Nonlinear Analysis Model of Fiber Plasterboard filled with reinforced concrete

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ABSTRACT: According to the material characteristic of fiber plasterboard filled with reinforced concrete, the computational macro-model basing on the multi-component element is proposed and stiffness matrix of the model is provided. The inner vertical springs of the model are located at the points of Gaussian integration to improve the computational accuracy and efficiency. Based on the experiment study, hysteretic models of the vertical and horizontal component are revised and revised skeleton curves of the components are given. The elasto-plastic push-over is carried out to the four fiber plasterboard filled with reinforced concrete with the macro-model. The computational results are close to those of experiments, which show that the model has relatively high accuracy and simple calculation. We can get some points that plasterboard and steel reinforcement greatly affect the strength performance of the board and that effect of the intensity of concrete is small.
KEYWORDS: plasterboard model, fiber plasterboard, skeleton curve, Gaussian integration

1. INTRODUCTION

Glass fiber reinforced plasterboard (GFRP) is an Australian developed and manufactured walling product used in the building industry to provide habitable enclosures for buildings. The 120 mm thick, lightweight, hollow-core panels are machine made using gypsum plaster reinforced with chopped glass fiber, as is shown in Fig.1. Because the construction progress of the plasterboard with concrete is rapid compared with concrete construction，the plasterboard is also called rapid wall. However, since the rapid wall is made of many materials, its mechanical property is cared by engineers. To study lateral bearing capacity, deformation capacity, ductility of the wall, low-cycle tests were carried out. Like other new material on construction, calculating model for the rapid wall must be founded to analyze the material's nonlinear performance which evaluates the earthquake resistance of rapid-wall building.

For many years, domestic and international scholars have presented many macro mechanics models for shear wall, such as equivalent beam model, wall column model, truss model, etc. Recently, macro-model basing on multi-component elements is often used to study non-linear property of shear walls. The model adopting Timoshenko beam theory consists of vertical components, which stimulate the axial and flexural deformation and axial force, and a horizontal component, which stimulates shearing deformation and shear. Therefore, based on relevant literatures, material characteristic and test results of GFRP with fully filled with reinforced concrete, macro-model on multi-component element for GFRP with fully filled with reinforced concrete and skeleton curves of components are proposed to study the earthquake-resistance performance on GFRP with fully filled with reinforced concrete in this paper.

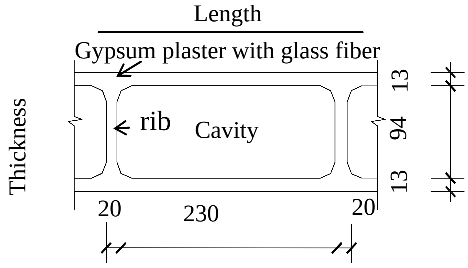

Fig.1. Cross section of a GFRGP

2.MACRO-MODEL OF GFRP WITH FULLY FILLED WITH REINFORCED CONCRETE

The macro-model of multi-component elements consists of four vertical springs (k_1, k_2, k_3, k_4) and a horizontal spring (k_h), which are connected with bold lines which denote stiffening bars, as are shown in Fig.2. k_1, k_4 springs denoting side columns bear vertical load and moment, k2, k3 springs denoting inner GFRP with fully filled with reinforced concrete bear vertical load and moment, and k_h bears horizontal shear. To improve accuracy of calculating results, according to Gauss-Legendre second-order integral, the distance l_m between k2 or k3 component and central point o is $l_m = \frac{1}{\sqrt{3}}(B/2-250)$, where B is width of GFRP with fully filled with reinforced concrete. The distance between central point o and bottom beam is 0.5h and the pint o is the rotational centre of GFRP with fully filled with reinforced concrete. Thus, the stiffness and bending resistance influenced by axial force of GFRP is considered, so the model has more merits than wall column model. The element stiffness matrix of GFRP with fully filled with reinforced concrete is deduced as following.

