 2\ f_{nyk}\ x_k \tag{2.5}$$

In this way, the co-ordinates of subsequent vertex (point "*k*") of the considered branch can be obtained if the co-ordinates of the preceding vertex (point "*k*–1") are known. Thus, the BST surface for the first and second quadrant can be computed using a simple plastic analysis concept. Because the BST surface is point-symmetric with respect to the origin, the co-ordinates of points for the other two quadrants can be easily obtained.

The slope of a tangent to the BST surface is equal to the location of the element in the building plan ($\text{tg}\alpha_k = \Delta M_{tk} / \Delta V_{nyk} = x_k$). This slope defines the centre of plastic rotation of the system. The BST surface has as many branches with finite slope as twice the number of resisting planes in the direction of ground motions. The first branch is associated with mechanisms that leave the leftmost resisting plane in the elastic range, the second branch the second farthest plane to the left, and so forth until we reach the rightmost resisting plane. Because the inelastic behaviour of a building is developed along the BST surface, its shape controls this behaviour. Therefore, even without static or dynamic non-linear analysis it is possible to compare the expected seismic performance of different structural configurations based on their BST surface. The factors that determine the shape of the BST surface and influence the inelastic behaviour are the strength eccentricity, lateral and torsional capacity of the system and planwise distribution of strength (Lađinović and Folić, 2006).

3. DEFORMATION DEMANDS

When structural performance relies on ductile response during a major seismic event, the relationship between inelastic deformations of the system affected by translational and torsional actions should be considered. But, as a general rule, in current design practise this is not done. The primary aim in the seismic design of buildings should be to address displacements corresponding with performance criteria. Considerations relevant to the ultimate limit state are governed by displacement ductility capacities that can be reliably provided for lateral force-resisting elements. Global displacement ductility factors, specified in codes for typical structural systems, are inappropriate when the geometry of the components of the system is different. Instead of estimating the ductility capacity of a system, the designer should identify the critical element and, to protect that element, reduce the design ductility demand on the system.

In a study Paulay (Paulay, 1997) deals with the seismic design for torsional response of ductile buildings, where buildings are classified into two categories: *torsionally restrained* and *torsionally unrestrained*. In a torsionally unrestrained building, all elements that resist torsion may be yielding during an earthquake, while in a torsionally restrained building the resisting planes orthogonal to the ground motion direction are still elastic. Fig. 4a illustrates an example of torsionally unrestrained structure. In the absence of eccentricities (in terms of both strength and stiffness), this structure is commonly referred to as "torsionally balanced". Paulay suggests that a small excess of strength will lead to inelastic twisting of the system. In this case, if the ductility capacity of element μ_{max} is not to be exceeded, than the displacement ductility demand of the system μ_Δ, measured at the centre of mass, is to be severally restricted:

$$\mu_\Delta = \frac{\mu_{\max} - 1}{1 + \omega_0} + 1, \qquad \omega_0 = \frac{k_2}{k_1}\left(\frac{0.5 + e_m / D}{0.5 - e_m / D}\right)^2 \tag{3.1}$$

where ω_0 is the structural parameter depending on the distribution of mass and stiffness in the building plan.

The assumptions (3.1) made by Paulay are highly conservative. Contrary to his assumption, the torsionally unbalanced building does not pivot about the stronger element during its dynamic response to the earthquake. To illustrate the foregoing points, the dynamic analysis of an unrestrained system with two resisting planes in *y*-direction, subjected to the El Centro earthquake is performed (Lađinović and Folić, 2005). To ensure the assumption made by Paulay, the strength of element 2 is increased by 20% in relation to the strength of element 1 (ω_0 = 1.20). The non-linear dynamic analysis is performed using Newmark's method. An event-to-event solution strategy is used, where the structure properties are re-formed each time there is a non-linear event (a change in stiffness).

