Suppression Of Vehicle-Induced Bridge Vibration By Using Tuned Mass Damper*

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ABSTRACT

The importance of bridge vibrations induced by moving vehicles, which act as oscillators on a bridge as well as time variant forces, has long been recognized. This vibration can amplify the propagation of existing cracks resulting in further damage to the bridge. It has become one of the causes of reduction in long-term serviceability of the bridge, although major bridge failures are not usually caused directly by moving vehicles. It is also a critical factor to bridge structure fatigue and rapid deterioration since, the vehicle-induced vibration is more critical to bridges with medium to small span; it is worthwhile to investigate the possibility of applying Tuned Mass Damper (TMD) on these bridges. A TMD is a passive type control device with variety of merits in that it has permanent service time and only requires easy management and maintenance efforts and no external power supply source. In order to achieve the above objective, a general formulation of the vehicle-induced bridge vibration controlled with a TMD system is developed here in this paper, which takes into account the road surface conditions. Then, a comprehensive investigation is made to investigate the efficiency of the TMD for suppressing vibrations of bridge under moving vehicles. Such a study is helpful in evaluating the control performance before real control devices are designed in practice. These analytical results will also be useful in carrying out further studies for control strategies suppressing the vehicle-induced bridge vibration

Keywords: Moving Oscillator, Tuned Mass Damper, Mode superposition technique, Runge-Kutta method.

INTRODUCTION

The new civil engineering structures are becoming quite strong but very flexible since the increase in the stiffness modulus of the new construction materials lag behind the increase in their strength. Using lighter and flexible construction materials also results in human discomfort and sometimes, unsafe conditions. Developments in design technology and material qualities in civil engineering enable the construction of more light and slender structure, which are vulnerable to variety of dynamic loads (winds, waves, earthquakes, moving load, etc.) that significantly affect the safety and serviceability of structure, material contents and human occupants thus have been a matter of great concern for a very long time.

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that significantly affect the safety and serviceability of structure, material contents and human occupants thus have been a matter of great concern for a very long time.

The extent of protection required for these structures may range from reliable operation and occupancy comfort to human and structural survivability. The standardized codes have given through a number of modifications as the aftermath of major events such as earthquake,
cyclone, etc. and have made these revisions essential. Though code revisions promise better safety in future structures provided the code specifications are followed properly, yet the stability of the structures already constructed following old codes of practice are in real threat. Therefore, control of the civil engineering structures such as tall buildings, long bridges and towers has generated much interest to be able to solve some of these problems.

Thus, vibration induced by heavy moving loads may significantly increase the maximum internal stresses of bridges in excess of those originally assumed by designer. As a result many bridges suffer a sharp decrease in service life and the existing are approaching the end of their service life, and even endanger the safety of supporting structure and therefore require extensive repairs and/or replacement unless other ways are found to reduce stresses and strains due to external loads (Earthquake, Wind, Moving Loads, etc.) and to sustain the safety of the bridges. One method of reducing the vibration of structures is to add an energy dissipative system to the primary structure to control the dynamic response. The **T**uned **M**ass **D**amper (**TMD**), which is a secondary vibration system connected to the primary structure at suitable points, is a classical device to dissipate a substantial amount of vibration energy of the main structure. A typical TMD generally consists of a mass, a spring, and a dashpot. Since Den Hartog firstly investigated the optimum values of TMD parameters using a two-degree of freedom model in 1950s, the TMDs have been extensively studied and applied to suppress vibrations of buildings and bridges. It is well known that the TMD is effective in suppressing the single-mode resonant vibration when its frequency is tuned to the modal frequency of the structure [(22), (10)]. Much of research efforts were focused on developing the design procedure and optimizing the TMD parameters. Although excessive studies [(8), (9), (24)] have been conducted on TMDs for suppressing vibration of structures under wind and seismic loads, little research has been done on applying TMDs to control vehicle-induced bridge vibration (28).

