# Displacement based seismic design of RC bridge piers: Method and experimental evaluation

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**ABSTRACT :**

A direct displacement based seismic design procedure of RC bridge piers fulfilled multiple performance objectives, which usually expressed as that designed structures can resist against minor earthquake without any damage, resist against moderate earthquake with repairable structural damage and resist against strong earthquake without collapse, is developed based on the improved capacity spectrum method. The procedure uses the yield displacement and displacement ductility factor as design parameters, uses inelastic seismic demand spectrum with yield spectral accelerations and yield displacements format to calculate seismic demands of the pier under different earthquake design levels. Seismic capacities of the pier are determined by acceptable structural damage states, which are estimated quantitatively by both of the strains of concrete and longitudinal steels in plastic hinge zone and expressed as displacements at top of the pier by transforming from relationship between curvature ductility factor and displacement ductility factor. Two specimens with 1:2.5scale are designed by the proposed method and another reference specimen with same scale is designed according to bridge seismic design code in China. The damage states, bearing capacities, ductility, and energy dissipation of specimens are compared when they are subjected to cyclic loading. Then four bridge specimens with 1:2 scale to the specimens in the completed cyclic test, 3 based on displacement-based seismic design method and 1 based on bridge seismic design code in China, are tested on shaking table. Results of cyclic test and shaking table test show that ductility capacities of bridge piers designed using displacement-based method are fulfilled seismic demands expected. The proposed displacement based seismic design method can be applied to the bridge design in the earthquake regions.

**KEYWORDS:** reinforced concrete bride piers, displacement based seismic design, multiple performance objectives, cyclic test, shaking table test

## 0. INTRODUCTION

With the advance of the idea of performance based seismic design, displacement based seismic design method of structure is made a rapid progress in recent years. In the field of seismic design of bridge structure, Kowalsky and Priestley et al replaced bridge pier with elastic system possessing effective damp to analyze its nonlinear seismic response, which is called “substitute structure method”, and proposed displacement based seismic design method for RC bridge pier. Other researchers together with them developed the method to the application of multi-degree bridge and continuous bridge. Chopra et al pointed out that the “effective elastic analysis” would greatly underestimate displacement response of bridge pier and suggested an alternate analysis method “elasto-plastic response spectrum”. Fajfar applied elasto-plastic response spectrum to capacity spectrum method and improved capacity spectrum method for evaluation of structural seismic performance or for displacement base seismic design. Xue Qiang proposed a “reduction coefficient of capacity spectrum figure” based on improved capacity spectrum method to transform graphic analysis to analytic solution and gave a design case of displacement base method for RC bridge pier. Some of civil researchers studied displacement based seismic design method too. Yang Yumin et al suggested a displacement based seismic design method for continuous bridge by assuming superstructure as rigid bar. Zhu Xi et al generalized displacement based seismic design of bridge structures and gave some research proposals. Zhu also studied displacement based method for RC bridge piers and for shock isolation continuous bridge, considering effect of near-fault ground motions.

Generally, most of present displacement based seismic design methods for RC bridge pier fulfill iterative design procedure by taking the displacement on the bridge pier top as design target and taking displacement ductility factor as auxiliary parameter to determine system period or stiffness. Some researchers tried to avoid iteration in seismic design, but the implicational assumption is that effective stiffness is constant. Present research show that effective stiffness of RC bridge pier is greatly related to the strength. Aschheim and Dai Junwu et al pointed out that system yield displacement depends little on strength and is more stable than traditional stiffness. Clavi et al emphasized again superiority of displacement based seismic design method in topic report about seismic bridge in 13WEE, and introduced their own method, which is different from other methods by taking concrete and steel strain as design parameters to determine top displacement. He also point out that yield displacement is stable.

Because of the uncertainty of occurring site and strength of earthquake, three levels of fortification which is described as "resisting against minor earthquake without any damage, resisting against moderate earthquake with repairable structural damage and resisting against strong earthquake without collapse" and corresponding multi-stage seismic design method have been adopted in many seismic design codes. Displacement based seismic design is inheritance and development of present "multi-levels multi-stage" design method, is refinement and quantification of present method. Common used displacement based seismic design methods mostly assume objective displacement directly and consider little about "multi performance objectives", including quantification criterion and how to realize.
In this study Ay-Dy format earthquake demand spectrum is adopt based on improved capacity spectrum method, yield displacement and displacement ductility factor are taken as reference design variables, and a direct displacement based seismic design for RC bridge pier to realize multi performance objectives is suggested.