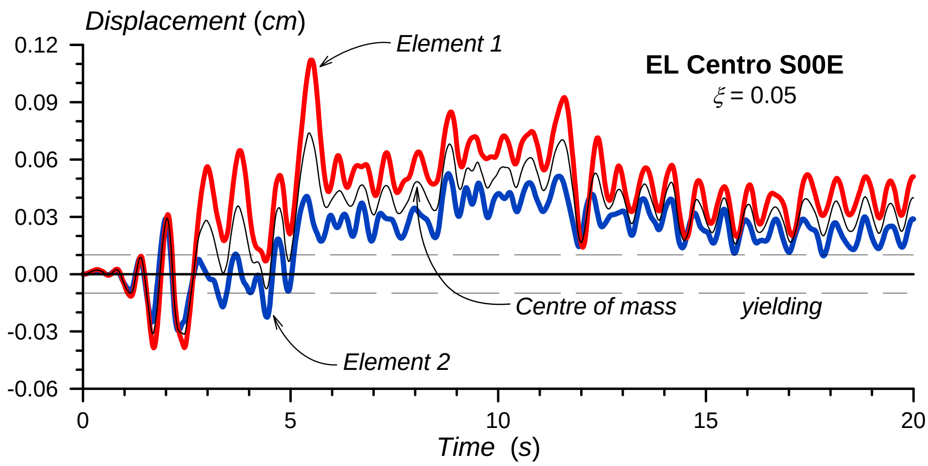

Figure 3. Displacement time history of the torsionally unrestrained system

Results of the analysis (Fig. 3) show that torsional motion produces a significant increase in the displacement of the flexible edge. The maximum displacement demands of resisting planes 1 and 2, and the centre of mass are: $\max u_{1y}$ = 11.26 cm, $\max u_{2y}$ = 5.26 cm, and $\max u_{cmy}$ = 7.40 cm, respectively. The maximum ductility demand of the weaker resisting plane (element 1) is μ_{max} = 11.26. According to Paulay's equation (3.1), the ductility demand of the system, measured at the centre of mass, should be μ_Δ = 5.66, but the actual value is just μ_Δ = 7.40. Obviously, Paulay's estimate is highly conservative. This is easily explained by observing displacement history, which shows that both element yield and pivoting do not take place about the yield position of the strong element (Fig. 3).
To study the response of torsionally restrained and unrestrained systems to real seismic action, the two single-storey buildings with the plan aspect ratio a / b of 2 and different structural configurations are considered

(Lađinović and Folić, 2006). The stiffness and lateral capacity of the both systems is the same (Fig. 4), and both systems have equal normalized stiffness and strength eccentricity ($e_{vx} = e_{rx} = 0.125\ a$). Both systems were subjected to the N-S component of the El Centro ground motion in the *y*-direction. Results of the analysis are shown in Fig. 5 for the torsionally unrestrained system, and in Fig. 6 for the restrained system.

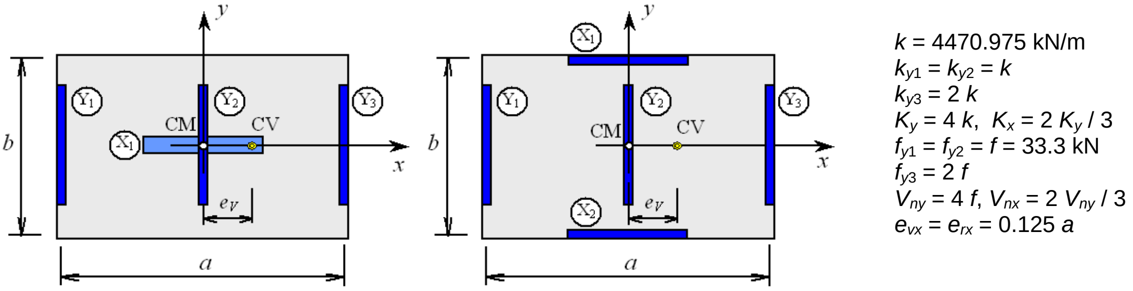

Figure 4. Characteristics of a) torsionally unrestrained, and b) restrained systems