Here in this paper, an attempt is made to study the possible effectiveness of TMDs for suppressing vibrations of bridges under vehicle loads. In order to achieve this objective, a general formulation of the vehicle-induced bridge vibration controlled with a TMD system is first developed, which takes into account the road surface conditions. Then, a comprehensive investigation is made to investigate the efficiency of the TMD for suppressing vibrations of different bridges under two vehicle load patterns, i.e. two trucks moving side by side and several trucks passing over the bridge in a traffic flow. Such a study is helpful in evaluating the control performance before real control devices are designed in practice. These analytical results will also be useful in carrying out further studies of control strategies in order to suppress the vehicle-induced bridge vibration.

VEHICLE-BRIDGE WITH TMD COUPLED SYSTEM

The present study developed a fully computerized approach to simulate the interaction of series of coupled vehicles with bridge system installed with a passive TMD at a desired position. . The vehicle model used in this study is a so-called 3-D suspension model (or full truck model) that includes a combination of vehicle bodies represented by several rigid bodies connected by both primary and secondary vehicle suspension systems (Fig. 1). The tires and suspension systems are idealized as linear elastic spring elements and dashpots. The contact between the bridge deck and the moving tire is assumed to be a point contact. The model can be used to simulate vehicles on highway roads or bridges with axle number varying from two to five. This is a more realistic model because it incorporates pitching, rotating, and yawing motions of the vehicle and also the variation of the axle force on the tires of each axle (7). The TMD with one vertical degree of freedom consist of a rigid mass with a suspension system idealized as elastic spring element and dashpot. The proposed approach uses the direct integration method to treat the interaction by updating the characteristic matrices according to the position of contact points of vehicle at each time step and the TMD position permanently at single Desired location. Therefore, the equations of motion are time dependent and they should be modified, updated, and solved by the Runge-Kutta method (5) at each time step. The road surface roughness $r(x)$ and the separation of the vehicle tire from the bridge due to large irregularities of the road surface is also taken into consideration in the analysis. In the present study, the vehicle-bridge coupled with TMD problem is firstly characterized by three sets of differential equations of motion, one for the bridge, the other for the vehicle and one that of TMD. Then the combined systems are coupled through the contact condition. For demonstration purposes a 3-axle articulated truck consisting of up to 11 independent degrees of freedom is shown in Fig.1. The equation of motion for the vehicle is derived based on the following matrix form

$$[M_v]\{\ddot{d}_v\}+[C_v]\{\dot{d}_v\}+[K_v]\{d_v\}=\{F_v^G\}+\{F_c\} \tag{1}$$

Where the vehicle mass matrix $[M_v]$, damping matrix $[C_v]$, and stiffness matrix $[K_v]$ are obtained by considering the equilibrium of the forces and moments of the vehicle system; $\{d_v\}$ is the displacement vector of vehicle; $\{F_v^G\}$ is the self-weight of the vehicle; and $\{F_c\}$ is the vector of wheel-bridge contact forces acting on the vehicle.

The bridge is modeled using the conventional finite element method, and the equations of motion for the bridge can be expressed as

$$[M_b]\{\ddot{d}_b\}+[C_b]\{\dot{d}_b\}+[K_b]\{d_b\}=\{F_b\}+\{F_T\} \tag{2}$$

Where $[M_b]$ is the bridge mass matrix, $[C_b]$ is the damping matrix, and $[K_b]$ is the stiffness matrix; $\{F_b\}$ is the wheel-bridge contact force on the bridge; and $\{F_T\}$ is the interacting force between the TMD and the bridge. The force exerted on the bridge by the vehicle acts at the very point of the vehicle passage and the inertia force by the

TMD at one single permanent location.

The equations of motion for the vehicle and bridge are coupled through the interaction forces, i.e. $\{F_b\}$ and $\{F_c\}$. $\{Fb\}$ and $\{F_c\}$ are action and reaction forces existing at the contact points of the two systems and are expressed as a function of the deformation of the vehicle's lower springs $\{\Delta_l\}$ (42):

$$\{F_b\} = -\{F_c\} = [K_l](Z_a - Z_b - r(x)) + [C_l](\dot{Z}_a - \dot{Z}_b - \dot{r}(x)) \tag{3}$$

in which $r(x)$ is the road surface profile of the bridge deck (Fig. 1) and it can be simulated using reversed Fourier transformation.