## 1. Seismic damage limit states of RC bridge pier

Bridge pier is the main structural member to resist lateral force. According to experience of earthquake disasters, many of seismic damages of bridge occur in bridge pier.

### *1.1. Seismic damage limit states*

According to damage degrees of RC bridge pier, damage limit states are divided into four stages:

(1) Elastic limit state: structure is elastic, longitudinal steel yield the first time and curvature ductility factor $\mu_\Phi$<1.0.
(2) Miner damage limit state: compressional strain of concrete $\varepsilon_{cu}\leq 0.004$, tensile strain of longitudinal steel $\varepsilon s\leq 0.015$, bridge can continue to work without any repair after earthquake.
(3) Damage control limit state: compressional strain of concrete $\varepsilon_{cu}\leq 1.5$ or $0.004+0.9\rho_s\ [f_y/300]$, bridge need repair to work after earthquake.
(4) Collapse control limit state: lateral bearing capacity of bridge pier decrease to 85 percent of its maximum capacity or longitudinal steel fractures. For I or II class steel, $\varepsilon_s=0.075$. If bridge damage is beyond this state, all the functions of bridge disappear.

It should be noted that no brittle failure will occur under the guarantee in capacity design.

### *1.2. The expression of damage state by means of displacement*

As shown in Fig. 1 that a single bridge pier bears horizontal force $F$, plastic hinge is formed at bridge pier bottom, effective distribution length is $Lp$, the pier height is $L$. In Fig. 2 moment-curvature relationship of bottom cross section and curvature corresponding to variant damage limit states is shown.
Yield force and displacement are expressed as:

$$F_y = M_y / L \qquad (1)$$

$$\delta_y = \phi_y L^2 / 3.0 \qquad (2)$$

Effective elastic stiffness $K_e$:

$$K_e = F_y / \delta_y \qquad (3)$$

Define displacement ductility factor $\mu_\triangle$ as ratio of maximum displacement $\delta$ and yield displacement $\delta_y$, curvature

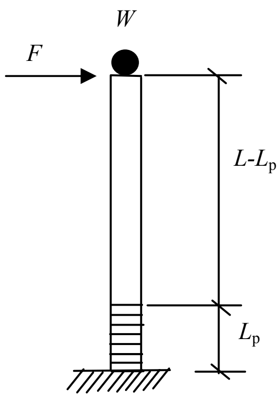


Fig.1 Single pier model

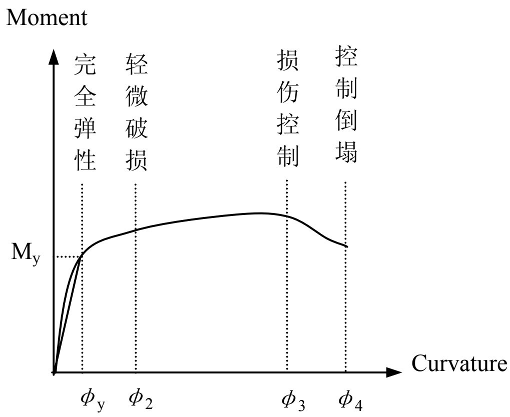


Fig.2 Bending Moment —curvature relationship of cross section at the bottom of the pier

ductility factor $\mu_\varphi$ as ratio of maximum curvature and yield curvature $\varphi_y$, then relationship between $\mu_\triangle$ and $\varphi_y$ is:

$$\mu_\Delta = 1.0 + 3.0\left[\mu_\varphi - 1.0\right]\frac{L_p}{L}\left[1.0 - 0.5\frac{L_p}{L}\right] \quad (4)$$

Where, $L_p$ =effective plastic hinge length.

$$L_p = 0.08L + 0.022 f_y d_s \quad (5)$$

Where, $f_y$=yield strength of longitudinal steel, $d_s$=diameter of longitudinal steel.
Then we can obtain force-displacement relationship from moment-curvature relationship by equation (4) and (5), and calculate displacements corresponding to variant damage limit states. Set section moment at bridge pier bottom as $M$, the corresponding curvature as $\varphi$, then relationship between lateral force $F$ and the corresponding displacement $\delta$ is:

$$F = M / L \quad (6)$$

$$\begin{cases} \delta = F / K_e & (\phi \le \phi_y) \\ \delta = \mu_\Delta \delta_y & (\phi > \phi_y) \end{cases} \quad (7)$$

Where, $\mu_\triangle$ is computed according to equation (4), $\mu_\varphi = \varphi/\varphi_y$.

