DYNAMIC LATERAL RESPONSE OF SINGLE PILES CONSIDERING SOIL INERTIA CONTRIBUTIONS

Gustavo Pacheco[1], Luis E. Suárez[2], and Miguel Pando[3]

[1] *Professor, Dept. of Civil and Environmental Engineering, Polytechnic University of Puerto Rico, PR, USA*
Email: gpacheco@pupr.edu
[2] *Professor, Department of Civil Engineering, University of Puerto Rico at Mayagüez, PR, USA*
Email: lsuarez@uprm.edu
[3] *Associate Professor, Department of Civil Engineering, University of Puerto Rico at Mayagüez, PR, USA*
Email: mpando@uprm.edu

ABSTRACT :

In the most widely used model for analyses of piles under lateral loads, the discrete Winkler model, the pile is modeled as beam elements and the soil is represented by disconnected, concentrated springs normal to the pile axis. The literature shows that for dynamic analyses of piles, the soil stiffness and damping properties can be adequately included through lumped springs and dashpots, whereas no lumped masses are added to represent the soil inertia effects. The objective of this paper is to present a simple model to represent the soil that includes its inertial effects and to investigate their importance on the dynamic response of single piles.

The proposed methodology is based on a lumped model consistent with the Winkler proposition. The parameters of the discrete model were obtained by approximating the continuous (plane strain) model developed by Novak. In the proposed approach, the pile-soil interaction is accounted for by three frequency independent elements: a spring with stiffness k_a, a mass m_a, and a dashpot with coefficient c_a. The spring-mass-dashpot coefficients representing the soil are defined by simple equations. The proposed model was used to demonstrate that a lumped soil mass is not required for small/medium soil Poisson's ratios. However, consideration of the soil mass was found to be important for soil deposits with high Poisson's ratios (e.g., undrained loading of saturated soils where $\nu = 0.5$). For the case of saturated soils the inertia contribution due to the soil lumped masses is of the same order of the pile mass.

KEYWORDS: Pile dynamic stiffness, pile dynamic lateral response, pile horizontal vibration, Winkler models.

1. INTRODUCTION

Piles are an extensively used foundation system typically employed to support structures placed over soft soil layers or where shallow foundations are not appropriate because they do not provide the required capacity or may experience excessive settlements or deformations. Pile foundations have to be designed to support not only gravity loads, but also lateral loads due to earthquakes, wind, and vehicle impact loads, among others. This paper will focus specifically on the response of piles to dynamic lateral loads where it is often required to perform dynamic analysis of the pile for transverse (lateral) vibrations.

There are three major approaches to perform dynamic analysis of piles (Poulos and Davis 1980; Fleming et al., 1992):

1) The Elastic Continuum approach, in which the soil is represented as a homogeneous elastic semi-space. The advantage of this approach is that it automatically includes the radiation of energy to infinity, known as radiation damping, through the complex expression of the pile impedance function.

The drawbacks are that it is only applicable to viscoelastic materials, the nonlinear behavior can only be accounted for by changing the elastic modulus of the full space (it does not allow for localized plastification), and the boundary conditions are limited to soil deposits with homogeneous layers.

2) The Finite Element (FE) approach in which a finite soil domain is discretized with FE elements and approximate boundary conditions are imposed. The Boundary Element and Finite Difference Methods can be also considered in this category. Due to the cost of the specialized software, the time consuming model generation and solution, the FE approach is usually only used for large projects.

3) The Winkler model, also known as Beam on Elastic Foundation, Beam on Winkler Foundation, and subgrade-reaction approach. This model was originally proposed by Winkler in 1867, and it considers that the deflection (*y*) at any point of the soil in contact with the pile is linearly related to the contact pressure (*p*) at that point. Although it is criticized because of the lack of continuity of the soil model due to the fact that the displacements at a point of a soil are not influenced by stresses and forces at other points, this approach has been widely employed in foundation engineering practice (e.g., Poulos and Davis, 1980; El Naggar and Novak, 1996; Wang et al., 1998; Mostafa and El Naggar, 2002). Some of the advantages of this approach are: 1) it is simple to implement; 2) it permits fast problem modeling and solution computation; 3) it can take into account the variation of soil stiffness with depth and the layering of the soil profile; 4) it has the ability to simulate nonlinearity (through *p-y* curves), and hysteretic degradation of the soil by changing the modulus of subgrade-reaction, 5) There is a large body of knowledge associated to the method (e.g., there are many empirical correlations to determine the modulus of subgrade-reaction, etc).