In which $\dot{r}(x) = \frac{dr(x)}{dx}\frac{dx}{dt} = \frac{dr(x)}{dx}V(t)$ and $V(t)$ is the vehicle velocity; Z_a is vehicle axle suspension displacement in vertical direction, and Z_b is the displacement of bridge at wheel-road contact points. The TMD is installed at a desired position of the bridge where its response becomes a maximum. The equations of motion, which represent the interaction between the TMD and the bridge, are

$$[M_T]\ddot{d}_T + [C_T](Z_T - Z_b) + [K_T](Z_T - Z_b) = 0 \tag{4}$$

Where $[M_T]$ is the mass of TMD, $[C_T]$ is the damping of TMD, $[K_T]$ is the spring coefficient, and Z_T is the displacement of TMD. At time t, the interacting force, $\{F_T\}$, between bridge and TMD is

$$\{F_T\} = [K_T](Z_T - Z_b) + [C_T](\dot{Z}_T - \dot{Z}_b) \tag{5}$$

Firstly, the design of damper and spring element is carried out under the theory of classical TMD optimization, which determinates optimal tuning ratio of frequency and damping ratio: A lot of proposed tuning conditions are available for TMD, but Den Hartog's (1962) optimum tuning conditions are most often used as suggested are given below

$$\varepsilon_T = \frac{M_T}{M_b} \quad ; \quad \omega_T = \frac{\omega_n}{1+\varepsilon_T} \quad ; \quad \left(\frac{C_T}{C_c}\right)^2 = \frac{3\varepsilon_T}{8(1+\varepsilon_T)^3} \, ; \quad C_c = 2M_T\omega_n$$

Where C_T and c_c are damping values corresponding to the TMD and critical damping values, respectively. The mass ratio ε_T, the mass of the bridge M_b, natural frequency of the fundamental mode ω_n .The dynamic responses of structures are affected more significantly by the TMD damping values than by the damping values of structures, one can use the critical damper damping proposed by Tsai (1993), which avoids beating phenomenon is given as below:

$$\xi_T = \xi_n + \sqrt{\varepsilon_T}$$

In this paper, Den Hartog's frequency tuning condition and Tsai's critical damper damping value function of modal damping ratio ξ_n, and mass ratio ε_T are adopted to achieve maximum vibration control efficiency. By substituting Eq. (3) and Eq. (5) into Eq. (1) and Eq. (2), the final equations of motion of bridge, TMD, and vehicle system can be rewritten in matrix form as in the following equation.

$$\begin{bmatrix} M_b & & \\ & M_T & \\ & & M_v \end{bmatrix}\begin{Bmatrix} \ddot{d}_b \\ \ddot{d}_T \\ \ddot{d}_v \end{Bmatrix} + \begin{bmatrix} C_b + C_{bb} + C_T & C_{bT} & C_{bv} \\ C_{Tb} & C_T & 0 \\ C_{vb} & 0 & C_v \end{bmatrix}\begin{Bmatrix} \dot{d}_b \\ \dot{d}_T \\ \dot{d}_v \end{Bmatrix} + \begin{bmatrix} K_b + K_{bb} + K_T & K_{bT} & K_{bv} \\ K_{Tb} & K_T & 0 \\ K_{vb} & 0 & K_v \end{bmatrix}\begin{Bmatrix} d_b \\ d_T \\ d_v \end{Bmatrix} = \begin{Bmatrix} F_b^r \\ 0 \\ F_v^r + F_v^G \end{Bmatrix} \tag{6}$$

