ON THE DYNAMICS OF UNILATERALLY SUPPORTED RIGID BLOCKS

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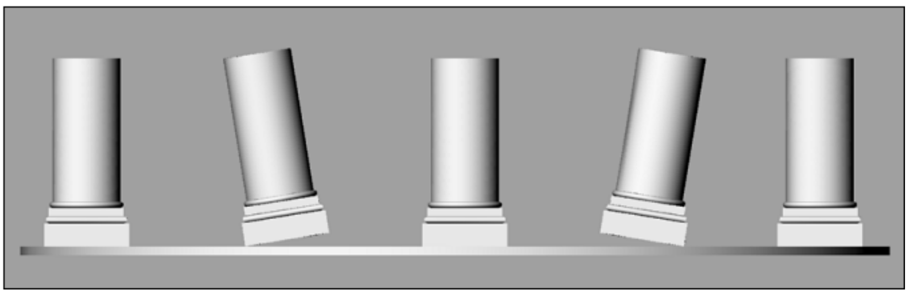
ABSTRACT :

The paper addresses the problem of the study of dynamic evolution of a rigid block under pure rocking, which, as well known, is strongly affected by the impacts of the block with the foundation base.
On one side, the paper sets up the problem from a theoretical point of view, writing down dynamic equilibrium conditions involving the impact problem by means of the Dirac- distribution. On the other side, an overview of some experimental tests performed on a shaking table is given for emphasizing peculiar feature of dynamic motion.

KEYWORDS: Rigid block, unilateral model, structural dynamics, theoretical set up, experimental tests.

1. INTRODUCTION

The rocking response and the possibility of overturning of rigid bodies during earthquakes represent important features in seismic safety problems.
During strong ground shaking, rigid structures (such as electrical equipments, retaining walls, liquid storage tanks, tall rigid buildings, tombstones, etc.) slide, bounce, rock or overturn, sometimes resulting in a final substantial damage (Yim et ali, 1980).
Despite the apparent simplicity, the motion of rigid blocks poses difficult problems to solve.
The analysis of rocking response of rigid blocks has attracted the attention of many researchers, especially from a theoretical point of view, and relevant dynamics are less understood than in many other non-linear vibration problems (Makris, 1998).
The cause of the interest in treating dynamics of rigid blocks is mainly due to possible applications: a broadly similar response is actually exhibited, during earthquakes, by ancient stone temples and sculptures saved in museums or in archaeological open spaces, as an example. In almost every destructive earthquake, free standing columns of monuments have survived undamaged in earthquakes that caused spectacular destruction around them.



a)

b)

Figure 2 Free standing columns and tombstones can be considered as rigid blocks resting on a rigid horizontal surface and rocking according to the action of a static horizontal force Milne (Milne, 1881).

Free-standing stone columns (Fig.1), that supported heavy structures, remained standing, although at the end of the earthquake they were surrounded by heaps of debris that had been seemingly more stable structures.
Tall, slender stone pillars in graveyards have survived strong ground motion whereas box-like electric power transformers have rocked and overturned (Yim et ali, 1980).
Another important aspect of the analysis of rigid blocks dynamics is its applicability to possible overturning of unanchored structures and equipments (Fig. 2) during earthquakes, leading a renewed interest to this problem (Makris, 1998).

Figure 2 An electrical transformer at the Sylmar Converter Station, California that overturned during the 1971 San Fernando earthquake (by Makris, 1998).

Analysis of rocking response of rigid blocks may be applicable also for a variety of other cases besides those previously cited. It is applicable, for example, in slender water tanks, petroleum cracking towers, stacks of graphite of nuclear reactors (Tso & Wong, 1989) and so on, where a broadly similar response may be also observed during earthquakes, as well as for engineering applications concerning the stability of water tanks (Housner, 1963).

2. DYNAMICS OF FREE STANDING RIGID BLOCKS UNDER PURE ROCKING

2.1. Description of the Rocking Motion of the Rigid Block Reference Model

The reference model consists of a rigid block, which is assumed to lie on a horizontal oscillating surface; the moving base-plane is considered to be sufficiently rough as to prevent sliding of the block and to allow pure rocking.
Depending on the base excitation signal, which is assumed to be horizontal, periodic and unidirectional, and to act orthogonally with respect to the block, the problem of analysis of dynamics of the rigid block may be reduced to a plane one.
Actually, generally speaking, with reference to Figure 3, if base accelerations act in the 1-2 plane keeping orthogonal to the block sides, rotations around the 1 and 2-axes can be neglected and a two-dimensional representation of the problem can be referred to.