To implement the beam on elastic foundation model for the case of dynamic loads, the soil stiffness and damping contributions are represented by a series of unconnected lumped springs and dashpots distributed along the pile length. Usually no lumped mass representing the soil inertia is considered. It was also proposed to use a dynamic stiffness (Novak, 1974), but its implementation is difficult because the expressions for the dynamic stiffness are complex and frequency dependent. This approach can only be implemented in the frequency domain analysis, and because it is based on the superposition principle, it cannot be applied to analyze nonlinear cases.

The objective of this work is to find simple, frequency independent expressions to represent the pile-soil interaction through a lumped model, consistent with the Winkler hypothesis, formed by a spring with stiffness *ka*, a dashpot with coefficient *ca* and a mass *ma*. The model parameters are obtained by approximating the continuous, plane strain model developed by Novak (1974). The simple, independent expressions derived in this paper allowed one to study the effect of soil properties and its inertial contributions to the pile-soil interaction problem. The model with lumped soil masses is used to parametrically study the importance of the soil inertial effects.

2. NOVAK'S DYNAMIC STIFFNESS OF SOILS

Novak (1974) presented an approximate analytical expression for the dynamic stiffness of piles based on linear elasticity. Novak (1974) assumed that the soil is composed of a set of independent, infinitesimally thin horizontal layers in plane strain state that extend laterally to infinity and experience small displacements. The soil layers are considered homogeneous, isotropic, and linear-elastic. The pile is assumed to be vertical, cylindrical and moving as a rigid body (a hypothesis that is consistent with the Navier-Bernoulli beam theory). The massless rigid circular disc that represents the pile cross section is considered to experience a harmonic vibration. No separation is allowed between the rigid cylinder and the soil medium.

This approach was later extended by Novak and his coworkers (Novak and Abloul-Ella, 1978; Novak et al. 1978) to viscoelastic materials with frequency independent material damping (also called hysteretic damping). The damping is considered by means of the *Complex Shear Modulus* $G^* = G + i\,G' = G\,(1 + i\,D)$, where the parameter D, known as the loss factor, is defined in terms of the loss angle δ:

$$D = \tan\delta = \frac{G'}{G} \tag{2.1}$$

where D is the loss factor; G and G' are, respectively, the real and imaginary part of the shear modulus.

The complex horizontal stiffness of the soil associated with a unit length of the cylinder, ku, (or the dynamic soil reaction per unit length of pile to a unit horizontal harmonic displacement of the rigid disc) is given by the following equation:

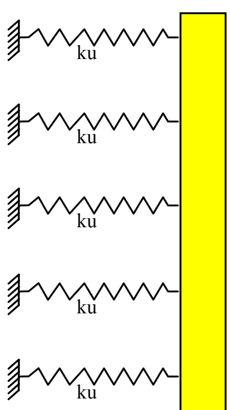
$$ku = G\,\pi\, f\left(a_o, \nu, D\right) \tag{2.2}$$

where:

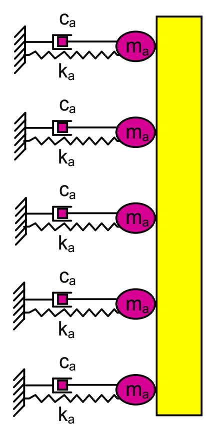
$$f\left(a_o,\nu,D\right) = -a_o^{\,2}\,\frac{4K_1\left(b_o^*\right)K_1\left(a_o^*\right) + a_o^*K_1\left(b_o^*\right)K_0\left(a_o^*\right) + b_o^*K_0\left(b_o^*\right)K_1\left(a_o^*\right)}{b_o^*K_0\left(b_o^*\right)K_1\left(a_o^*\right) + a_o^*K_1\left(b_o^*\right)K_0\left(a_o^*\right) + b_o^*a_o^*K_0\left(b_o^*\right)K_0\left(a_o^*\right)} \tag{2.3}$$