The additional terms C_{bb}, C_{bv}, C_{vb}, K_{bb}, K_{bv}, K_{vb}, and in Eq. (6) are due to the expansion of the contact force vector expressed by Eq. (3) and Eq. (5). As a vehicle passes over a bridge not only the position but also the magnitude of the contact force are changing with respect to time. This is caused by the fact that the position the vehicle, the response of bridge and vehicle, and the road roughness *r(x)* at the wheel and bridge deck contact points no longer remain the same. The change of contact force with time indicates that the additional terms in Eq. (6) are time-dependent and will change as the vehicle moves across the bridge. To simplify the modeling procedure, the bridge mode superposition technique is used based on the obtained bridge mode shapes and the corresponding natural circular frequencies. Since the ratio of the TMD mass to the bridge mass is assumed to be very small, the attachment of TMD does not cause a meaningful change to the static equilibrium of the bridge, and the mode shapes of the bridge remain the same as those of the original bridge without the TMD. The mode superposition makes it possible to separate the bridge modal analysis from the vehicle-bridge coupled model. Consequently, the number of equations in Eq. (6) and the complexity of the entire procedure are greatly reduced. A MATLAB program is developed based on the above methodology. The mass matrix, stiffness matrix, and damping matrix of vehicle and bridge are automatically assembled using the fully computerized approach. The equations of motion are solved in time domain by using the Runge-Kutta method. By solving the equations, the dynamic response of the vehicle, bridge, and TMD can be obtained in time history.

NUMERICAL ANALYSIS AND DISCUSSION

Two numerical examples of different truckload cases, i.e. two trucks moving side by side and several trucks moving one following another, are investigated by using the developed method. Different bridges are also considered to study the effectiveness of TMD. Three typical simply supported rectangular short span concrete slab bridge of normal range of span 6m of 0.30m slab thickness, 8m of 0.40m slab thickness and 10m of 0.50m slab thickness and slab-on-girder bridges, with span lengths of 16.76 m (55 ft), 24.38 m (80 ft) and 30.48 m (100 ft) designed for HS20-44 loading are investigated in this paper. The slab-on-girder bridges consist of seven girders that are simply supported with girder spacing of 2.13 m (7 ft). The bridges have a roadway width of 14.32 m (47 ft) and a bridge deck thickness of 0.20 m (8 in). The primary data of the bridges and the first modes of each bridge are listed in Table 1. The damping ratio is assumed to be 0.02. The AASHTO HS20-44 truck is used in the numerical analysis and its sketch is shown in Fig. 1. The geometry, mass distribution, damping, and stiffness of the tires and suspension systems of this truck are listed in Table 2 (37). The static wheel loads for the first, second, and third axle are 17.8 KN, 71.2 KN, and 71.2 KN, respectively, which make the total weight of this truck 320 KN. Modal frequencies of the vehicle are calculated as 1.52, 2.14, 2.69, 5.94, 7.74 and17.95 Hz. The truck was assumed to pass a step up of 0.0381 m, which is used to simulate the differential faulting between the bridge deck and approach slab (6), and then move on a smooth surface of bridge deck at a constant speed of 20 m/s (45mph). Since the bridges considered in the present study have two traffic lanes, two vehicles, one in each lane, is considered at any given moment. In this case, two trucks are assumed to move side by side when passing over bridges, which agrees also with the critical live load specifications of AASHTO codes [(1), (2)]. According to the preliminary analyses, the bridge's first mode is dominant in the dynamic response for all eight bridges. To reduce the bridge dynamic response, it is desirable to tune the TMD to the fundamental frequency (the first frequency) of the bridge. The TMD is positioned at the center of the bridge (Fig. 1) where the first mode response is at the maximum. The mass ratio of 1% is selected in this study, though ratios between 1% and 5% have generally been used in other studies [(34), (29), (24)]. Fig. 2 and Fig. 3 display the time history of the deflection of all the bridges with and without TMD. It is observed that the maximum dynamic deflection when the vehicle is on the bridge (forced vibration period) is slightly reduced by TMD. For instance, the maximum dynamic deflections of the 8 m slab bridge and the 30.48 m girder bridge without TMD are 5.42 mm and 6.90 mm, respectively. After the installation of the TMD, the deflections of these two bridges are reduced to 5.16 mm and 6.99 mm, respectively. This indicates that the reducing effect of the forced vibration is only 4.86% for the 8 m slab bridge, and 0.60% for the 30.48m girder bridge. It is evident that the TMD does not significantly suppress the forced vibration in these bridges. However, it is obvious from Fig. 2 and Fig. 3 that the vibration level of all bridges is greatly reduced during the free vibration period, which means the TMD is effective in reducing the free vibration although it is difficult to control the forced vibration. The forced vibration for the passage of two side-by-side trucks is very short for the bridges considered in this study. In this short period of passage of vehicle, the bridge only vibrates for very few cycles (Fig. 2 and Fig.3). Although the TMD increases the overall damping of the bridge, it needs time to respond to the vibration before it can effectively absorb vibration energy from the main structure and then suppress the vibration. Table 3 summarizes the reduction of displacement for different bridges, which displays that the suppression effect for the shorter bridges (B1, B2, B3, and B4) is generally better than that for relatively longer bridges (B5 and B6). This is probably due to the fact that the vibration of the shorter bridges is more active (i.e., with higher frequencies) than the longer bridges. Since the fundamental frequency of B1 to B4 is relatively high, during the forced vibration period, there are more excited vibration cycles in the TMDs for the shorter bridges than for the longer bridges. On the other hand, compared with B5 and B6, the multi-axle truck loads applied on the relatively short bridge (B1, B2, B3, and B4) can be considered as repeated loads (similar to the train load on short bridges) although the number of axles of the truck is small, because the bridge span is short compared to the 4.26 m spacing of the HS20 truck axles. Whereas, for longer bridge, since the bridge span is relatively long compared to the axle spacing there is only one half-cycle of loading as the truck crosses the span. Thus, the loading frequency for shorter bridge is higher than that of the longer bridge, which causes more actively vibration in the shorter bridges.

