EVALUATION OF MOMENT REDISTRIBUTION DEMAND IN CONTINUOUS CONSTRUCTIONS UNDER GRAVITY AND SEISMIC LOADS

M.S. Marefat[1] and M.Sh. Farzanian[2]

[1] *Professor, Dept. of Civil Engineering, University of Tehran, Tehran, Iran*
[2] *M.Sc. Student of Earthquake Engineering, University of Tehran, Tehran, Iran*
Email: mmarefat@ut.ac.ir, shafee_f@yahoo.com

ABSTRACT:

In the current practices of structural design, the concept of moment redistribution in conjunction with linear analysis is well known and widely accepted. There are numerous studies and various references that deal with the issue and different codes propose various provisions for the amount of permissible redistribution. Despite numerous advantages, there are some limitations in the available work about this subject. Nearly all of these studies are restricted to symmetric multi span continuous beams subjected to static gravity load. In reality, continuous beams lie in the structural frames, symmetric configuration may not happen, and more importantly, lateral load also exist. Most of available works assume that a single plastic hinge to form at the center of joints while this study suggests that two distinct plastic hinges to take shape on both sides of an internal support of a frame. Consequently, relevant conclusions of available works suffer serious limitations and shortcomings. In this study, a continuous frame structure is considered, the effect of lateral load has been taken into account, and two distinct plastic hinges have been assumed. An attempt has been made to evaluate the redistribution demand of a continuous construction without the mentioned restrictions. For this aim, a simple solution based on virtual work theory has been used to determine the demand of plastic rotation. Problems associated with the current methods of determination of required moment redistribution are assessed and the necessary amendments are proposed.

KEYWORDS: Continuous reinforced concrete constructions, Moment redistribution demand, Seismic design, Plastic rotation

1. INTRODUCTION

Linear analysis with moment redistribution is one of the prominent and ordinary methods used in continuous structures design. This approach is to use a linear elastic analysis for calculating the bending moment and shear force distributions in a reinforced concrete structure, and then moment redistribution is performed under code provisions. This has the virtue of simplicity as well as permitting results from a series of analysis to be combined using the principle of superposition and the easy elastic analysis at the ultimate limit state (ULS). Because of non-linear behavior in continuous structures, redistribution is practical approach. RC beams subjected to intensive loading (such as earthquake), are affected by the non-linearity of the materials results from the concrete cracking and the steel yielding. This procedure causes the structure to have distance from elastic and modifies the distribution of moments, so moment redistribution occurs. The code provisions for redistribution permit linear analysis, then decrease or increase the different sections' stress -within certain limits- and consequently consider this effect in other sections. Based on this process, the amount of design moment after redistribution is determined.

2. MOMENT REDISTRIBUTION

In a structure, the moment diagram resulting from a linear elastic analysis generally exhibits some peaks representing the sections with maximum negative or positive moments. The so-called plastic hinges (due to concrete cracking and steel yielding) can appear in that region of the structures. In fact, the mentioned section is yielding and plastic rotation occurring when the moment resistance of the section is lower than the amount of the moment obtained from the linear analysis, so it makes it possible to direct the moment of a given section to other section, which behavior is called redistribution of moment. Different codes provide the designers with the ability of making use of the ductility of the reinforced concrete beams in changing the curve of the moment resulted from the analysis of the linear elasticity (with the application of special limits). Usually, such change and correction of moments result in the reduced maximum of the negative moment and the consequent increase of the positive moment (if the static equilibrium is established along with the imposed loads). According to the definitions, the percentage of the moment redistribution is stated as follows:

$$\beta = \frac{M_{el} - M_{red}}{M_{el}} \times 100 \qquad (2.1)$$

In which relation, M_{el} is the moment resulted from the elastic analysis (before redistribution) and M_{red} is the moment after redistribution. Based on definition, β is the Percentage of moment redistribution.