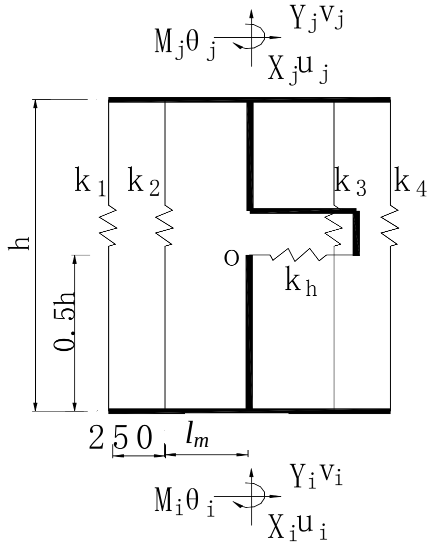

Fig.2 Macro-model of GFRP with fully filled with reinforced concrete

The top and bottom displacements of GFRP with fully filled with reinforced concrete are defined as $\{\Delta\}^T = \{u_i, v_i, \theta_i, u_j, v_j, \theta_j\}$, where u_i, v_i, θ_i denote horizontal displacement, vertical displacement and angle of i end of the model , respectively and u_j, v_j, θ_j denote horizontal displacement, vertical displacement and angle of j end, respectively. Fore vector at the top and bottom of the model is $\{F\}^T = \{X_i, Y_i, M_i, X_j, Y_j, M_j\}$, where X_i, Y_i, M_i denote shear, axial force and moment of i end of the model, respectively and X_j, Y_j, M_j denote shear, axial force and moment of j end of the model, respectively. The direction sign of vectors is shown in Fig.2.

The horizontal displacements of i and j ends are respectively u_i^s, u_j^s under shear deformation and the central point of rotation of the model is o point. Thus, following displacements u_i and u_j are obtained in accordance with geometry.

$$u_i = u_i^s - 0.5h\sin\theta_i \tag{1}$$

$$u_j = u_j^s + 0.5h\sin\theta_j \tag{2}$$

Based on the small deformation, there are $\sin\theta_i \approx \theta_i, \sin\theta_j \approx \theta_j$. Eq. (1) is subtracted from Eq. (2) and the new Equation is expressed as

$$u_j^s - u_i^s = u_j - u_i - 0.5h(\theta_j + \theta_i) \tag{3}$$

Therefore, the deformation of horizontal shear spring is listed as follow

$$\delta u = u_j - u_i - 0.5h(\theta_j + \theta_i) \tag{4}$$

Moreover, if letter *m* denotes any spring, the axial displacement of i end of number *m* spring is expressed as

$$v_{im}^s = -\theta_i l_m + 0.5h(1-\cos\theta_i) + v_i \tag{5}$$

The axial displacement of j end of number *m* spring is expressed as

$$v_{jm}^s = -\theta_j l_m - 0.5h(1-\cos\theta_j) + v_j \tag{6}$$

Where l_m is distance between number *m* spring and center axis of figure; l_m is positive value when a spring locates in the right of center axis; else, l_m is negative value.

Eq. (5) is subtracted from Eq. (6), the axial deformation of *m* spring is expressed as

$$\delta v_m = (\theta_i - \theta_j) l_m + v_j - v_i - 0.5h(1-\cos\theta_j) - 0.5h(1-\cos\theta_i) \tag{7}$$

Based on the small deformation, there are $\cos\theta_i = 1, \cos\theta_j = 1$. The above equation can be simplified to

$$\delta v_m = (\theta_i - \theta_j) l_m + v_j - v_i \tag{8}$$

If the virtual displacement of model is $\{\Delta^*\}^T = \{u_i^*, v_i^*, \theta_i^*, u_j^*, v_j^*, \theta_j^*\}$, virtual work under outside force is expressed as

$$W = \{\Delta^*\}^T\{F\} \tag{9}$$

Otherwise, virtual work under internal force and virtual displacement is expressed as

$$U = k_h \delta u \delta u^* + \sum_{m=1}^{n} k_{vm} \delta v_m \delta v_m^* \tag{10}$$

Where k_h is the stiffness of horizontal spring; k_{vm} is axial stiffness of m spring. According to virtual work principle, the following equation is obtained.

$$\{\Delta^*\}^T\{F\} = k_h \delta u \delta u^* + \sum_{m=1}^{n} k_{vm} \delta v_m \delta v_m^* \tag{11}$$

By introducing Eq.(4) and Eq.(8) into Eq.(11) and integrating, Eq.(11) is then simplified to

$$\{F\} = [K]\{\Delta\} \tag{12}$$

In which element stiffness matrix is shown as follow:

$$[K_e] = \begin{bmatrix} k_h & 0 & 0.5hk_h & -k_h & 0 & 0.5hk_h \\ & \sum_{m=1}^{n} k_{vm} & -\sum_{m=1}^{n} k_{vm} l_m & 0 & -\sum_{m=1}^{n} k_{vm} & \sum_{m=1}^{n} k_{vm} l_m \\ & & 0.25h^2 k_h + \sum_{m=1}^{n} k_{vm} l_m^2 & -k_h rh & \sum_{m=1}^{n} k_{vm} l_m & 0.25h^2 k_h - \sum_{m=1}^{n} k_{vm} l_m^2 \\ & & & k_h & 0 & -0.5hk_h \\ & & & & \sum_{m=1}^{n} k_{vm} & -\sum_{m=1}^{n} k_{vm} l_m \\ & & & & & 0.25h^2 k_h + \sum_{m=1}^{n} k_{vm} l_m^2 \end{bmatrix} \tag{13}$$