## 2. Ay-Dy format earthquake demand spectrum and its properties

According to elastic response spectrum theory, when damping ratio is small, the relation between elastic acceleration spectrum $S_{a,e}$ and elastic displacement spectrum $S_{d,e}$ is:

$$S_{a,e} = \frac{4\pi^2}{T^2} S_{d,e} \quad (8)$$

Where, $T$=structural period.
According to elasto-plastic response spectrum theory, the relation between elasto-plastic displacement spectrum $S_{d,p}$ of effective ductility factor and elastic displacement spectrum $S_{d,e}$ is:

$$S_{d,p} = \mu \frac{S_{d,e}}{\overline{R}} \quad (9)$$

Where, $\mu$=displacement ductility factor, and it is assumed to be constant, $\overline{R}$ =ration of assembly average of elastic displacement spectrum of a number of earthquake waves and reduced assembly average of elastic displacement spectrum of effective ductility factor.

$\overline{R}$ is different from the usually mentioned reduced strength factor spectrum of effective ductility $R(\mu,T)$, the latter is assembly average of ratio of elastic displacement spectrum of a number of earthquake waves and reduced elastic displacement spectrum of effective ductility factor. The relationship of $\overline{R}$ and $R(\mu,T)$ is:

$$\overline{R} = \frac{R(\mu,T)}{\phi} \tag{10}$$

Where, $\phi$=correction factor with site condition, displacement ductility factor and period etc considered, $R(\mu,T)$= reduced strength factor spectrum of average effective ductility, in this study the simplified formula Vidic suggested by Fajfar is adopted.

Both sides of equation (9) are divided by displacement ductility factor $\mu$, then:

$$D_{\mathrm{y}} = \frac{S_{\mathrm{d,p}}}{\mu} = \frac{S_{\mathrm{d,e}}}{\overline{R}} \tag{11}$$

Where, $D_y$=yield displacement.

With equation (8) is considered, yield acceleration spectrum corresponding to yield strength of system is:

$$A_{\mathrm{y}} = \frac{4\pi^2}{T^2} D_{\mathrm{y}} = \frac{S_{\mathrm{a,e}}}{\overline{R}} \tag{12}$$

An earthquake demand spectrum is established by taking $D_y$ as abscissa and $A_y$ as vertical coordinates. Slope coefficient of the line connecting zero and any point on the spectrum curve is period.

Strength demand and displacement demand of single degree system with the mass $W$ are:

$$F_{\mathrm{y}} = WA_{\mathrm{y}} \tag{13}$$

$$D = \mu D_{\mathrm{y}} \tag{14}$$

From equation (11) and (12) we can obtain a property of $A_y$ - $D_y$ format earthquake demand spectrum: A ray from zero intersect with some displacement demand spectrum curves with variant displacement ductility factor, and the periods corresponding to every intersections are the same, which facilitate the realization of displacement based seismic design considering multi performance objectives.

## 3. Displacement based seismic design for RC bridge pier

### *3.1 Performance objectives and displacement design criterion of bridge pier*

Performance objectives are acceptable greatest structural damage degree under anticipated seismic risk level. How to determine performance objectives of RC bridge pier is beyond this study. Performance objectives can be generalized as "resisting against minor earthquake without any damage, resisting against moderate earthquake with repairable structural damage and resisting against strong earthquake without collapse". Displacement design criterion in this study corresponding to performance objectives mentioned above is:

$$\delta_{\mathrm{i}} \leqslant [\delta_{\mathrm{i}}]/\gamma_{\mathrm{i}} \tag{15}$$

Where, $i$ represent minor, moderate and strong earthquake actions respectively. $\delta_i$=maximum top displacements of bridge pier corresponding variant earthquake actions, and minor, moderate and strong earthquake correspond to elastic, damage control and collapse control limit states respectively. $\gamma_i$=correction factor to consider the difference between monotonic loading and cyclic loading. Equal sign of equation (15) does not satisfy at the same time.