in which K_n is the modified Bessel function of the second kind of order n, a_o is a dimensionless frequency = $\omega r_o/V_S$, ω is the vibration frequency in rad/s, r_o is the pile radius, Vs is the shear wave velocity of the soil; $a_o^* = a_o\, i / \sqrt{(1+ i\, D)}$; $b_o^* = a_o^* / \eta$; $\eta = \sqrt{(2\,(1-\nu)/(1-2\nu))}$; and ν is the Poisson's ratio of the soil.

The three dimensional nature of the pile-soil system under horizontal vibrations can now be simplified to a planar model, using beam elements to represent the pile and linear springs with a dynamic stiffness ku to represent the soil, as shown schematically in Figure 1(a).



b) Novak's pile-soil model (after Novak, 1974)



a) Proposed lumped-mass pile-soil model (This paper)

Figure 1 Two dynamic pile-soil models

3. PROPOSED LUMPED MASS MODEL

Consider a single degree of freedom (SDOF) system consisting of a rigid body with mass M constrained to move along one axis, and attached to a fixed support by a spring of stiffness K and a dashpot of constant C. By definition, the *dynamic stiffness* or *impedance* $Kd(\omega)$ is the amplitude of the harmonic force of frequency ω, $F(t)= Kd(\omega)\, e^{i\omega t}$, that must be applied to the SDOF system to obtain a unit steady state harmonic displacement of frequency ω, $X(t) = 1\, e^{i\omega t}$. It is straightforward to show that the dynamic stiffness of the SDOF system is:

$$Kd(\omega) = K - \omega^2 M + i\,\omega\, C \tag{3.1}$$

By grouping the real part and the imaginary part, Novak's soil dynamic stiffness *ku* (Eqn. 2.2) can be rewritten as follows:

$$ku = G\,\pi\, f(a_o,\nu,D) = G\,\pi\left\{\text{Real}\left[f(a_o,\nu,D)\right] + i\ \text{Imag}\left[f(a_o,\nu,D)\right]\right\} \tag{3.2}$$

In the proposed model, the Novak complex stiffness, *ku,* is approximated as a quadratic polynomial in a_o. In this way, *ku* is equivalent to the dynamic stiffness of a SDOF system (Eqn. 3.1). This is done by introducing the following approximations in the real and imaginary parts of the complex function $f(a_o,\nu,D)$:

$$\begin{aligned} &\text{Real}\left[f(a_o,\nu,D)\right] \approx \alpha_k - \alpha_m\, a_o^{\,2} \\ &\text{Imag}\left[f(a_o,\nu,D)\right] \approx \alpha_c\, a_o \\ &ku \approx G\,\pi\left(\alpha_k - \alpha_m\, a_o^{\,2} + i\,\alpha_c\, a_o\right) \end{aligned} \tag{3.3}$$

Comparing *ku* in Eqn. 3.3 with the dynamic stiffness of a SDOF system given by Eqn. 3.1, one can obtain the stiffness coefficient k_a, the lumped mass m_a, and the lumped viscous damper coefficient c_a (representing the radiation damping):

$$\begin{aligned} &ku \approx k_a - m_a\,\omega^2 + i\,c_a\,\omega \\ &k_a = G\,\pi\,\alpha_k \\ &m_a = \pi\, r_o^2\,\rho\,\alpha_m \\ &c_a = \pi\, r_o\, V_s\,\rho\,\alpha_c \end{aligned} \tag{3.4}$$

The coefficients defined by the simple expressions in Eqn. 3.4 constitute an approximation to Novak's model. Note that the coefficients are defined per unit of pile length and are frequency independent. The proposed lumped mass model is shown schematically in Figure 1(b).