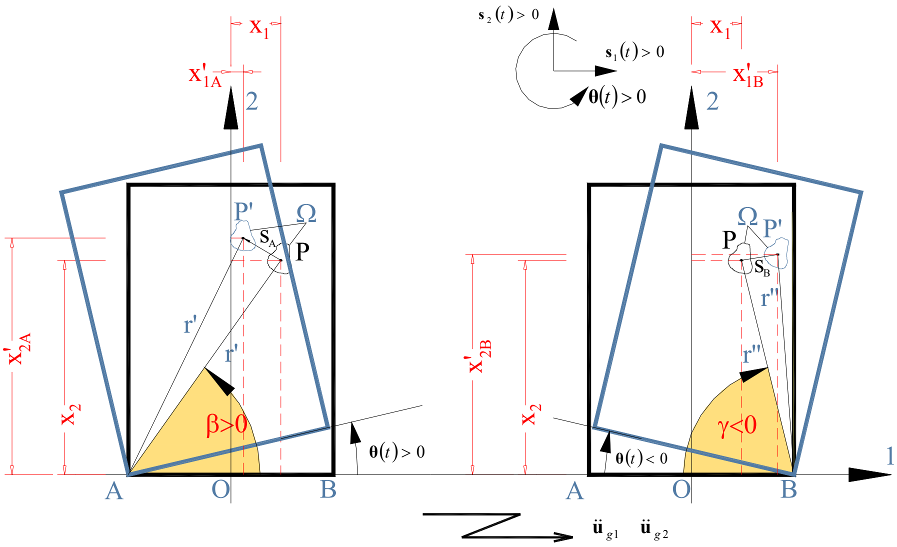


Figure 3 Plane rocking of a rigid block model around its base corners A and B.

During the motion, the block keeps in constant contact with the floor, either standing on its base or rocking around one of its base corners. Therefore two situations of rotation are individuated: counter-clockwise (positive) rotation of the block around the left base corner (A), and clockwise (negative) rotation of the block around the right base corner (B), as shown in Figure 3.

2.2. Counter-clockwise Motion

One first considers the counter-clockwise motion around the base corner A, denoting by the index index $(\cdot)^A$ any related quantity or variable.
During the motion, the instantaneous position of the generic point P of the block with respect to the counter-clockwise reference system (O,1,2) is described by

$$\mathbf{x}'^A(t) = \mathbf{x} + \mathbf{s}^A(\mathbf{x},t) = \begin{bmatrix} x'^A_1(t) \\ x'^A_2(t) \end{bmatrix} = \begin{bmatrix} x_1 + s^A_1(\mathbf{x},t) \\ x_2 + s^A_2(\mathbf{x},t) \end{bmatrix} \tag{2.1}$$

where $\mathbf{x}'^A(t)$, $\mathbf{x}$ represent respectively the instantaneous and original position vectors of P, $\mathbf{s}^A(\mathbf{x},t)$ its displacement and t is the time-variable. Vectors components in the reference system are denoted by the indexes $(\cdot)_1$ and $(\cdot)_2$ respectively.
After introducing the rotation angle θ (t), the instantaneous position of P, under small displacements, can be expressed as follows

$$\mathbf{x}'^A(t) = \begin{pmatrix} -b + r'\cos[\beta + \theta(t)] \\ r'\sin[\beta + \theta(t)] \end{pmatrix} = \begin{pmatrix} -b + r'\cos\beta - r'\sin\beta\,\theta(t) \\ r'\sin\beta + r'\cos\beta\,\theta(t) \end{pmatrix} = \begin{pmatrix} -x_2 \\ b + x_1 \end{pmatrix}\theta(t) + \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \tag{2.2}$$

where r' is the modulus of the vector **AP**, β is the angle formed by **AP** with the horizontal axis and b is the half breadth of the block (Fig. 4).
Whence one can deduct the displacement vector expression with the related velocity and acceleration fields, as follows

$$\mathbf{s}^A(\mathbf{x},t) = \mathbf{x}'^A(t) - \mathbf{x} = \begin{pmatrix} -x_2 \\ b + x_1 \end{pmatrix}\theta(t), \qquad \dot{\mathbf{s}}^A(\mathbf{x},t) = \begin{pmatrix} -x_2 \\ b + x_1 \end{pmatrix}\dot{\theta}(t), \qquad \ddot{\mathbf{s}}^A(\mathbf{x},t) = \begin{pmatrix} -x_2 \\ b + x_1 \end{pmatrix}\ddot{\theta}(t) \tag{2.3}$$

where first and second order time derivatives are denoted by superimposed dots.