### *3.2 Procedure of displacement based seismic design for bridge pier*

The detail procedure is as following:

(1) Determination of initial design parameters: length of bridge pier, mass of superstructure, and mechanical parameters of concrete and reinforced steel.

(2) Determination of earthquake action: first determine peak acceleration value of minor, moderate and strong earthquake, and then compute $A_y$ —$D_y$ format earthquake demand spectrum.

(3) Conceptual design: determine sectional dimension and stirrup ratio according to experience and constructional demand.

(4) Evaluate yield displacement $\delta_y$ and assume displacement ductility factor $\mu$: yield displacement is calculated

according to equation (2), and yield curvature is evaluated by formula of Priestley et al.

For rectangular section: $$\phi_y = 2.14\varepsilon_y / H \quad (16)$$

For circle section: $$\phi_y = 2.45\varepsilon_y / D \quad (17)$$

Where, $\varepsilon_y$=yield strain of longitudinal steel, *H*=computational section height, *D*=diameter.

(5) Determination of design strength: yield spectrum acceleration $a_y$ corresponding to $\delta_y$ and $\mu$ can be found in $A_y-D_y$ format earthquake demand spectrum, and horizontal earthquake action $F_y$ is calculated according to equation (13). Then design axial force: $N=Wg$, design moment: $M=F_yL$.

(6) Design of bridge pier section: calculate longitudinal steel ratio according to design axial force and design moment. Computation of section strength corresponding to initial yield of longitudinal steel adopts analysis results of moment-curvature.

(7) Determination of top displacements corresponding to variant damage limit states: at first moment-curvature of bridge pier section is analyzed to obtain curvature or curvature ductility factor corresponding to variant damage limit states, and then top displacements are computed.

(8) Calculating seismic displacement response of bridge pier employing capacity spectrum method: according to computed yield spectrum acceleration and yield displacement, top displacement of bridge pier corresponding to minor, moderate and strong earthquake is calculated employing $A_y-D_y$ format earthquake demand spectrum.

(9) Checking up of displacement design criterion: put results of equation (7) and (8) into equation (15). If inequality does not satisfy, keep yield displacement and top displacement invariant and recompute displacement ductility factor corresponding to moderate and strong earthquake until equation (15) satisfy. Note: longitudinal steel ratio is between 0.4% and 4%. Results out of the range indicate improper determination of section parameters or wrong design of stirrup ratio, and then the procedure should be restart form step (3).

(10)Checking up of shear strength according to capacity design principle: shear strength should be greater than flexural strength to guarantee the formation of plastic hinge to dissipate earthquake energy. Shear strength should satisfy:

$$Q \geqslant \gamma_0 Q_0 \quad (18)$$

Where, $Q_0$=shear force corresponding to design moment, $\gamma_0$=super-strength factor.

If shear strength does not satisfy equation (18), stirrup ratio should be increased. If equation (18) is satisfied, the design procedure is finished.

## 4. Design cases

Three design cases that satisfy the displacement design criterion of equation (15) are introduced in the following. Damage limit states of the three cases are different. The rational determination of correction factor $\gamma_i$ is beyond the study. In this paper $\gamma_i$=1.0, 1.5 and 2.0 corresponding to minor, moderate and strong earthquake. Super-strength factor $\gamma_0$=1.6

### *4.1. Design conditions*

(1) Initial parameters of bridge pier: $L$=6.0 m, $W$=315 T, $f_y$=340 MPa, $f_{yh}$=240 MPa, $f_c$=21 MPa, $E_s$=2.1×10$^5$ MPa, $E_c$=3.0×10$^4$ MPa.

(2) Earthquake action: peak value of acceleration corresponding to minor, moderate and strong earthquake is 0.14 g, 0.4 g and 0.8 g respectively. $A_y-D_y$ format earthquake demand spectrum is employed corresponding to acceleration peak value 0.4 g.

### *4.2. Design schemes*

Only case 1 of the three are introduced here.

(1) Conceptual design: section of bridge pier is circle. Diameter of bridge pier $D$ =1100 mm, stirrup Φ12@80, stirrup ratio $\rho_s$=0.51%>0.4%, axial compression ratio $\eta_k$=0.1.