3. CAPACITY OF MOMENT REDISTRIBUTION

In the moment redistribution at the considered section, the member with a plastic rotation provides the possibility of transferring moment to the other sections. As a result, if the amount of the plastic rotation capacity of the considered section is sufficient for the considered section, redistribution is allowed. Therefore, the plastic rotation capacity is one of the fundamental parameters in the execution of the moment redistribution depending on different factors [1]. For determining the plastic rotation capacity, we may make use of theoretical relations, empirical relations, and results of experiments.

3.1. Theoretical Relations for the Determination of the Plastic Rotation Capacity

Based on the mechanical properties of materials, plastic rotation capacity may be calculated as the integral of curvatures after reinforcement yielding along the hinge as follows:

$$\theta_{pl} = \int_{L_{pl}} \frac{1}{r}.dx = \int_{L_{pl}} \frac{\varepsilon_s - \varepsilon_{sy}}{d - x}.dx \qquad (3.1)$$

where θ_{pl} is the plastic rotation capacity, $\frac{1}{r}$ is the curvature at critical section, L_{pl} is the length of the plastic hinge, x is the neutral axis depth, d is the effective depth of a cross section, ε_s is the steel strain, and ε_{sy} is the steel strain at the instant of the steel yielding. This equation is not, however, easy to apply because the curvature has a strongly non-linear development along the length of the beam owing to the variation of the bending stiffness between the cracked and non-cracked sections. Therefore, the approximate theoretical relations are used as follows [3]:

$$\begin{cases} \theta_{pl} = (\phi_u - \phi_y)L_{pl} \quad ,\phi_y = \dfrac{f_y}{E_s(1-k_y)d} \quad ,\phi_u = \dfrac{\varepsilon_{cu}}{C} \; ,L_{pl} = 0.08L + 0.022d_b.f_y\ (\text{MPa}) \\ k_y = \sqrt{(n^2A^2 + 2nB)} - nA \quad , \quad A = \rho + \rho' \quad , \quad B = \rho + \rho'\delta' \quad , \quad n = \dfrac{E_s}{E_c} \quad , \quad \delta' = \dfrac{d'}{d} \end{cases} \qquad (3.2)$$

Where ϕ_u and ϕ_y are the curvatures at the critical section at yield and ultimate respectively, L_{pl} is the effective hinge length suggested by Paulay [4] (in which L is the distance of the maximum moment from the Inflection Point, d_b is the diameter of steel, and f_y is the yield strength of steel), k_y is the compression zone depth at yield normalized to d, ρ and ρ′ are the reinforcement ratios of the tension and compression (normalized to bd) respectively, ε_{cu} is the ultimate concrete strain at extreme compression fiber, and C is the depth of the neutral axis.

3.2. Empirical Relations for the Determination of the Plastic Rotation Capacity

There are various empirical expressions for the determination of the plastic rotation capacity presented through different codes and studies carried out by researches. Following on from semi-empirical expressions proposed by Panagiotakos and Fardis [5], EC 8 and GRECO provide the following empirical relationships for the ultimate plastic rotation capacity in beams and columns with a rectangular cross-section [6]:

$$\begin{cases} \theta_{pl}^{u} = 0.0145(0.25)^{\nu}\left(\dfrac{\max(0.01,\omega')}{\max(0.01,\omega)}\right)^{0.3}(f_c)^{0.2}\left(\dfrac{L_s}{h}\right)^{0.35}\times 25^{\left(\alpha\rho_s\frac{f_{yw}}{f_c}\right)}(1.275^{100\rho_d}) \\ \nu=\dfrac{N}{bhf_c},\omega=\rho\dfrac{f_y}{f_c},\omega'=\rho'\dfrac{f_y}{f_c},\rho_s=\dfrac{A_{sx}}{b_wS_h},\alpha=\left(1-\dfrac{S_h}{2b_c}\right)\left(1-\dfrac{S_h}{2h_c}\right)\left(1-\dfrac{\sum b_i^2}{6b_ch_c}\right) \end{cases} \quad (3.3)$$