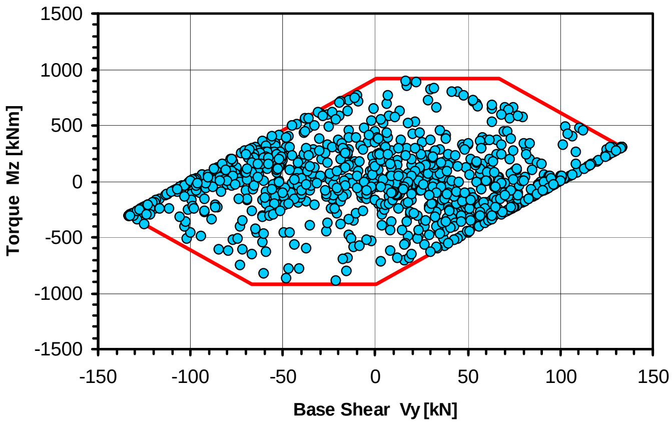

Figure 5a. Unrestrained system – base shear and torque response histories

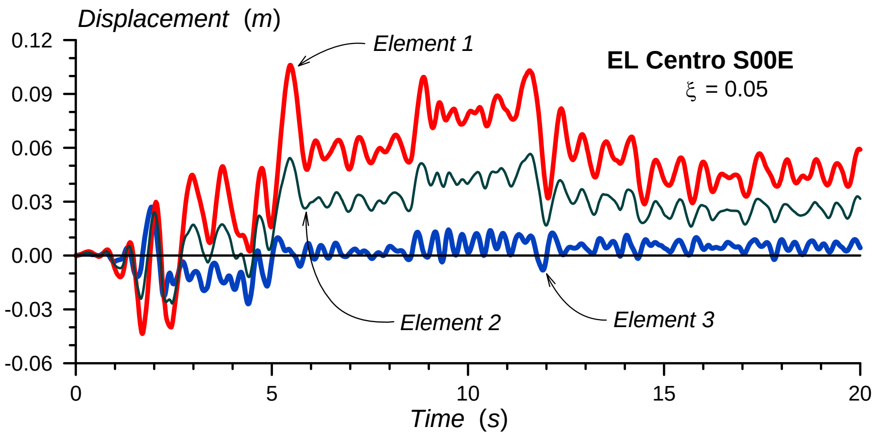

Figure 5b. Unrestrained system – displacement time history

The results referring to the torsionally unrestrained system show that most of the inelastic behaviour occurs along the two parallel branches with the positive slope (Fig. 5a). This implies that resisting plane 3, i.e. the strongest element in the *y*-direction, remains elastic in many instants during the response. As a consequence, the instantaneous centre of plastic rotation is located in resisting plane 3 (the stiff edge of the building) during most of the inelastic response. Because of that, resisting plane 1 (the farthest plane from element 3) will experience a significant increase in displacements relative to plane 3 due to the plan rotation (Fig. 5b). The seismic response

of the restrained system is substantially favourable, because a larger proportion of the inelastic behaviour migrates from the branches of the BST surface with the positive slope to the constant base shear branches (Fig. 6a). It implies that in this case (for the restrained system) a number of plastic mechanisms
that involve yielding of all *y*-direction planes was developed. Thus, more uniform displacement demands are expected for the resisting planes in such systems (Fig. 6b).