4. DETERMINATION OF COEFFICIENTS OF THE LUMPED MODEL

The coefficients α_k, α_m, and α_c in Eqn. 3.3 were determined by a least square approximation of the Novak's dynamic stiffness function $f(a_o, \nu, D)$ defined by Eqn. 2.3. The curve fitting was done for a loss factor $D = 0$ and for different soil Poisson's ratios ν, in order to obtain an adequate approximation of the function for a range of the dimensionless frequency a_o from 0 to 3. The coefficients α_k and α_m were obtained from the real part of $f(a_o, \nu, D)$, while the coefficient α_c is established from the imaginary part of $f(a_o, \nu, D)$. An example of such curve fitting for $\nu = 0.5$ is presented in Figure 2. Table 1 and Figure 3 present the variation of the regression coefficients for soil Poisson's ratios ν, ranging, in a descendent order, from 0.5 to 0. It is evident from this figure that the coefficient α_m, and hence the lumped mass m_a, can have significant values for high Poisson's ratios. For ν near 0.5 (i.e., saturated soils under undrained loading) α_m tends to 1, and the resulting m_a will have a value of the same order as the pile mass (if the soil density is considered to be of the same magnitude than the pile material). This situation may have a significant impact on the pile-soil system response, i.e., location of the peaks in a frequency response analysis. Therefore the effect of the soil mass should be accounted for in these cases.

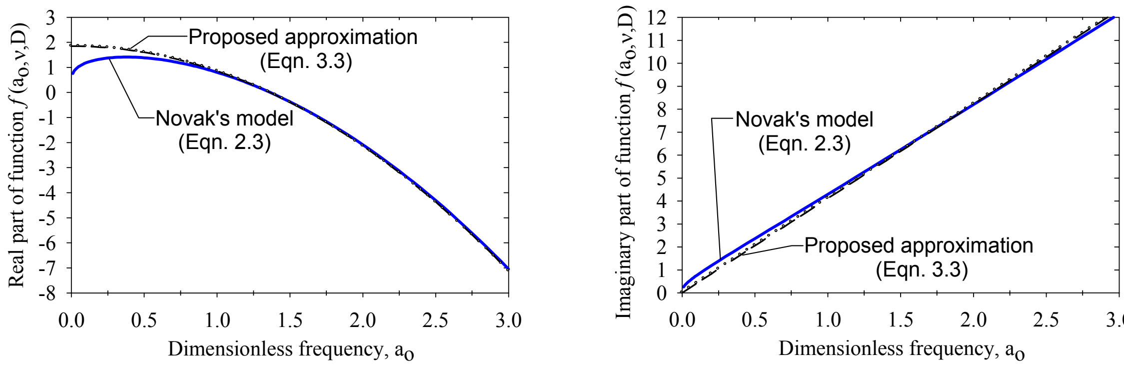


Figure 2 Example of the proposed curve fitting for the real and the imaginary parts of $f(a_o, \nu, D)$ for $\nu = 0.5$

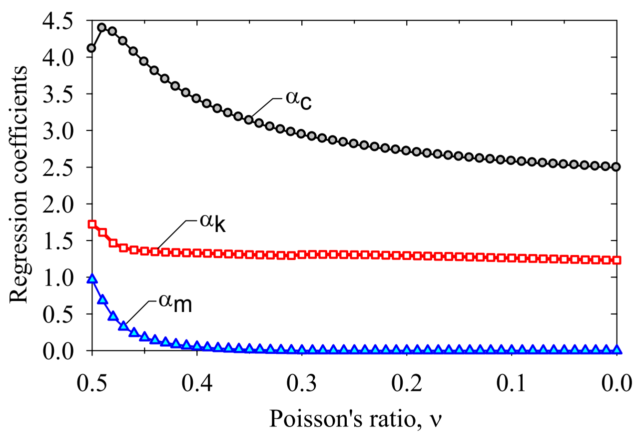


Figure 3 Variation of the regression coefficients with Poisson's ratio from 0.5 to 0.0.

Table 1 Values of the regression coefficients for Poisson's ratio from 0.5 to 0.0.