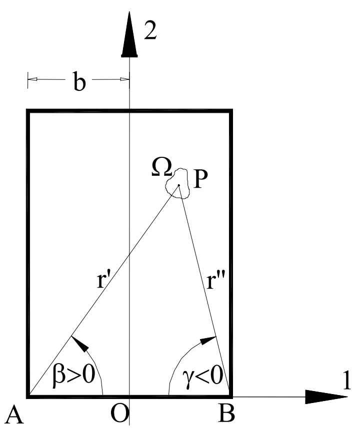


Figure 4 Geometric characteristics of the block.

After evaluating the forces and the relevant moments calculated with respect to the reference origin O involved in the motion equation, i.e. the vectors of inertial forces $\mathbf{F}^A_I(t)$, base acceleration $\mathbf{F}_a(t)$, self-weight $\mathbf{F}_g(t)$, and their relevant moments (moduli) $M^A_{I,O}(t)$, $M^A_{a,O}(t)$, $M^A_{g,O}(t)$, one may impose equilibrium to translation and rotation as follows

$$\mathbf{F}_I^A(t)+\mathbf{F}_a(t)+\mathbf{F}_g(t)=0 \qquad \Rightarrow -\int_\Omega \mu(\mathbf{x})\ddot{\mathbf{s}}^A(\mathbf{x},t)dV-\int_\Omega \mu(\mathbf{x})\ddot{\mathbf{u}}_g(t)dV+\int_g \mu(\mathbf{x})\cdot\mathbf{g}dV=0 \tag{2.4}$$

$$M^A_{I,O}(t)+M^A_{a,O}(t)+M^A_{g,O}(t)=0 \qquad \Rightarrow -\int_\Omega \left\|\left[\mathbf{x}+\mathbf{s}^A(\mathbf{x},t)\right]\times\left[\ddot{\mathbf{s}}^A(\mathbf{x},t)+\ddot{\mathbf{u}}_g(t)-\mathbf{g}\right]\right\|\mu(\mathbf{x})dV=0 \tag{2.5}$$

where $\mu(\mathbf{x})$ is the material density at $\mathbf{x}$, the base and gravity accelerations are denoted respectively by $\ddot{\mathbf{u}}_g(t)$ and $\mathbf{g}$, $S_{1,O}=\int_\Omega x_2\mu(\mathbf{x})dV$ and $S_{2,O}=\int_\Omega x_1\mu(\mathbf{x})dV$ are the static moments of the block with respect to the 1 and 2 axes, and its mass is $m=\int_\Omega \mu(\mathbf{x})dV$.

2.2. Clockwise Motion

One now considers the clockwise motion around the base corner B, denoting by the index index $(\cdot)^B$ any related quantity or variable. During the motion, the instantaneous position of the generic point P of the block with respect to the reference system (O,1,2) is described by

$$\mathbf{x}'^B(t)=\mathbf{x}+\mathbf{s}^B(\mathbf{x},t)=\begin{bmatrix} x'^B_1(t) \\ x'^B_2(t)\end{bmatrix}=\begin{bmatrix} x_1+s^B_1(\mathbf{x},t) \\ x_2+s^B_2(\mathbf{x},t)\end{bmatrix} \tag{2.6}$$

where $\mathbf{x}'^A(t)$, $\mathbf{x}$ represent respectively the instantaneous and original position vectors of P, $\mathbf{s}^A(\mathbf{x},t)$ its displacement and t is the time-variable. Vectors components in the reference system are denoted by the indexes $(\cdot)_1$ and $(\cdot)_2$ respectively.
After introducing the rotation angle θ (t), the instantaneous position of P, under small displacements, can be expressed as follows