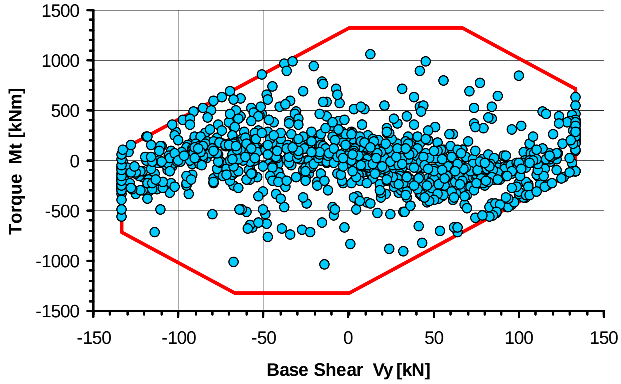

Figure 6a. Restrained system – base shear and torque response histories

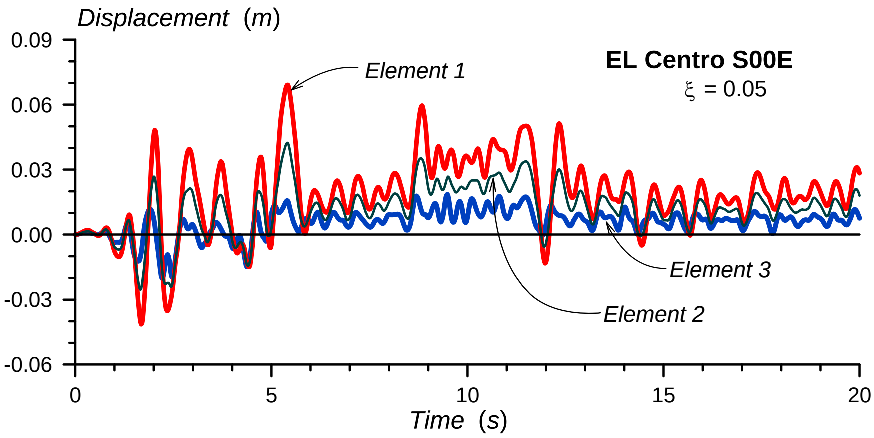

Figure 6b. Restrained system – displacement time history

The structural response naturally depends a lot on the applied excitation, which can be seen from the results of the analysis given in Fig. 7. In Fig. 8, a history of displacement is shown for the flexible side of the building for the restrained system with the eccentricity $e_v = 0.125a$, for different ground motions – in the analysis are applied records of El Centro S00E (ELC), San Fernando EQ (SFN), CastaicN69W, and Montenegro 1977 earthquake BarNS (BAR) and Petrovac EW (PET).

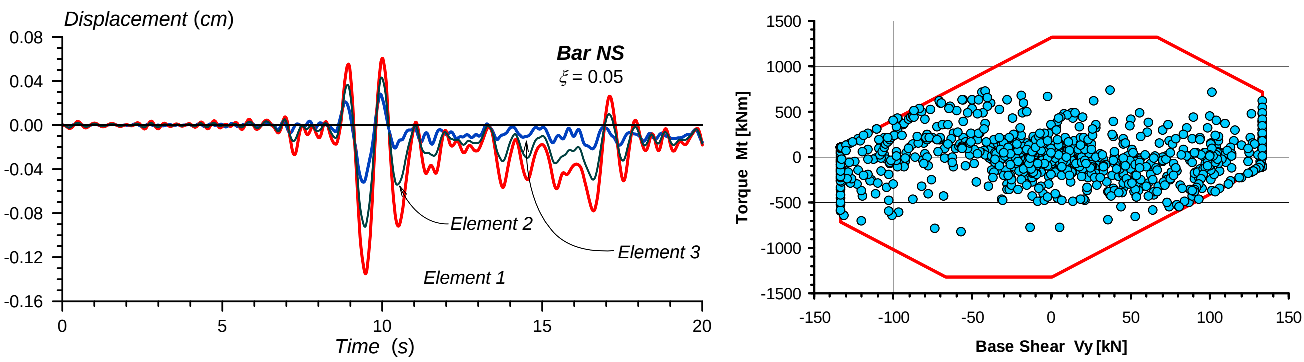

Figure 7. Structural response of torsionally restrained system for Bar NS record (Montegro 1977 EQ)

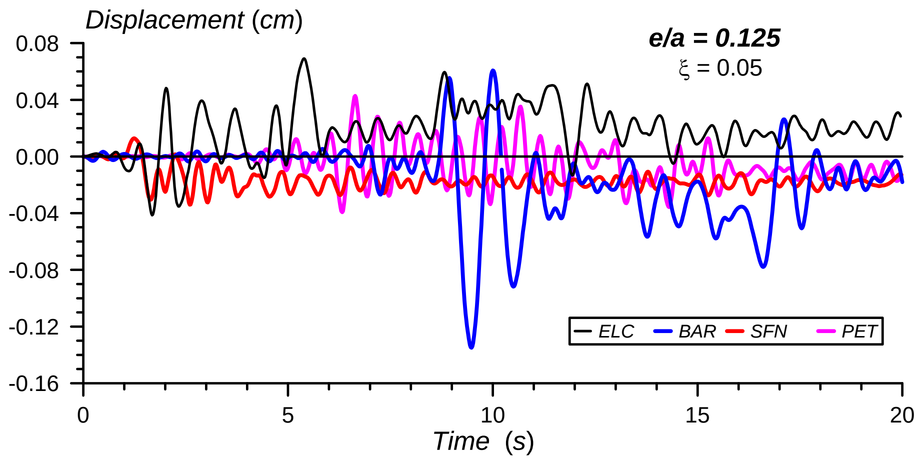

Figure 8. Displacement time history of torsionally restrained system for different ground motions

The seismic response of the asymmetric structure significantly depends on the size of eccentricity. The behaviour of the previously discussed torsionally restrained system (Fig 4b), but with a smaller eccentricity (e_r = e_v = 0.025a), is essentionally more favourable (Fig. 9). On the basis of the results of the analysis can be seen that all bearing elements translate at the same time into a postelastic area, hence, the torsion of this system is significantly less expressed. Fig.10 presents the history of displacement of the flexible side of the building (element 1) exposed to the action of El Centro for different eccentricities. With the increase of eccentricity, the increase of motion is noticeable.