Where b_c and h_c denoting the width and depth of the core respectively, b_i is the distances of successive longitudinal bars laterally restrained at stirrup corners or by 135 degree hooks, ρ_s is the ratio of transverse steel parallel to the direction x of loading, f_{yw} is the yield stress of transverse steel, ρ_d is the steel ratio of diagonal reinforcement in each diagonal directions, L_s is the shear span (distance from the inflection point up to the maximum moment), and b_w is the width of the cross-section. It must be noted that making use of Eqn. 3.3., for the determination of permissible amount of the redistribution, the safety factor of 1.8 is practiced for the consideration of the uncertainties existing in samples. Besides the above-mentioned relationship, various relations have been presented for the determination of the plastic rotation capacity, the most famous relations among which are presented in Reference [7].

3.3. Results of experiments for the determination of the Plastic Rotation Capacity

One of the methods for the determination of the rotation capacity is the execution of loading on the reinforced concrete elements. Based on the results achieved through the laboratory studies carried out by KhanMohammadi [8], the rotation capacity of five beam specimens is presented in Table 3.1. It also must be stated that considering the fact that the loading practiced on samples is cyclic and for the difference in the amount of tension and compression reinforcement ratios of every sample, two plastic rotation capacity quantities is achieved for the sake of every section. In addition, the plastic rotation capacity of five beam specimens is determined based on Eqn. 3.3.

Table 3.1 Comparison of plastic rotation capacity of Eqn. 3.3. and experimental results

specimen	Deflection at yield		Deflection at C.P.		Plastic Rotation Capacity of Eqn. 3.3.		Plastic Rotation Capacity of experiment		Ratio of experiment to Eqn. 3.3.	
	U_y^+	U_y^-	U_{cp}^+	U_{cp}^-	θ_{pl}^+	θ_{pl}^-	θ_{pl}^+	θ_{pl}^-	Direction +	Direction -
SBC – 1	16	24	105	96	0.062	0.049	0.0712	0.0576	1.149	1.176
STC – 2	28	28	95	92	0.0818	0.051	0.0536	0.0512	0.655	1.004
SBC – 3	16	23.3	112	96	0.0671	0.0528	0.0768	0.0582	1.145	1.102
NBC – 4	17.5	24	94	85	0.0684	0.043	0.0612	0.0488	0.894	1.135
NTC – 5	23.5	21.5	94	89	0.0555	0.055	0.056	0.054	1.009	0.982

Amounts of plastic rotation capacity resulted from the experimental results and empirical Eqn. 3.3 (without the practice of 1.8 as safety factor) are presented in Table 3.1. As it is clear from the results of such test, the achieved amount of plastic rotation capacity have remarkable correspondence with Eqn. 3.3 without the practice of safety factor. Since the cyclic loading is practiced in the carried out tests and since the empirical Eqn. 3.3 is valid for the parts subject to the cyclic loads, this relation can be used for seismic concerns and we use this relation for the determination of the rotation capacity in this study.

4. REQUIRED MOMENT REDISTRIBUTION

One of the basic troubles in the determination of the permissible moment redistribution is the plastic rotation demand. The past studies have been limited to gravitational loading and focused on symmetric multi span continuous beams. Effect of the seismic load, frame behavior, and effect of column, existence of dissymmetrical, and formation of two distinct plastic hinges on both sides of an internal support of a frame on redistribution has been ignored. In this section, a method will be presented for the evaluation of the plastic rotation demand of the symmetrical and dissymmetrical continuous beams under the effect of the seismic and gravitational load and with the consideration of the frame behavior.