$$\mathbf{x}'^B(t)=\begin{pmatrix} b-r''\cos[\gamma+\theta(t)] \\ r''\sin[\gamma+\theta(t)]\end{pmatrix}=\begin{pmatrix} b-r''\cos\gamma\cos\theta(t)+r''\sin\gamma\sin\theta(t) \\ r''\sin\gamma\cos\theta(t)+r'''\sin\theta(t)\cos\gamma\end{pmatrix}=\begin{pmatrix} x_2 \\ b-x_1\end{pmatrix}\theta(t)+\begin{pmatrix} x_1 \\ x_2\end{pmatrix} \tag{2.7}$$

where r'' is the modulus of the vector **BP**, γ is the angle formed by **BP** with the horizontal axis and b is the half breadth of the block (Fig. 4).
Whence one can deduct the displacement vector expression with the related velocity and acceleration fields, as follows

$$\mathbf{s}^B(\mathbf{x},t)=\mathbf{x}'^B(t)-\mathbf{x}=\begin{pmatrix} x_2 \\ b-x_1\end{pmatrix}\theta(t), \qquad \dot{\mathbf{s}}^B(\mathbf{x},t)=\begin{pmatrix} x_2 \\ b-x_1\end{pmatrix}\dot\theta(t), \qquad \ddot{\mathbf{s}}^B(\mathbf{x},t)=\begin{pmatrix} x_2 \\ b-x_1\end{pmatrix}\ddot\theta(t) \tag{2.8}$$

where first and second order time derivatives are denoted by superimposed dots.
After evaluating the forces and the relevant moments calculated with respect to the reference origin O involved in the motion equation, i.e. the vectors of inertial forces $\mathbf{F}_I^B(t)$, base acceleration $\mathbf{F}_a(t)$, self-weight $\mathbf{F}_g(t)$, and their relevant moments (moduli) $M^B_{I,O}(t)$, $M^B_{a,O}(t)$, $M^B_{g,O}(t)$, one may impose equilibrium to translation and rotation as follows

$$\mathbf{F}_I^B(t)+\mathbf{F}_a(t)+\mathbf{F}_g(t)=0 \qquad \Rightarrow -\int_\Omega \mu(\mathbf{x})\ddot{\mathbf{s}}^B(\mathbf{x},t)dV-\int_\Omega \mu(\mathbf{x})\ddot{\mathbf{u}}_g(t)dV+\int_g \mu(\mathbf{x})\cdot\mathbf{g}dV=0 \tag{2.9}$$

$$M^B_{I,O}(t)+M^B_{a,O}(t)+M^B_{g,O}(t)=0 \qquad \Rightarrow -\int_\Omega \left\|\left[\mathbf{x}+\mathbf{s}^B(\mathbf{x},t)\right]\times\left[\ddot{\mathbf{s}}^B(\mathbf{x},t)+\ddot{\mathbf{u}}_g(t)-\mathbf{g}\right]\right\|\mu(\mathbf{x})dV=0 \tag{2.10}$$

2.3.Description of Global Rocking Motion

Let assume the global displacement field $\mathbf{s}(\mathbf{x},t)$ as suitably given by superposition of the two displacement fields introduced in the above, relevant to the two clockwise and counter-clockwise rocking mechanisms around the two base corners of the block, as follows

$$\mathbf{s}(\mathbf{x},t)=H[\theta(t)]\cdot\mathbf{s}^A(\mathbf{x},t)+H[-\theta(t)]\cdot\mathbf{s}^B(\mathbf{x},t) \tag{2.11}$$

where $H(\cdot)$ represents the step function, with the Dirac function $\delta(\cdot)$ its first derivative.
Remembering that $H[-\theta(t)]=1-H[\theta(t)], H'[-\theta(t)]=\{1-H[\theta(t)]\}'=-\delta[\theta(t)], \delta[-\theta(t)]=\delta[\theta(t)]$ and $\delta'[-\theta(t)]=-\delta'[\theta(t)]$, after time derivation, the global velocity and acceleration fields can be inferred.