CONCLUSIONS

The vehicle-induced bridge vibration may affect the durability of the structure and the safety and comfort of passengers. It also can lead to deterioration and reduction in service life of the bridge. Although the major bridge failures are not normally caused by vehicle-induced vibration, it causes more subtle problems and contributes to fatigue, surface wear, and cracking of concrete deck and beams, which leads to corrosion. In this study, the TMD is investigated for the purpose to suppress the vehicle-induced vibration of bridges by a finite element approach. Based on the numerical analyses of short and mediate bridges, the following conclusions can be drawn: A conventional TMD control approach usually focuses on suppressing the resonant vibration by supplying additional damping to the concerned modes. However, in the case of two trucks passing the bridge side by side, it was found that the addition of damping provided by the TMD does not result in an appreciable reduction of the maximum dynamic displacement during the forced vibration period (i.e. when the vehicle is on the bridge) due to the reason that the forced vibration period is too short and that the TMD does not have enough time to respond. Although this approach could be inefficient for forced vibration, it is evident from the analysis results that the TMD is effective in reducing the vibration level in free vibrations. On the other hand, for all the bridges investigated in this study, the reduction of acceleration is larger than that of the displacement.

It is emphasized that the performance of TMD may be influenced by the dynamic characteristics of bridge vibrations induced by moving vehicles. The properties of the bridge, the moving oscillator and the TMD were chosen to simulate the maximum interaction between a bridge and a vehicle. When the natural frequency of the vehicle matches that of the bridge, the bridge behaves like a vibration absorber. As is seen in TMD, the first mode is greatly reduced, where second and third, higher modes remain nearly unchanged. This is due to the fact that the TMD control basically minimized the RMS responses of the structure due to the broad bound excitations. Thus the first mode will be the largest contributions to the RMS response and will be primarily modified to reduce the RMS response. The decay of vibration is rapid in all control cases, so much as that the vibration level is almost zero by the time the vehicle moved off the bridge and effectively reduces the duration of exposure. This effect is important for two reasons; it keeps vibration level down when the bridge is excited by a continuous stream of vehicles and because annoyance is function of both vibrational level and duration. Although not specifically pursued by the TMD systems, important reductions in the vehicle relative displacement were achieved Similar to the case of the vehicle's displacement, accelerations of the bridge were greatly reduced with respect to the uncontrolled case, even though the TMD systems were not focused on reducing this aspect of the response.