4.1. Required Plastic Rotation under Gravitational Loading

For the determination of the plastic rotation demand under non-symmetrical loading in general state, we analyze figure 2 without the consideration of the side force resulted from earthquake. Based on the elastic analysis, the moment distribution curve (M_i) before redistribution is represented in figure 3.a. considering the equilibrium of forces and compatibility conditions, the formation place of hinge may be realized in the indicated locations. As a result, the moment distribution curve (M_i^R) after redistribution becomes in the form of figure 3.b. Each one of the (M_i^R) moments compared to (M_i) may be reduced or increased. If in the first state it is assumed that in the support of B capacity of the section in both sides of the column is decreased, two indicated hinges in both sides of the median columns are formed in figure 2. Therefore, we will have two distinct hinges. In this situation, making use of the virtual work theory, we can reach the rotation demand of θ_{rBC} and θ_{rBA} joints based on the change in the rotation of each one of the said sections through the following relation:

$$P^{*}.\,\theta_r = \theta_r = \sum \int M^{*} \cdot \delta \, dx \tag{4.1}$$

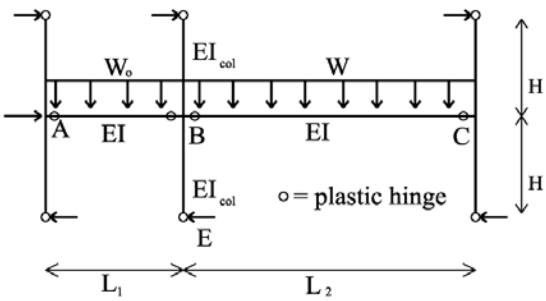

Figure 2 Arrangement of the non-symmetrical frame under the gravity and seismic loads

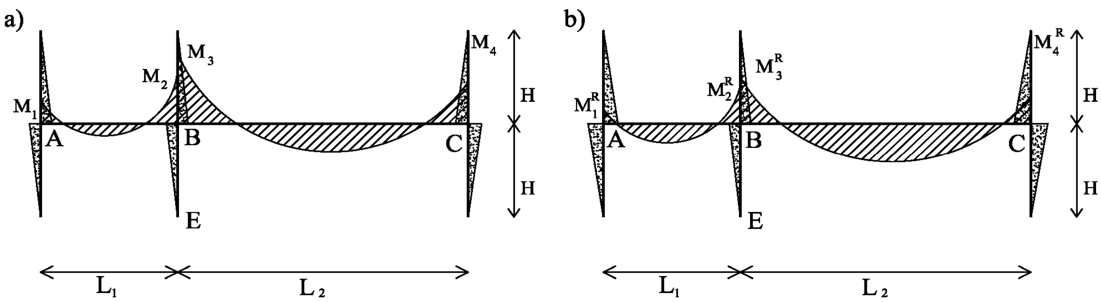

Figure 3 Moment distribution curve of the non-symmetrical frame under the gravity load
a) before redistribution b) after redistribution

As it is seen figure 4, P^* is the unit virtual moment in the direction of the element's slope at the place of the plastic hinge, M^* is the resulting distribution of virtual moment, and δ is the elastic distribution of curvature due to the applied loads on the structure. In this expression the * reflects the virtual force system and the integration of $M^* \cdot \delta$ applies over the length of each member.

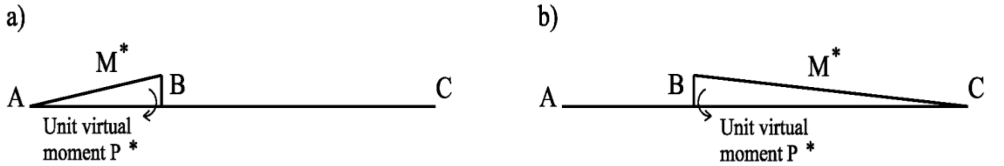

Figure 4 Virtual moments distribution curve a) for rotation of θ_{BA} b) for rotation of θ_{BC}