ν	α_k	α_m	α_c
0.50	1.721	0.965	4.107
0.49	1.611	0.682	4.389
0.48	1.463	0.457	4.338
0.47	1.399	0.322	4.209
0.46	1.369	0.235	4.066
0.45	1.354	0.177	3.929
0.40	1.327	0.051	3.425
0.30	1.307	-	2.941
0.20	1.294	-	2.712
0.10	1.261	-	2.579
0.00	1.230	-	2.491

5. VERIFICATION AND VALIDATION OF THE LUMPED MODEL

A brief description of the model verification and validation is presented in this section. For more details the reader may consult Pacheco-Crosetti (2007).

As described in Section 3, the proposed lumped-mass model is based on using approximate polynomial functions that are frequency independent. The first verification undertaken was to assess the accuracy of the approximate polynomial functions. This was done by comparing these polynomial functions with Novak's original formulation. Comparisons presented in Pacheco-Crossetti (2007) in terms of the coefficient of determination R^2 and the coefficient of variation showed that the proposed polynomial approximations yielded very accurate estimates. Another more detailed verification was performed, in which the dynamic response of a pile-soil system obtained using Novak's model and the proposed lumped model were compared. The particular response quantity compared was the dynamic flexibility (or frequency response function) of the pile head. This verification involved using a wide range of soil and pile properties. The results presented by Pacheco-Crossetti (2007) showed a very good agreement between the results of the two models.

These two tests lead to the conclusion that the proposed lumped mass model is a very good approximation of Novak's dynamic stiffness. The proposed simpler model can be reliably used to account for the effect of the stiffness and inertia of the soil surrounding the pile and the radiation of energy on the lateral vibrations of the structure.

In addition to the tests described before, three additional tasks were carried out to further validate the proposed lumped mass model:

First the damping and the stiffness coefficients of the approximate model were compared to values found in the literature and obtained with other methods. In particular, the damping coefficient c_a was compared to the one proposed by Berger et al. (1977), as reported by Wang et al. (1998), and by Gazetas and Dobry (1984). Figure 4 presents the ratio between the proposed coefficient c_a and the coefficient c_B from Berger et al. It is evident that both models predict similar radiation damping coefficients for all Poisson's ratio values.

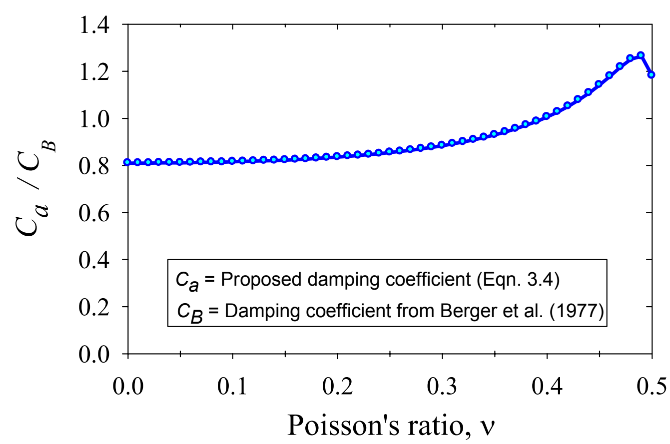


Figure 4 Ratio between damping coefficients c_a and c_B for different Poisson's ratios

The stiffness coefficient k_a was compared to the parameter k_h proposed by Klar et al. (2004) for buried pipes, consisting in two times the coefficient of subgrade reaction for beams resting on isotropic elastic medium presented by Vesić (1961). Figure 5 shows the ratio between the coefficients k_a and k_h, as a function of the soil modulus of elasticity for different pile modulus of elasticity and for a soil Poisson's ratio equal to 0.3. It can be appreciated from this figure that the proposed model predicts a higher stiffness than Vesić's model.