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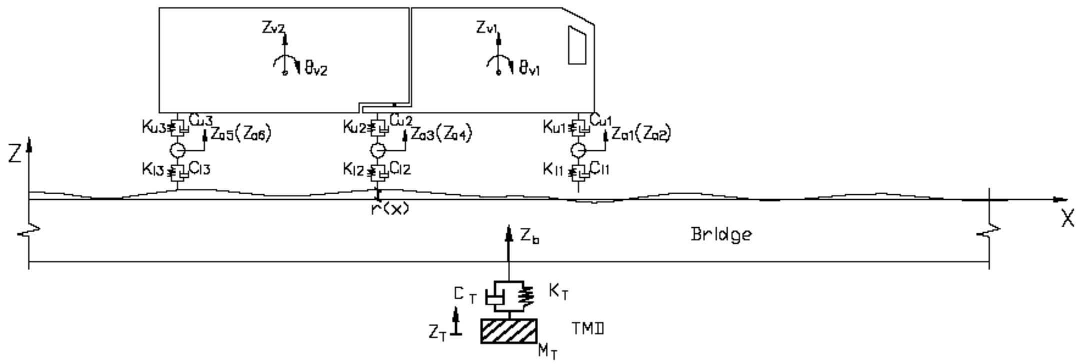


Fig 1A: Dynamic Interaction Analytical Model Of The Vehicle-Bridge System - Longitudinal View.

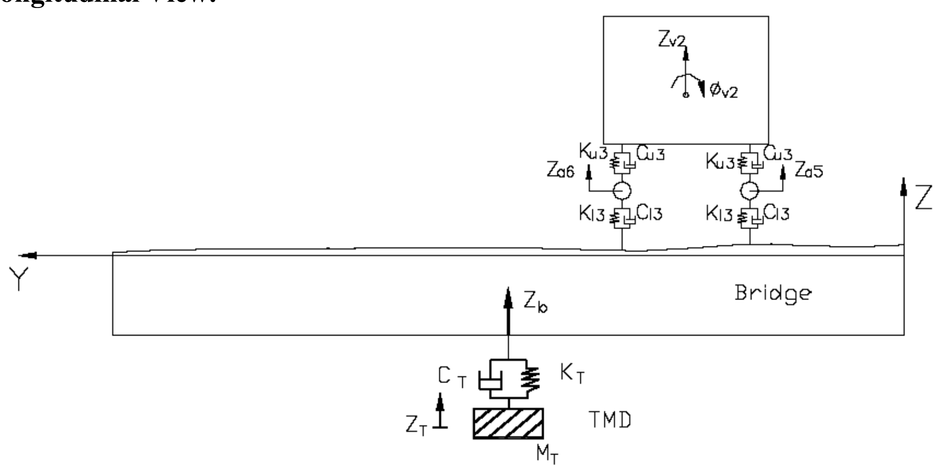
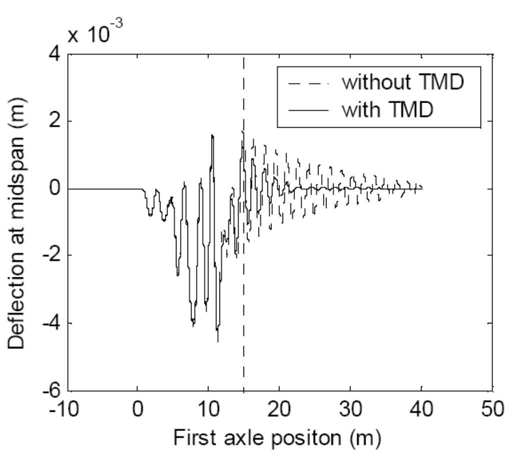
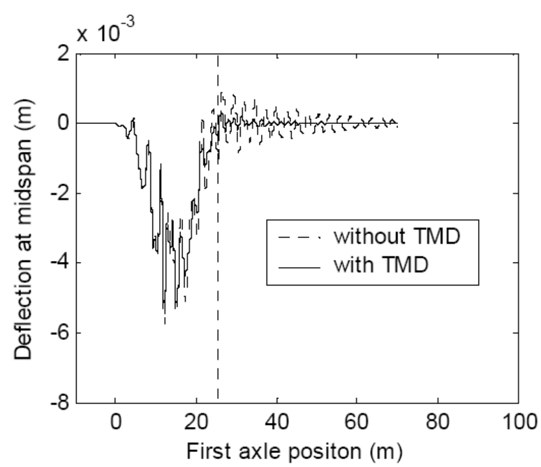


Fig 1B: Dynamic Interaction Analytical Model Of The Vehicle-Bridge System - Sectional View.