Now, rotation of each one of the sections before redistribution through integrating the product of multiplying the virtual moments of figure 4 and the elastic distribution of curvature related to loading and moment curve of figure 3.a in the form of Eqn. 4.2. and rotation of each one of the sections after redistribution through integrating the product of virtual moments of figure 4 and the elastic distribution of curvature related to loading and the moment curve of figure 3.b is determined according to Eqn. 4.3., and finally with the calculation of the difference of these two rotations in every section, the plastic rotation demand of that section is achieved in the form of Eqn. 4.4. as follows:

$$\theta_{BA}^{E} = \left(\frac{2M_2 + M_1}{6EI_{eff}}\right) L_1 \tag{4.2}$$

$$\theta_{BA}^{R} = \left(\frac{2M_2^R + M_1^R}{6EI_{eff}}\right) L_1 \tag{4.3}$$

$$\theta_{rBA} = \theta_{BA}^{E} - \theta_{BA}^{R} \tag{4.4}$$

where, θ_{BA}^{E} is the rotation of the section prior to redistribution, θ_{BA}^{R} is the rotation of the section after redistribution, EI_{eff} is the effective stiffness of beam, θ_{rBA} is the rotation required by beam under the effect of the considered redistribution in the place of the hinge. Doing so, we may calculate the rotation demand of the right hinge of the support of B. In the second situation, only the capacity of the section in one side of support B is reduced. So, we will have just one hinge in the joint. The demand for rotation is achieved from the combination of the changes of the two segments, i.e. change in the rotation of the section with the record of capacity reduction and no reduction. Values of θ_{rBA} and θ_{rBC} of these two sections can be calculated from Eqn. 4.2., 4.3. and 4.4. Finally, we may reach the required plastic rotation of the concentrated hinge as follows:

$$\theta_{rB} = \theta_{rBA} + \theta_{rBC} \tag{4.5}$$

4.2. Required Plastic Rotation under gravitational and seismic loading

A dissymmetrical frame is being considered under the gravitational and seismic loading according to figure 2. Considering the elastic analysis and distribution of the shear force resulted from earthquake in the inflectional point of the column, the moment distribution curve along the continuous elements become in the form of figure 5.a (M_i). According to the redistribution principles over storey, the moment distribution curve will be achieved in the form of figure 5.b after redistribution.

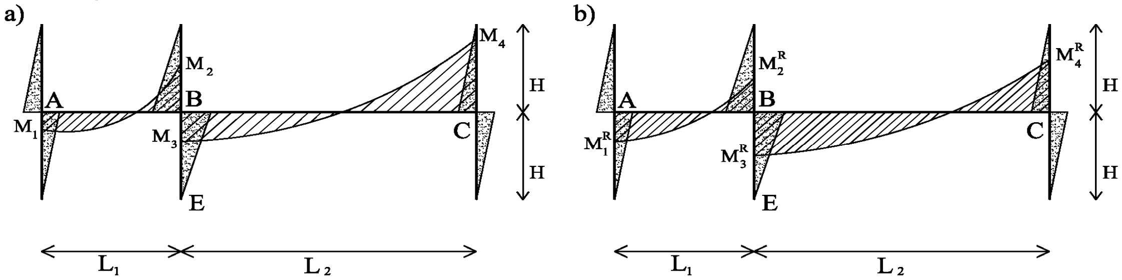

Figure 5 Moment distribution Curve of the Dissymmetrical Frame under gravitational and seismic loads
a) prior to redistribution b) after redistribution

In this situation, similar to the gravitational loading, the formation of the hinge in the indicated locations of figure 2 is possible but since one of the redistribution conditions is that the magnitude of any or all beam end moments may be changed as long as the sum of beam end moments remains unchanged and the use of the maximum capacity dictated by the code provisions, usually, the reduction of the maximum negative moment is accompanied by the positive moment increase. As a result, section of one side of the support, depending on the seismic direction, becomes plasticized and the other side remains elastic. In this situation, one concentrated hinge is formed at the side of the joint. Similar to the second mood in gravitational loading, we can calculate the required plastic rotation in this situation making use of Eqn. 4.2. up to 4.5. In the meantime, if the designing element is such that it results in the reduction of the capacity of the moment of the section in both sides of the B joint compared to the elastic analyze, we will have two separate hinges in both sides of the joint and it can be calculated similar to the first situation in the gravitational loading section.