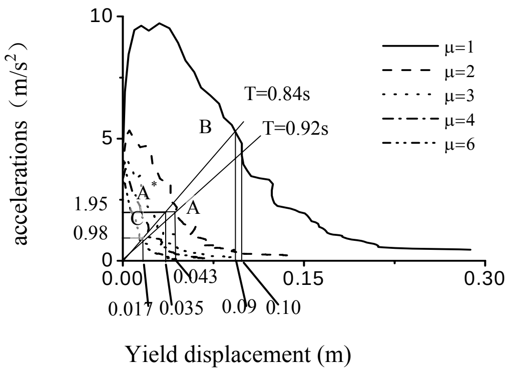


Fig.3 Estimating displacement response of the pier by inelastic seismic demand spectrum with $A_y-D_y$ format (case 1)

(2) Yield displacement $\delta_y$=4.3cm, displacement ductility factor is assumed to be 2.0.
(3) Determination of design strength: find spectrum acceleration $a_y$=2.0 m/s$^2$ (Point A in Fig. 3) in $A_y-D_y$ format earthquake demand spectrum corresponding $\delta_y$= 4.3 cm and $\mu$=2. Horizontal earthquake action $F_y$ is calculated to be 629 kN, design axial force $N$=3086kN, design moment $M$=3778kNm.
(4) Section design: longitudinal 24Φ32, longitudinal ratio $\rho_t$=2%. Yield displacement $\delta_y$=3.5cm. yield moment $\delta_y$=3.5cm, and the corresponding spectrum acceleration $a_y$=1.95 m/s$^2$.
(5) Computation of top displacement of bridge pier corresponding to variant damage limit states: according to the method introduced in section 1.2, displacement corresponding to elastic, damage control and control collapse limit states are 3.5 cm, 16.2 cm and 51.8 cm respectively.
(6) Calculating displacement response of bridge pier using capacity spectrum method: according to spectrum acceleration $a_y$ and pier top yield displacement $\delta_y$ corresponding to real yield moment, a point A$^*$ can be found in Fig. 3. A ray is formed to connect zero and A$^*$ and the corresponding period $T$=0.84 s. Generally bridge structure keeps elastic under minor earthquake action. The ray intersects with spectrum curve corresponding to displacement ductility factor $\mu$=1 at point B. Then the displacement response under minor earthquake action can be calculated by means of abscissa of point B: $D_s$=9.3cm/$a_m$×$a_s$=3.3cm<3.5cm, which satisfies the elastic assumption. Displacement response under moderate earthquake is obtained using displacement ductility factor corresponding to point A$^*$. As is shown in Fig. 3 that A$^*$ is between displacement ductility factor 2 and 3. According to vertical coordinates on the ray corresponding to $\mu$=2, $\mu$=3 and point A$^*$, the displacement ductility factor corresponding to A$^*$ can be computed μ=2.55 by means of linear interpolation. Then displacement response under moderate earthquake is $D_m$=2.55×3.5 cm=8.9 cm. When displacement response under strong earthquake is computed, yield displacement need to be narrowed $a_m/a_l$ times ($a_m$ is peak acceleration corresponding to Fig. 3): $\delta_y^*$=3.5×0.4/0.8 cm=1.75 cm. The corresponding point on the ray is C, displacement ductility factor of which is $\mu$=5.2 (the way is similar to point A$^*$). Then displacement response is $D_l$=5.2×3.5cm=18.2cm.
(7) Checking up of displacement design criterion: put results of equation (5) and (6) into (15):

| | |
|---|---|
| 3.3cm<3.5cm/1.0=3.5cm | *for minor earthquake* |
| 8.9cm<16.2cm/1.5=10.8cm | *for moderate earthquake* |
| 18.2cm<51.8cm/2.0=25.9cm | *for strong earthquake* |

Note: yield displacement approaches its limit value under minor earthquake, which satisfies displacement design criterion.
(8) Checking up of shear strength: design shear strength is $Q$=1.6×($M_y/L$)=980 kN, which is less than real shear strength 1131 kN.
Case 1 is designed under control of minor earthquake strength.

### 4.3. Comparison of design cases

Summary of design schemes is shown in table 1, in which case 4 is designed according to current seismic design specifications of high way engineering (JTJ004-89). Elastic earthquake response spectrum accords to Fig. 3.

Correction factor of importance is 1.2, and integrated influence coefficient is 0.3. Stirrup is selected as ϕ 12@100.