3. STIFFNESS OF VERTICAL AND HORIZONTAL SPRINGS

According to Timoshenko beam theory and virtual work principle, stiffness of vertical and horizontal springs can be deduced. Timoshenko beam has flexural, transverse and axial deformation [9]. Transverse deformation perpendicular to neutral axis keeps plane and the relation of stress and strain follows small deformation theory. Thus, axial displacement of random point in the beam is expressed as

$$u(x,z)=u_0(x)-z\cdot\theta(x) \tag{14}$$

Where, $u_0(x)$ and $\theta(x)$ are the displacement and angle in the neutral axis of the point, respectively. Besides, the relative angle of the point is expressed as

$$dw/dx=\theta(x)+\beta \tag{15}$$

Where β is the angle caused by shear deformation.

According to small deformation theory, the axial strain ε_x and shear strain γ_{xz} is respectively expressed as

$$\varepsilon_x=\frac{\partial u}{\partial x}=\frac{du_0}{dx}-z\frac{d\theta}{dx} \tag{16}$$

$$\gamma_{xz}=\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}=-\theta+\frac{dw}{dx}=\beta \tag{17}$$

According to energy conservation law, equation of strain energy on beam is shown as follow

$$\int_v(\delta x\cdot\sigma_x+\delta\beta\cdot\tau_{xz})dv=\int_0^h(\delta w\cdot q)dx \tag{18}$$

Where q is the distribution load of beam.

The transverse displacement w, axial displacement u and angle θ are expressed by linear interpolation function. Inner displacements of an element can be expressed as

$$\begin{Bmatrix} w \\ u \\ \theta \end{Bmatrix}=\begin{bmatrix} N_1 & 0 & 0 & N_2 & 0 & 0 \\ 0 & N_1 & 0 & 0 & N_2 & 0 \\ 0 & 0 & N_1 & 0 & 0 & N_2 \end{bmatrix}\Phi=N\Phi \tag{19}$$

Where, N is matrix of shape function and $N_1=(1-\xi)/2$, $N_2=(1+\xi)/2$, ξ is nature coordinate of an element. Here, displacement vector of a node is expressed as $\Phi=\begin{bmatrix} w_i & u_i & \theta_i & w_j & u_j & \theta_j \end{bmatrix}^T$.

Axial and shear strain of any point of a element is respectively expressed as

$$\varepsilon_x=\frac{du_0}{dx}-z\frac{d\theta}{dx}=\begin{bmatrix} 0 & \frac{dN_1}{dx} & -z\frac{dN_1}{dx} & 0 & \frac{dN_2}{dx} & -z\frac{dN_2}{dx} \end{bmatrix}\Phi=B_a\Phi \tag{20}$$

$$\beta=\frac{dw}{dx}-\theta=\begin{bmatrix} \frac{dN_1}{dx} & 0 & -N_1 & \frac{dN_2}{dx} & 0 & -N_2 \end{bmatrix}\Phi=B_s\Phi \tag{21}$$

Based on Eq.（18）-（21）, the following equation is obtained as follow.

$$\int_v\delta(\Phi^T B_a^T EB_a\Phi+\Phi^T B_s^T GB_s\Phi)dv-\int_0^h\delta(\Phi^T N^T q)dx=0 \tag{22}$$

Based on Eq.（22）, the stiffness matrix of an element is obtained as follow.

$$[K_e]=\int_v B_a^T EB_a dv+B_s^T GB_s dv=K_a+K_s \tag{23}$$

Where, K_a is axial and flexural stiffness matrix，K_s is shear stiffness matrix.

With disperse sum method instead of integral method，the tension stiffness k_{vm} and shear stiffness k_{vm} of m layer are expressed as

$$k_{vm}=E_m A_m/l \tag{24}$$

Where A_m is the sum area of concrete, steel bar and plasterboard.

$$k_s = GA_s / l \quad (25)$$

Where A_s is effective shear area.