4.3. Effect of Column on the Required Plastic Rotation in beam

If two separate hinges are formed at both sides of the column in the continuous beam under the effect of the gravitational or seismic loading, change in the rotation of the column before and after redistribution will result in the increase or decrease of the plastic rotation demand at the place of the hinge of the continuous beam. For the change in the distribution of the moment of the column in both pre-redistribution and post-redistribution states, the amount of joint rotation will be changed at the site of the hinge. Making use of Eqn. 4.1. and based on figure 6, we can reach the quantity of rotation of column(s) (existence of column under and/or over the beam level at B joint), the θ_{BE} related to the imbalanced moment imposed on them (difference of the beam moments at the both sides of the column in the gravitational state or the sum of the beam moments at both sides of the column in the seismic state) before redistribution of $(M_B)_{col}$ and after redistribution of $(M_B^R)_{col}$, and then through reaching the difference between these two quantities, θ_{rBE} causes increase of the demand for the plastic rotation of the beam at one side of the column and decrease at the other side of the hinge (negative (-) or positive (+) mark):

$$(M_B)_{col} = M_3 \pm M_2 \tag{4.6}$$

$$(M_B^R)_{col} = M_3^R \pm M_2^R \tag{4.7}$$

$$\pm\theta_{rBE} = \left[\frac{(M_B)_{col} - (M_B^R)_{col}}{6(EI_{eff})_{col}}\right] H \tag{4.8}$$

If one concentrated hinge is formed instead of the formation of two separate hinges, the rotation of the column will have no effect on the rotation demand.

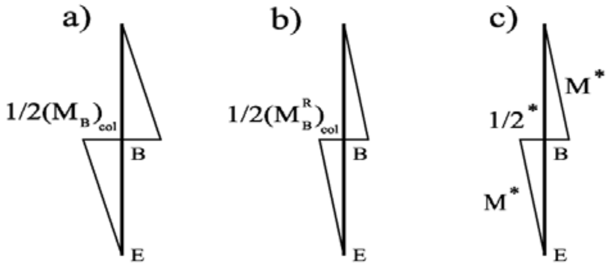

Figure 6 Moment distribution Curve of the column at B joint a) before redistribution
b) after redistribution c) unit virtual moment

4.4. Effect of Dissymmetry and Length of Span on the Required Plastic Rotation

We assume the form of figure 2 under the gravitational and seismic loading with two kinds of dissymmetric arrangement. In the first arrangement, we assume the length of the L_1 span at 5 m and length of the adjacent span at 3, 4, 8, 10 and 12m, and in the second arrangement, we consider the length of the adjacent span at 3, 5, 8, 10 and 12m. Then we consider three values of 10, 20, and 30 percent for the moment redistribution. Considering the range of the changes of the elastic moment for the change of the length of the L_1 span, i.e. 5 and 12m, and change of the length of the L_2 span from 3 up to 12m, the values of the designed moment are different. Therefore, for the precise study of the demand for the plastic rotation, we design the concrete sections such that the value of (ρ) will remain constant approximately in the concrete sections. In these two arrangements, we reduce the capacity of the section at the left hand of B joint and in the other side of the B joint for the consideration of the redistribution conditions we increase the capacity of the section with the same amount. As a result, we will have a concentrated hinge in B joint. Figures 7 and 8 show the demand for the plastic rotation in the hinge of the B joint in terms of different values of the length of the L_2 span for the first and second arrangement.