$$\begin{aligned}
&\mathbf{s}(\mathbf{x},t)=H[\theta(t)]\cdot\mathbf{s}^A(\mathbf{x},t)+H[-\theta(t)]\cdot\mathbf{s}^B(\mathbf{x},t)\\
&\dot{\mathbf{s}}(\mathbf{x},t)=\delta[\theta(t)]\cdot\dot\theta(t)\cdot\left[\mathbf{s}^A(\mathbf{x},t)-\mathbf{s}^B(\mathbf{x},t)\right]+H[\theta(t)]\cdot\left[\dot{\mathbf{s}}^A(\mathbf{x},t)-\dot{\mathbf{s}}^B(\mathbf{x},t)\right]+\dot{\mathbf{s}}^B(\mathbf{x},t)\\
&\ddot{\mathbf{s}}(\mathbf{x},t)=\delta'[\theta(t)]\cdot\dot\theta^2(t)\cdot\left[\mathbf{s}^A(\mathbf{x},t)-\mathbf{s}^B(\mathbf{x},t)\right]+\delta[\theta(t)]\cdot\left\{\ddot\theta(t)\cdot\left[\mathbf{s}^A(\mathbf{x},t)-\mathbf{s}^B(\mathbf{x},t)\right]+2\,\dot\theta(t)\cdot\left[\dot{\mathbf{s}}^A(\mathbf{x},t)-\dot{\mathbf{s}}^B(\mathbf{x},t)\right]\right\}+H[\theta(t)]\cdot\left[\ddot{\mathbf{s}}^A(\mathbf{x},t)-\ddot{\mathbf{s}}^B(\mathbf{x},t)\right]+\ddot{\mathbf{s}}^B(\mathbf{x},t)
\end{aligned} \tag{2.12}$$

comparing the above reported equilibrium conditions for the counter-clockwise and clockwise motion, the global equilibrium conditions can be written in a similar form

$$\mathbf{F}_I(t)+\mathbf{F}_a(t)+\mathbf{F}_g(t)=0 \qquad \Rightarrow -\int_\Omega \mu(\mathbf{x})\ddot{\mathbf{s}}(\mathbf{x},t)dV-\int_\Omega \mu(\mathbf{x})\ddot{\mathbf{u}}_g(t)dV+\int_g \mu(\mathbf{x})\cdot\mathbf{g}dV=0 \tag{2.13}$$

$$\begin{aligned}
M_{I,O}(t)+M_{a,O}(t)+M_{g,O}(t)=0 \qquad &\Rightarrow -\int_\Omega \left|\left[\mathbf{x}+\mathbf{s}^B(\mathbf{x},t)\right]\times\left[\ddot{\mathbf{s}}^B(\mathbf{x},t)+\ddot{\mathbf{u}}_g(t)-\mathbf{g}\right]\mu(\mathbf{x})dV=0\\
&\Rightarrow -\int_\Omega\left[\ddot{s}_2^B(\mathbf{x},t)\cdot x_1+\ddot{s}_2^B(\mathbf{x},t)\cdot s_1^B(\mathbf{x},t)\right]\mu(\mathbf{x})dV+\int_\Omega\left[\ddot{s}_1^B(\mathbf{x},t)\cdot x_2+\ddot{s}_1^B(\mathbf{x},t)\cdot s_2^B(t)\right]\mu(\mathbf{x})dV+\\
&\quad+\ddot{u}_{g1}(t)\cdot\left[S_{1,O}+\int_\Omega s_2^B(\mathbf{x},t)\mu(\mathbf{x})dV\right]-\left[\ddot{u}_{g2}(t)+g\right]\cdot\left[S_{2,O}+\int_\Omega s_1^B(\mathbf{x},t)\mu(\mathbf{x})dV\right]=0
\end{aligned} \tag{2.14}$$

whence after substitutions of global displacements, velocity and accelerations, gives