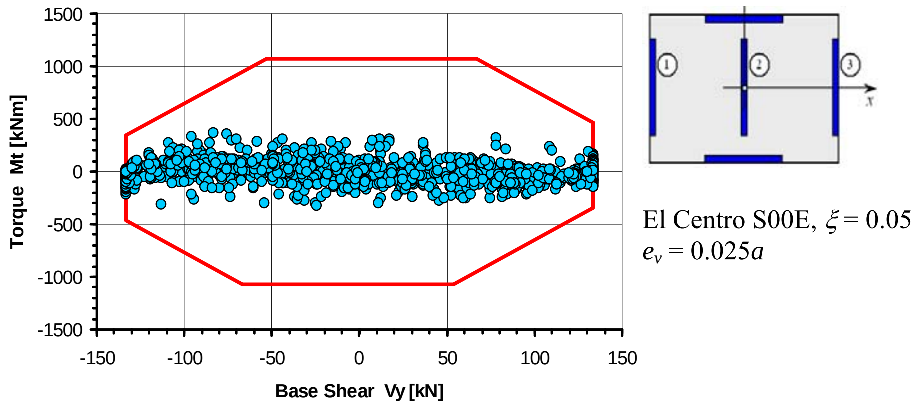

Figure 9. Base shear and torque response histories of torsionally restrained system with e_v = 0.025a

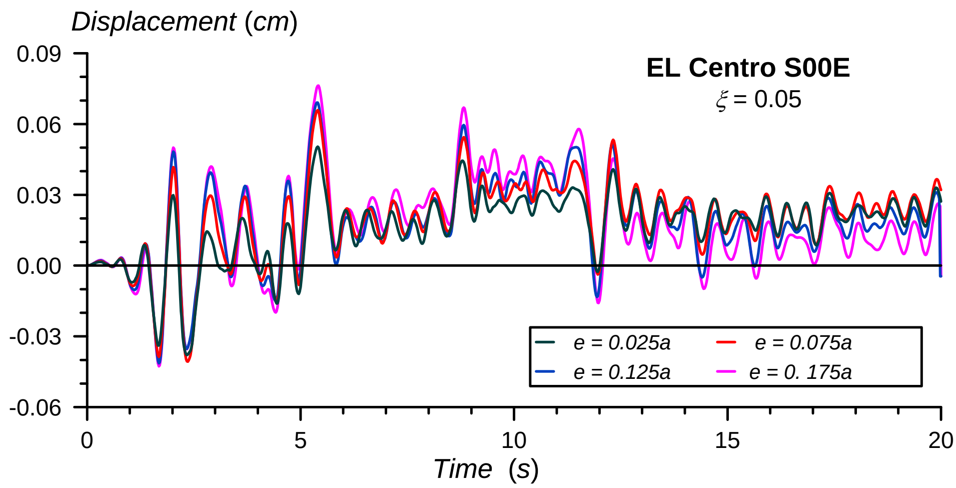

Figure10. Displacement time history of torsionally restrained system for different eccentricity

4. CONCLUSIONS

Torsional stiffness and resistance are characteristics of building structures that significantly influence their response to the seismic action. Responses in which the translational motion is dominant are preferable to those in which the torsional motion is significant because they tend to stress in different structural elements in a more uniform way (Fardis et al.). For the purpose of ensuring adequate torsional stiffness and resistance, the structural elements resisting the seismic action should be adequately distributed in plan. They should be close to the periphery of the building and oriented along two directions. The BST response histories, especially with the BST surface, may be a useful tool for a conceptual seismic design of asymmetric-plan buildings. The factors that determine the shape of the BST surface and influence the inelastic behaviour are the strength eccentricity, lateral and torsional capacity of the system and planwise distribution of strength. Stiffness eccentricity does not affect the shape of the BST surface, but it controls where on this surface the system develops its inelastic behaviour. The BST surface contains most of the information necessary to describe the inelastic properties of a system. Its shape is directly related to the yielding mechanisms of the structure and, thus, controls the relative displacement demand among resisting planes. The inelastic behaviour of the system is represented in this force space as motions along the surface. However, the inelastic deformation cannot be computed from the BST surface unless a non-linear static or dynamic analysis is performed. Results of the performed analysis show that the seismic response of the restrained system is substantially more favourable than in the unrestrained one. Also, in this case more uniform displacement demands are expected for the lateral load resisting planes.

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