Table 1 Seismic design of bridge piers

| case | diameter D(mm) | Longitudinal steel | stirrup | Response of displacement ductility factor | | | Design control |
|---|---|---|---|---|---|---|---|
| | | | | minor | moderate | strong | |
| 1 | 1100 | 24Φ32 (2%) | ϕ 12@80 (0.51%) | <1 | 2.5 | 5.2 | Minor earthquake control |
| 2 | 1000 | 24Φ25 (1.5%) | ϕ 12@80 (0.56%) | <1 | 2.5 | 5.9 | Control together |
| 3 | 1000 | 24Φ32 (2.5%) | ϕ 12@100 (0.45%) | <1 | 2.0 | 4.0 | Strong earthquake collapse |
| 4 | 1000 | 24Φ36 (3.1%) | ϕ 12@100 (0.45%) | <1 | 1.9 | 3.8 | Strength design |

It is shown in table 1 that with the increasing of stirrup ratio, displacement ductility capacity of bridge piers increase, and with the decreasing of longitudinal ratio, strength of bridge piers with the same diameter decrease (case 2 and case 3). It may change design control from deformation control under strong earthquake to strength control under minor earthquake to increase diameter of bridge pier. As to case 3 (displacement based design) and case 4 (designed according to current specification), longitudinal ratio of the latter is 26 percents higher than the former. As a result, strength of the latter increases 12 percents, and response of displacement ductility reduces only 5 percents, which proves that displacement based seismic design is rational in economic aspect.

## 5. Test verification

### *5.1. Quasi-static test*

Three bridge pier specimens scaled 1:2.5 are designed, two according to displacement based seismic design method and one according to current seismic design specification. Prototypes are shown in table 1. Cyclic loading is applied to the specimens to study the seismic performance.

Test results indicate:

(1) Three specimens all show evident flexural failure.

(2) Three specimens all satisfy displacement ductility demand under moderate and strong earthquakes.

(3) When displacement ductility factor is less than 3.5, normalized hysteretic energy dissipations of three specimens are similar. Then with the increasing of displacement ductility factor, normalized hysteretic energy dissipations of the specimen design according to current specification increases rapidly, which indicates the high capacity of absolute energy dissipation of the specimen.

(4) When displacement ductility factor is less than 3.5, capacity of relative hysteretic energy dissipations (compared with perfect elastoplasticity model) are a little better. Then with the increasing of displacement ductility factor, capacity of relative hysteretic energy dissipations are similar.

(5) Compared with elastic stiffness of bridge pier, performances of stiffness degradation of all specimens are similar.

### *5.2. Shaking table test*

Four bridge pier specimens scaled 1:2 are designed and fabricated with specimens in quasi-static test taken as prototypes and a shaking table test is performed in Institute of Engineering Mechanics of China Earthquake Bureau.

Test results indicate:

(1) Horizontal cracks are mainly damage modes, and specimens do not incline evidently, which shows specimens are in repairable stage.

(2) After input of moderate earthquake waves, displacement ductility factors are about 2.0. After input of strong earthquake waves, displacement ductility factors are about 4.0.

(3) Under moderate earthquakes, earthquake input energies of specimens designed according to displacement based seismic design method are greater than those of specimens designed according to current specification. While under strong earthquake, phenomenon reverses.

## 6. conclusion

(1) $A_y - D_y$ format earthquake demand spectrums of acceleration and displacement are suggested based on improved capacity spectrum method. The superiority is: the intersections of line pass zero and earthquake demand spectrum curves with variant displacement ductility factors correspond to the same period, which facilitates implementation of multi performance objectives in seismic design according to displacement based seismic design method.

(2) A direct displacement based seismic design method to take yield displacement and displacement ductility factor as reference design parameters that can realize multi performance objectives, which is expressed as "designed structures can resist against minor earthquake without any damage, resist against moderate earthquake with repairable structural damage and resist against strong earthquake without collapse", is proposed by taking strains of concrete and reinforced steel as quantified criterions under variant damage limit states, and capacity design principal is employed to guarantee shear strength demand.

(3) Quasi-static test and shaking table test are performed to verify the proposed design method. Failure modes, displacement ductility factors and energy dissipations are analyzed to study seismic performances of the bridge pier specimens. Test results show that bridge pier specimens designed by proposed method satisfy anticipated ductility demand. The method may be applied to bridges in earthquake regions.

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