According to stress integral, the axial force N, moment M and shear Q of an element are respectively expressed as

$$N = \int_v \sigma_x dv = \int_v EB_a \Phi dv = \sum k_m (u_{mj} - u_{mi}) + \sum k_m Z_m (\theta_i - \theta_j) \quad (26)$$

$$M = \int_v \sigma_x z dv = \int_v EB_a \Phi z dv = \sum k_m z_m (u_{mj} - u_{mi}) + \sum k_m Z_m^2 (\theta_i - \theta_j) \quad (27)$$

$$Q = \int_v \tau_{xz} dv = \int_v GB_s dv = k_h [(w_j - w_i) - 0.5h(\theta_i + \theta_j)] \quad (28)$$

Eq. (27)-(28) shows that the axial force N and moment M of an element are only relevant to axial stiffness, displacement and angle of an element and irrelevant to shearing deformation.

4 CONSTITUTIVE MODEL OF MATERIAL

4.1Constitutive Model of Steel Reinforcement

Bilinear elasto-plastic model of steel reinforcement is adopted and the ratio of tangent elastic modulus after reinforcement yield to initial elastic modulus is 0.01.

4.2 Constitutive Model of Concrete

Constitutive model of uniaxial stress on plain concrete is used to analyze concrete’s contribution to stiffness of vertical springs in computational model. Constitutive model of biaxial compression of concrete is used to analyze the shear of horizontal spring.

4.3 Constitutive Model of Plasterboard

Constitutive model of plasterboard is based on experimental results. The relation between stress and strain of plasterboard is expressed as:

(1) The relation between stress and strain under compressive state is listed as follow

$$\begin{cases} \sigma^p = -2\times 10^6 \varepsilon^p + 6600\varepsilon^p - 0.0161 & (\varepsilon^p \le 0.0015) \\ \sigma^p = 5 & (0.0015 < \varepsilon^p < 0.0025) \end{cases} \quad (29)$$

(2) The relation between stress and strain under tension is listed as follow

$$\begin{cases} \sigma^{pt} = 3712\varepsilon^{pt} & (\varepsilon^p \le 0.0003) \\ \sigma^{pt} = 79\varepsilon^{pt} + 1.09 & (0.0003 < \varepsilon^{pt} < 0.008) \end{cases} \quad (30)$$

5 SKELETON CURVES OF GFRP FULLY FILLED WITH REINFORCED CONCRETE

5.1 Skeleton Curve of Vertical Spring of GFRP Fully Filled with Reinforced Concrete

Skeleton curve of vertical spring of model is multi-line, as shown in Fig.4, parameters of which are defined as follow.

5.1.1 Parameters of vertical spring in tension

(1) Stiffness of initial stage is expressed as

$$K_{pe}=\frac{E_s A_s}{\psi_0 h}+\frac{E_p A_p}{h}\times 0.8 \tag{31}$$

$$d_{py}=-\varepsilon_p h \tag{32}$$

In which, ψ is inhomogeneous coefficient of reinforcement between tension cracks, E_s is elastic modulus of steel, k_{pe} and d_{py} are respectively elastic stiffness and deformation of vertical springs before tension destroy of plasterboard, E_p is the elastic modulus of plasterboard, A_p is transverse area of plasterboard， $\varepsilon_p=3\times10^{-4}$ is the strain under tension destroy,0.8 is reduction coefficient with cooperative work between plasterboard and concrete columns.

(2) Stiffness after plasterboard failure is expressed as

$$K_{se}=\frac{E_s A_s}{\psi_0^{'} h} \tag{33}$$

$$d_{sy}=\frac{\psi_0^{'} h f_y}{E_s} \tag{34}$$

Where $\psi^{'}$ is asymmetry coefficient of reinforcement between tension cracks after plasterboard failure.

(3) Stiffness after reinforcement yield is expressed as

$$K_{sy}=0.01K_{se} \tag{35}$$

5.1.2 Parameters of vertical spring in compression

(1) If concrete, plasterboard and reinforcement yield at the same time, the stiffness of vertical spring is expressed as

$$K_c=(A_c\frac{f_c}{f_y}+A_p\frac{f_p}{f_y}+A_s)E_s/h \tag{36}$$

$$d_{cy}=-\varepsilon_{sy}h=-\frac{f_y}{E_s}h \tag{37}$$

Where K_c and d_{cy} are respectively elastic stiffness and yielding deformation of vertical spring in compression, f_y is yield strength of reinforcement, A_c is transverse area of concrete columns; A_p is transverse area of plasterboard.

(2) Compression stiffness of vertical springs after reinforcement yields.

$$K_{cy}=0.02K_c \tag{38}$$

5.2 Skeleton Curve of Horizontal Spring of GFRP Fully Filled with Reinforced Concrete

Skeleton curve of horizontal spring of GFRP fully filled with reinforced concrete is symmetrical multi line, as shown in Fig.5.