$$\begin{aligned}
&\mathbf{F}_I(t)+\mathbf{F}_a(t)+\mathbf{F}_g(t)=0 \qquad \Rightarrow -\int_\Omega \mu(\mathbf{x})\ddot{\mathbf{s}}(\mathbf{x},t)dV-\int_\Omega \mu(\mathbf{x})\ddot{\mathbf{u}}_g(t)dV+\int_g \mu(\mathbf{x})\cdot\mathbf{g}dV=0\\
&\Rightarrow\begin{cases}2\delta'[\theta(t)]\cdot\dot\theta^2(t)\cdot\theta(t)\cdot S_{1,O}+2\delta[\theta(t)]\cdot S_{1,O}\cdot\left[\ddot\theta(t)\cdot\theta(t)+2\,\dot\theta^2(t)\right]+2H[\theta(t)]\cdot\ddot\theta(t)\cdot S_{1,O}-\ddot\theta(t)\cdot S_{1,O}=m\ddot{u}_{g1}(t)\\
2\delta'[\theta(t)]\cdot\dot\theta^2(t)\cdot\theta(t)\cdot S_{2,O}+2\delta[\theta(t)]\cdot S_{2,O}\cdot\left[\ddot\theta(t)\cdot\theta(t)+2\,\dot\theta^2(t)\right]+2H[\theta(t)]\cdot\ddot\theta(t)\cdot S_{2,O}-\ddot\theta(t)\cdot S_{2,O}+mb\ddot\theta(t)=m\left[\ddot{u}_{g2}(t)+g\right]\end{cases}
\end{aligned} \tag{2.15}$$

$$\begin{aligned}
&M_{I,O}(t)+M_{a,O}(t)+M_{g,O}(t)=0\\
&\Rightarrow -2\delta'[\theta(t)]\cdot\dot\theta^2(t)\cdot\theta(t)\left[(J_1+J_2)+\theta(t)bS_{1,O}\right]+\\
&\quad -2\delta[\theta(t)]\cdot\left\{\ddot\theta(t)\cdot\theta^2(t)bS_{1,O}+2\cdot\dot\theta^2(t)\cdot\theta(t)bS_{1,O}+\ddot\theta(t)\cdot\theta(t)(J_1+J_2)+2\dot\theta^2(t)(J_1+J_2)\right\}+\\
&\quad +2H[\theta(t)]\cdot\left\{-\ddot\theta(t)(J_1+J_2)+\theta(t)S_{2,O}\ddot{u}_{g1}(t)+\theta(t)S_{1,O}\left[\ddot{u}_{g2}(t)+g\right]\right\}+\\
&\quad +\ddot\theta(t)(J_1+J_2)+b\,\theta(t)m\ddot{u}_{g1}(t)+S_{1,O}\ddot{u}_{g1}(t)-S_{2,O}\left[\ddot{u}_{g2}(t)+g\right]+\\
&\quad -\ddot\theta(t)\cdot bS_{2,O}-\theta(t)\cdot S_{2,O}\ddot{u}_{g1}(t)+b\,\theta(t)m\ddot{u}_{g1}(t)+S_{1,O}\ddot{u}_{g1}(t)-\theta(t)\cdot S_{1,O}\left[\ddot{u}_{g2}(t)+g\right]-S_{2,O}\left[\ddot{u}_{g2}(t)+g\right]=0
\end{aligned} \tag{2.16}$$

where J_1 and J_2 represent the second order moments with respect to the reference axes.

3. SHAKING TABLE TESTS ON RIGID BLOCKS UNDER PURE ROCKING

3.1. Description of the Rocking Motion of the Rigid Block Reference Model

In the following one refers to experiments executed at the Laboratory of the Department of “Scienza delle Costruzioni” of the University of Naples “Federico II” (I.Corbi and R.Orefice, 2004), on a series of aluminium blocks placed on a unidirectional shaking table whose motion is purely transactional in the horizontal direction. The dimensions of the table plane are 1.0m×1.0m.

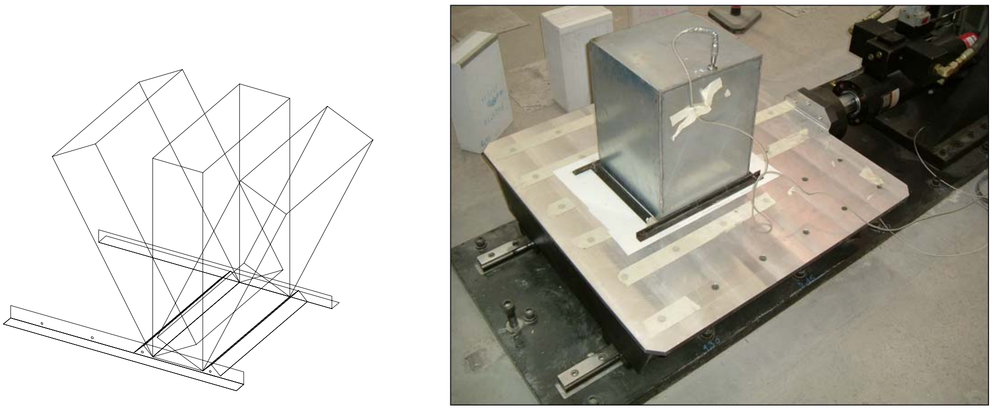


Figure 5. Pure rocking motion of the blocks during experiments.