Next a comparison with the experimental results of pile head dynamic stiffness presented by De Napoli (2006) was carried out. Figure 6 shows the real component of the dynamic stiffness obtained with the proposed model and the experimental results by De Napoli (2006). The prediction shown was done using a simple model that did not attempt to capture layering and detailed stiffness variations with depth. As can be seen there is a reasonable agreement between the two sets of results, particularly given that the results by De Napoli (2006) involved impact load tests on full-scale large diameter drilled shafts installed in a natural loess deposit. More details of this study can be found in Pacheco-Crossetti (2007).

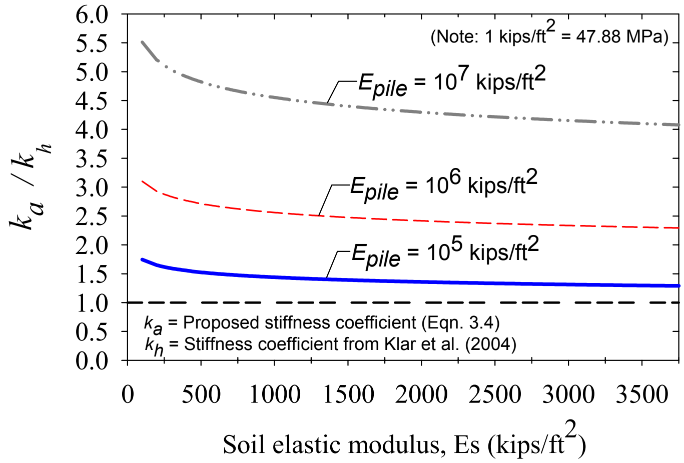


Figure 5 Ratio k_a/k_h for different moduli of elasticity of the soil and the pile, and Poisson's ratio $\nu = 0.3$

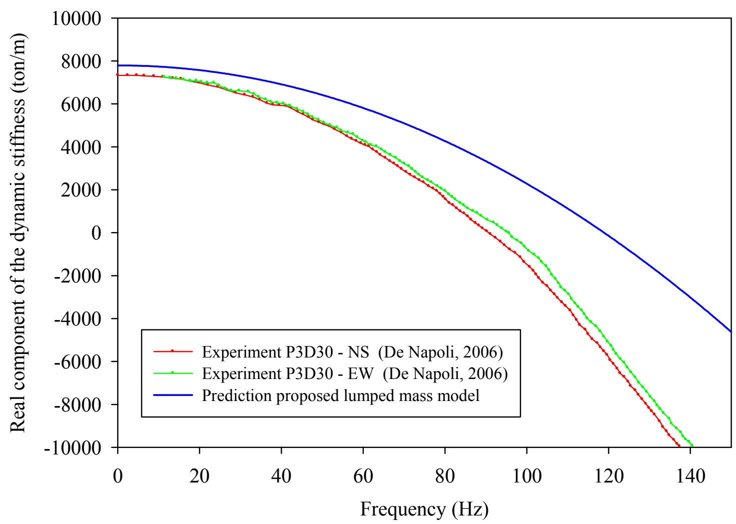


Figure 6 Experimental vs numerical pile dynamic stiffness, real component

Finally a comparison of the response obtained by 3D FE models and the proposed model was undertaken. Once again, the results presented in Pacheco-Crossetti (2007) showed reasonable agreement between the two approaches.

6. CONCLUSIONS

A simplified lumped model for the analysis of piles in lateral vibration, consistent with the Winkler hypothesis, was obtained by performing a regression analysis of the complex dynamic stiffness of the continuous (plane strain) model developed by Novak. The proposed lumped model has been proven to be a reliable approximation to the frequency dependent formulation of Novak.

In the proposed approach, the pile-soil interaction is taken into account through three frequency independent elements: a spring with stiffness k_a, a lumped mass with value m_a, and a dashpot with coefficient c_a. The spring-mass-dashpot coefficients k_a, m_a, and c_a that represent the soil can be obtained by means of simple equations. These equations include three parameters α_k, α_m, and α_c that depend on the soil Poisson's ratio.

The proposed lumped model was used to demonstrate that it is not necessary to include lumped soil masses for soils with low or medium values of the Poisson's ratio ($\nu \leq 0.3$). However, accounting for the soil mass is important for high Poisson's ratios, as it occurs in saturated soils under undrained loading. For $\nu = 0.5$, the soil mass contribution is of the same order than the pile mass.