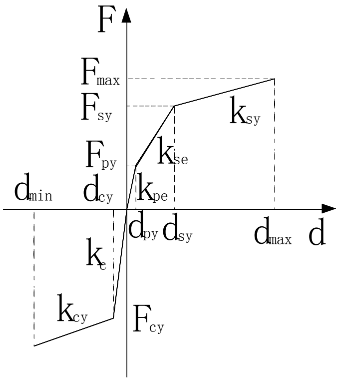

Fig.4 Skeleton curve of vertical

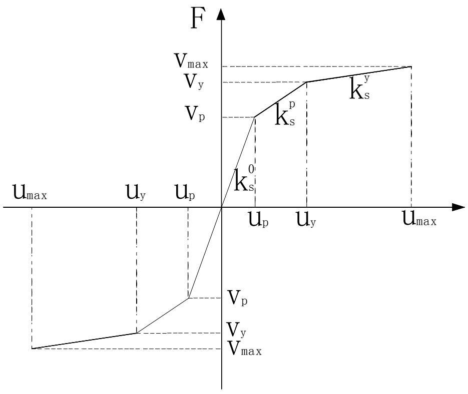

Fig.5 Skeleton curve of horizontal spring

（1） Initial shearing stiffness of a horizontal spring is expressed as

$$k_s^0 = \frac{GA_c}{h} \times \frac{f_{vp}}{f_v} \tag{39}$$

In whichG is elastic shearing modulus of concrete; A_c is transverse area of concrete; h is story height; f_{vp} is shearing strength of GFRP fully filled with reinforced concrete; f_v is shearing strength of GFRP.

（2）shearing stiffness after plasterboard cracking

$$k_s^p = \frac{GA_c}{h} = \frac{V_y}{U_y} \tag{40}$$

which , $V_y = GA_c\gamma_p$ is yield shear of GFRP fully filled with reinforced concrete, $U_y = \gamma_y h$ is yield shearing displacement, $\gamma_y = 0.9\gamma_p$ is yield shearing strain, γ_p is peak value of shearing strain; G is initial shearing modulus of concrete.

(3)Shearing stiffness of GFRP fully filled with reinforced concrete after reinforcement yields.

$$k_s^y = 0.01k_s^p \tag{41}$$

6 TEST PROOF

According to experimental result of literature [1, 3], push-over analysis of GFRP fully filled with reinforced concrete is carried out with the above multi-component model. The relation of experimental and numerical values between horizontal force and displacement at the top of GFRP fully filled with reinforced concrete is shown in Fig.6.

Fig.6 shows that calculating results are close to experimental results and, especially, the match of experimental result and numerical value at the initial stage is good. Partial experimental results deviate from calculating results at the limit load stage. As a whole, calculating results can meet requirement of engineering calculation. Adopting multi component model in analyzing mechanical properties of GFRP fully filled with reinforced concrete is effective.

We also find in calculation and test that the change of concrete strength produces small influence on shear capacity of GFRP fully filled with reinforced concrete. GFRP plays an important in resistance shear. The change of reinforcement area brings obvious change to the skeleton curve of GFRP fully filled with reinforced concrete.

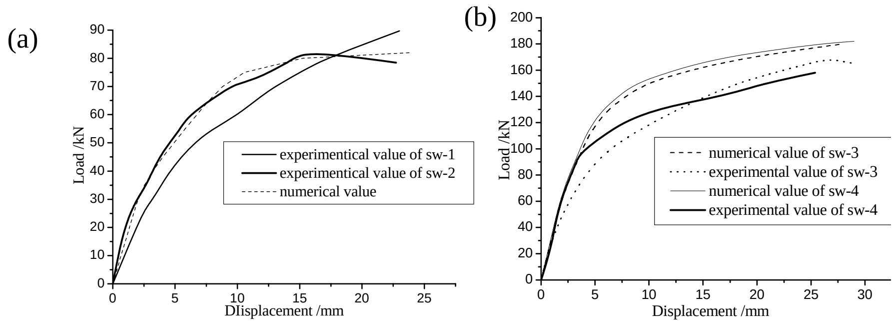

Fig.6 Relations between experimental result and numerical value
on sw-1,sw-2 ,sw-3,and sw-4

7 CONCLUSIONS

（1）Adopting improved multi component model in analyzing mechanical properties of GFRP fully filled with reinforced concrete is effective. The calculating elements can be adjusted in accordance with calculating precision. Multi component model can be used to study other material composite wall.

（2）Inner springs are located at the Gauss point which can improve calculating efficiency and precision.

（3）The change of concrete strength produces small influence on shear capacity of GFRP fully filled with reinforced concrete. GFRP plays an important in resistance shear. The change of reinforcement area brings obvious change to the skeleton curve of GFRP fully filled with reinforced concrete.

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