The whole prototype was fixed in such a way to have the impact or pivotal points between the block and the base plane at well defined positions, which resulted in the two requirements that no sliding of the model should occur on the base and the blocks should be sufficiently stiff such to be considered rigid. Under these assumptions, only rotations around the two base edges of the blocks were permitted (Fig. 5) according to the motion of the shaking table.
Experiments were executed on aluminum parallelepipeds of various sizes and dimension ratios (width/height), monitored by means of an accelerometer placed on the top of the blocks in order to read their accelerations.
The experiments were executed (I.Corbi and R.Orefice, 2004; Baratta et al., 2006), for different spans and by varying the frequency value of the sine base-excitation inferred by the shaking table between 1 Hz and 50 Hz with intervals of 1 Hz.
As an example, one reports in Figure 6 , some results of the experimental investigation concerning two blocks. The diagrams depict peak accelerations (m/s^2) of the blocks versus frequencies (of the unidirectional base acceleration) (Hz) for various span levels.

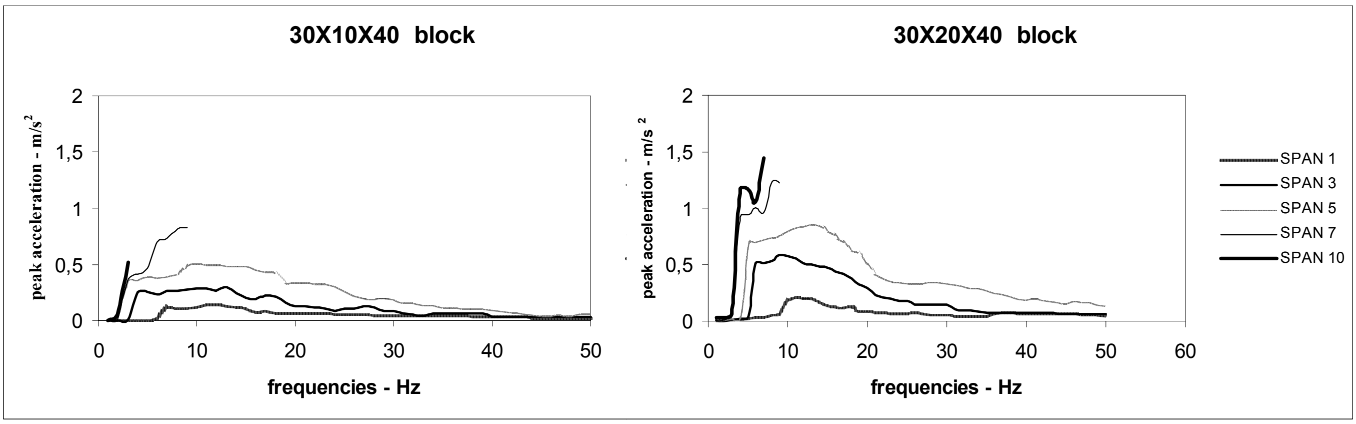


Figure 6. Peak accelerations versus frequencies for different spans.

One can notice that the rocking motion begins after a frequency of 4 – 6 Hz. Since the span is kept fixed, the value of the input peak acceleration increases with the square of the frequency.
Generally speaking, peak accelerations were higher on stockier and apparently more stable blocks, showing an unexpected effect which made the larger of two geometrically similar blocks less stable than the smaller one.

ACKNOWLEDGEMENTS

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CONCLUSIONS

The paper gives a general overview and set up of the theoretical problem of study of dynamics of rigid blocks under pure rocking. Such dynamics are not at all trivial, because of the non linear nature of the model due to the unilateral constraint, and also because they are deeply affected by the impact of the block against the basement, which requires some treatment and should be embedded in the fundamental dynamic equilibrium conditions. Some experimental investigation is summarized, as well, for illustrating some peculiar characters of the rocking motion on such models.

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