The model presented has the following advantages: 1) The coefficients k_a, m_a, and c_a are obtained by the simple independent expressions; 2) The coefficients are frequency independent, allowing the user to perform analyses in the time domain; 3) The model is easy to program in the user's sofware, and it can also be used with general purpose FEM software, since the frequency independent concentrated spring-mass-dashpot elements are available in most commercial analysis packages; 4) The model could be extended to perform nonlinear dynamic analysis of pile-soil systems, by replacing the spring constant k_a by a nonlinear static soil response, such as the one provided by the *p-y* method; 5) Having independent expressions for the stiffness coefficient (k_a), the dashpot coefficient (c_a), and the mass (m_a), instead of a complex variable equation for the dynamic stiffness, permits to perform sensitivity analyses, i.e. to evaluate the impact of the variation of each coefficient in the system response; 6) The time required for pre-processing, running the analysis, and post-processing is orders of magnitude smaller than the one required by more refined methods (such as a 3D FEM); and 7) A simple model is more prone to be adopted by the professional community, and may even be used to introduce pile dynamics and soil-structure interaction problems at the undergraduate level.

REFERENCES

Berger, E., Mahin, S.A., and Pyke, R. (1977). Simplified method for evaluating soil-pile structure interaction effects, Proceedings of the 9th Offshore Technology Conference, OTC Paper 2954, Houston, Texas, 589-598.

De Napoli, V. E. (2006). Análisis numérico y experimental de pilotes excavados de hormigón armado cargados lateralmente, Ph.D. Thesis, Facultad de Ciencias Exactas, Físicas y Naturales, Universidad Nacional de Córdoba, Argentina.

El Naggar, M. H. and Novak, M. (1996). Nonlinear analysis for dynamic lateral pile response, *Soil Dynamics and Earthquake Engineering*, **15**, 233-244.

Fleming, W.G.K., Weltman, A.J., Randolph, M.F., and Elson, W.K. (1992). Piling Engineering, Blackie & Son, Glasgow, and Halsted Press (imprint of John Wiley & Sons), New York.

Gazetas G. and Dobry R. (1984). Horizontal response of piles in layered soils, *Journal of Geotechnical Engineering, ASCE*, **110:1**, pp. 20-40.

Klar, A., Vorster T. E. B., Soga, K., and Mair, R.J. (2004). Soil-pipe-tunnel interaction: comparison between winkler and elastic continuum solutions, Report CUED/D-SOILS/TR 332, University of Cambridge, UK.

Mostafa, Y. E. and El Naggar, M. H. (2002). Dynamic analysis of laterally loaded pile groups in sands and clay, *Canadian Geotechnical Journal*, **39**, 1358-1383.

Novak, M. (1974). Dynamic Stiffness and Damping of Piles, *Canadian Geotechnical Journal*, **11**, 574-598.

Novak, M., and Aboul-Ella F. (1978). Impedance functions of piles in layered media, *Journal of Engineering. Mechanics*, *ASCE*, **104:EM3**, 643-661.

Novak, M., Nogami T., and Aboul-Ella F. (1978). Dynamic soil reaction for plane strain case, *Journal of Engineering. Mechanics*, *ASCE*, **104:EM4**, 953-595.

Pacheco-Crosetti, G. E. (2007). Dynamic lateral response of single piles considering soil inertia contribution, Ph.D. Dissertation, Civil Engineering & Surveying Department, University of Puerto Rico, Mayagüez Campus.

Poulos, H. G., and Davis E. H. (1980). Pile Foundation Analysis and Design, John Wiley & Sons, New York.

Vesić, A. B. (1961). Bending of beams resting on isotropic elastic solid, *Journal of the Engineering Mechanics Division*, *ASCE*, **87:2**, 35-53.

Wang, S., Kutter B. L., Chacko M. J., Wilson D. W., Boulanger R. W., and Abghari A. (1998). Nonlinear soil-pile structure interaction. *Earthquake Spectra*, **14:2**, 377-396.