(a) L = 6 m



(a) L = 16.76 m

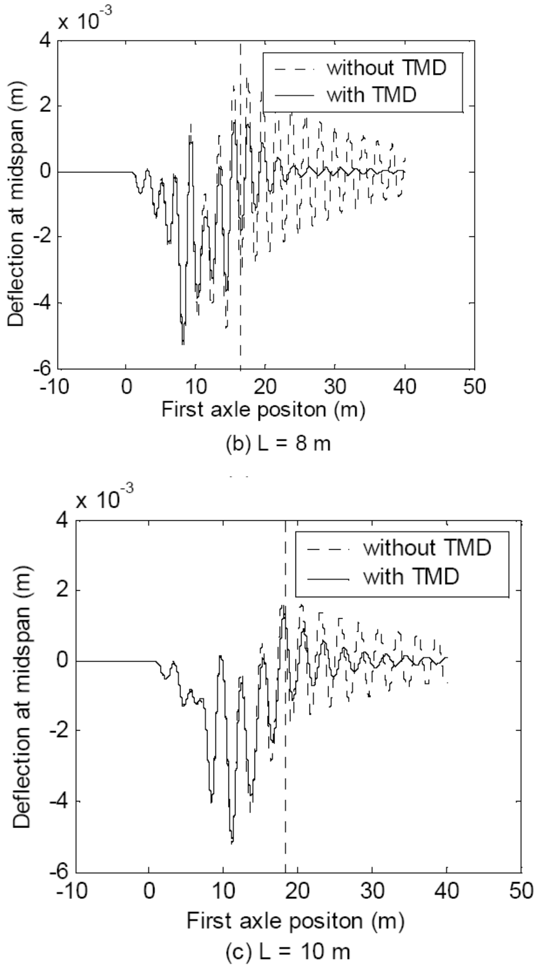


Fig. 2: Time History Of Deflection At Mid-Span Of Slab Bridges With And Without TMD

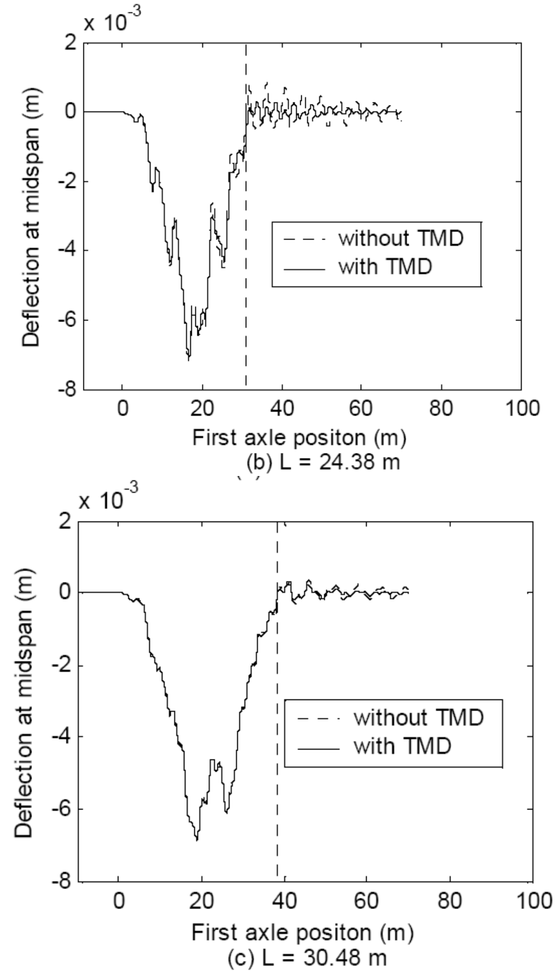


Fig. 3: Time History Of Deflection At Mid-Span Of Girder Bridges With And Without TMD