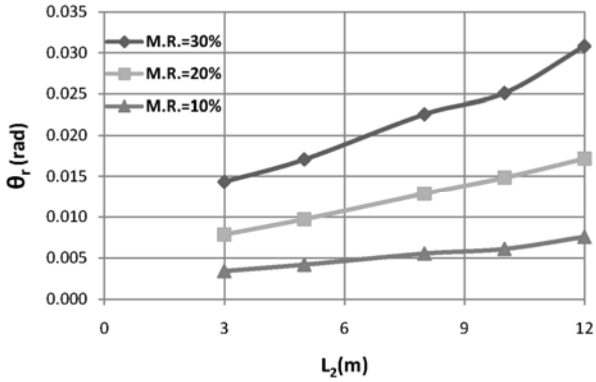

Figure 7 Effect of the span length on the required plastic rotation in the unequal spans under the effect of the gravitational and seismic loading, L_1= 5m

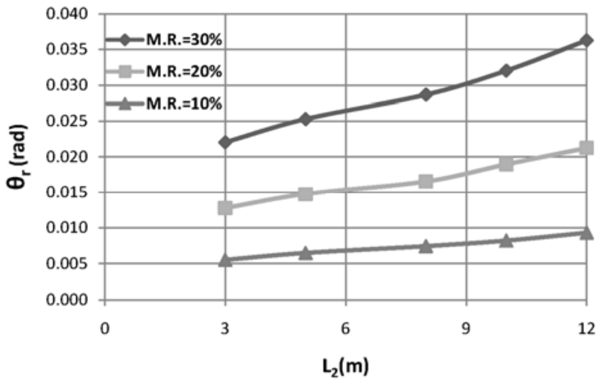

Figure 8 Effect of the span length on the required plastic rotation in the unequal spans under the effect of the gravitational and seismic loading, L_1= 12m

The above figures (7 & 8) indicate that the plastic rotation demand is increased with the increase of the length of the L_2 span. In addition, the redistribution value for the arrangement of the said frame at times that L_2 is fixed will result in the increase of the required rotation. In addition, curves of figures 8 and 9 indicate that effect of the length of the span in the arrangement related to the bigger redistribution is higher. In addition, the rate of the rotation required in the second arrangement is bigger in the equal redistribution and equal L_2. As a result, increased length of both L_1 and L2 spans will result in the increased amount of the demand.

5. PERMISSIBLE MOMENT REDISTRIBUTION

For the determination of the permissible moment redistribution in figure 5, it is assumed that the external moments of the continuous beam (M_1 and M_4) will be constant after the redistribution. If the left moment of the B support (M_2) is reduced with the rate of β percent, for the establishment of the redistribution conditions in the level of the storey, the right hand beam of the B joint (M_3) will be increased with the same rate. As a result, the demand for rotation in B joint (formation state of a concentrated hinge in joint) can be calculated as follows through the Eqn. 4.2. and 4.5.:

$$\theta_{r,B} = \frac{\beta M_2}{3EI_{eff}}(L_1 + L_2) \tag{5.1}$$

In cases that the rotation capacity existing in the formation section of hinge meets the required rotation of that section, the practiced redistribution will be allowable. Therefore, the allowable rate of moment redistribution (β) is achieved through the following relation through making the demand and plastic rotation capacity equal to each other as follows:

$$\beta = \frac{\theta_{c,B}}{\theta_{c,B} + \left[\frac{\phi M_n(L_1 + L_2)}{3EI_{eff}}\right]} \tag{5.2}$$

where, M_n is the nominal capacity of the section, ϕ is the reduction coefficient of the bending capacity and $\theta_{c,B}$ is the bending capacity of the section based on Eqn. 3.3.

6. COMPARISON AND CONCLUSION

Since the plastic hinge and the nonlinear behavior are created under the strong seismic effects, there is no reason for limiting the redistribution norm to the sole gravitational loading and the determination of the rotation quantity under the seismic load is necessary. There are some problems and limitations in case of the existing different studies. Analysis of the continuous beams is done mostly under the effect of the gravitational loading and in the geometrical symmetric conditions of the structure and load. In addition, in these studies, the plastic hinge is considered in the concentrated form and at the central place of the joint, while it is represented in this study that the formation of two separate hinges in both sides of the joint is possible and the symmetry condition may not be met. In this study, besides the gravitational loading, the seismic loading is also analyzed under the dissymmetric geometrical conditions and simple relations have been presented for the calculation of the required plastic rotation. In addition, it shows that this impact is higher in case of the bigger redistribution values.

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