

## NONLINEAR SEISMIC RESPONSE OF ISOLATED BRIDGES WITH LRB

X.L. Du<sup>1</sup> and Q.Han<sup>1,2\*</sup> J.D.Zhan<sup>1</sup>

<sup>1</sup> Professor, The Key Lab. Of Urban Security and Disaster Engineering of Ministry of Education, Beijing University of Technology, Beijing, China

<sup>2</sup> Dept. of Civil Engineering, Tsinghua University, Beijing, China

\*Email: qhan@yahoo.cn

### ABSTRACT :

In order to investigate the nonlinear seismic responses of isolated bridges with lead rubber bearings (LRB) under bi-directional horizontal earthquake excitation, bi-directional Bouc-Wen hysteretic model of LRB was adopted to describe the constitutive law of devices. Based on the bi-directional coupled Bouc-Wen model of LRB, an analytical model of nonlinear seismic responses of continuous multi-span isolated bridges with LRB and its solving method were carried out. Multi-degrees-of-freedom mathematical model of seismically isolated bridge is right and bi-directional coupled Bouc-Wen model of LRB is reasonable by shaking table tests of continuous girder isolated bridges model with LRB. Experiment results agreed expectably well with the results obtained from theoretically analytical results of displacement and acceleration of deck, vertical force of LRB, displacement and force-displacement of isolator roughly. It is verified that computation method of seismically isolated bridge given this paper is right and effective when analyzing nonlinear earthquake response of continuous girder isolated bridges with LRB. It should consider the bi-directional coupled interaction of the restoring forces of LRB, which has considerable effects on the seismic responses of the isolated bridge. Vertical tension of LRB should be considered if vertical earthquake amplitude is greater in isolator design.

**KEYWORDS:** isolated bridges, nonlinear seismic response, shaking table test, lead rubber bearings

### 1. INTRODUCTION

There are two modern design approaches intended for reducing destructive effects on bridge structures caused by strong earthquakes. One is used isolation device, which is a strategy that attempts to reduce the seismic forces to or near the elastic capacity of the structural member, thereby eliminating or reducing the inelastic deformations. The main concept in isolation is to reduce the fundamental frequency of structural vibration to a value lower than the predominant energy-containing frequencies of the earthquake. The other is used energy dissipation device or increased damping, which is reducing the amount of seismic energy input into the structure. In this way it is possible to limit large plastic deformations caused by the natural period enlargement. This stiffness decreases because of higher elongations due to nonlinear response, decoupling deck from pier under strong seismic events, and then increasing protection efficiency.

The lead-rubber bearing (LRB) is the well-developed seismic isolation devices for practical use. There had been several studies in the past investigating the aseismic design of isolated bridges with LRB. Li (1989) and Pagnini and Solari (1999) studied the stochastic response of a typical three-span bridge structure with the seismic isolation system consisting of rubber bearings and hysteretic dissipaters using the equivalent linearization technique. Hwang and Chiou (1996) and Hwang et al. (1996) established an equivalent linear model for the seismic analysis of base-isolated bridges with lead-rubber bearings using an identification method. Ghobarah and Ali (1998) and Turkington et al. (1989) showed that the LRB is quite effective in reducing the seismic response of bridges. Saiidi et al. (1999) studied the effectiveness of seismic isolators in reducing the force and displacement of the superstructure of a six-span bridge, and found that the use of isolators does not necessarily increase the displacement of the superstructure. Tan and Huang (2000) developed. Jangid (2004) analyzed the seismic response of isolated bridges by LRB under bidirectional earthquake excitation regarding the restoring forces relations in two orthogonal horizontal directions as Park model.

The most previous research is conducted assuming the force deformation behavior of the LRB as bilinear

with a single component of earthquake excitation. However, when the system is subjected to bidirectional excitation, the assumption of bilinear behavior may not be valid. As a result, there is a need to study the behavior of bridges isolated with LRB. In this paper, based on the two horizontal orthogonal directional coupled Park hysteretic model of LRB, an analytical model of nonlinear seismic responses of continuous multi-span isolated bridges with LRB and its solving method were carried out under bi-directional horizontal earthquake excitation. Furthermore, to verify MDOF mathematical model of seismically isolated bridge presented in this paper is right, bi-directional coupled Park hysteretic model of LRB is reasonable, and considering bi-directional interaction of restoring forces of LRB to assess the effects of seismic isolation on the peak response of the bridges under earthquake excitation is necessary, shaking table tests of continuous girder isolated bridges model with LRB were carried out.

## 2. MODELING

### 2.1. Mechanical Modeling of Isolation bearings

The uniaxial Bouc-Wen model (Wen 1975,1976) is used extensively in random vibration studies of inelastic system. Casciati (1989) considered Bouc-Wen model as a smoothed form of the rate independent plasticity model and generalized it to the bi-directional case. The Casciati model is considered as a smoothed form of the rate-independent plasticity model. The horizontal restoring force of isolation bearings,  $F = [F_x, F_y]^T$  consists of an elastic-hardening component and a hysteretic component given by

$$F = K_2 u + F_p \quad (2-1)$$

where  $K_2$  is the post-yield hardening stiffness,  $u = [u_x, u_y]^T$  is the translational deformation and  $F_p$  is the hysteretic force (the assumption of restoring force depends only on translational shear deformation of isolation bearings). The yield surface,  $\Phi(F_p)$  is assumed to be a circular interaction surface,

$$\Phi(F_p) = \|F_p\| - Q_D \quad (2-2)$$

where is  $Q_D$  the zero-displacement force intercept. The hysteretic  $F_p$  can be computed from the constitutive equation:

$$\dot{F}_p = (K_1 - K_2)(\dot{u} - \dot{u}_p) \quad (2-3)$$

Where  $K_1$  is the pre-yield elastic stiffness,  $\dot{u}_p$  is the plastic displacement increment.  $\dot{u}_p$  is governed by associative plastic flow rule:

$$\dot{u}_p = \gamma \cdot \frac{\partial \Phi(F_p)}{\partial F_p} = \gamma \cdot \frac{F_p}{\|F_p\|} \quad (2-4)$$

Where  $\gamma \geq 0$  is the plasticity multiplier. The Kuhn-Tucker loading/unloading condition (Simo and Hughes 1998) is

$$\gamma \geq 0, \quad \Phi(F_p) \leq 0, \quad \gamma \cdot \Phi(F_p) = 0 \quad (2-5)$$

The consistency condition is satisfied as:

$$\gamma \cdot \dot{\Phi}(F_p) = \gamma \frac{F_p^T}{\|F_p\|} = \gamma^T \dot{F}_p = 0 \quad (2-6)$$

The return-mapping algorithm for plasticity proposed by Simo and Hughes (1998) is used to compute the restoring force  $F$  for an isolation bearing under a given displacement history  $u$ .

During a plastic regime, the rate of plastic force is

$$\dot{F}_p = (K_1 - K_2)(\dot{u} - \dot{u}_p) = (K_1 - K_2)\dot{u} - \left( (K_1 - K_2) \cdot \frac{F_p F_p^T}{\|F_p\|^2} \right) \dot{u} \quad (2-7)$$

Equation (2-7) can be written as

$$\dot{F}_p = (K_1 - K_2)\dot{u} - \left( (K_1 - K_2) \cdot \frac{F_p^T \dot{u}}{\|F_p\|^2} \right) F_p \cdot H(\Phi) \cdot H(\dot{\Phi}) \quad (2-8)$$

where  $H(\cdot)$  is the Heaviside function,  $H(\Phi)$  can be approximated with a smoothed function:

$$H(x) = \begin{cases} 0 & x < 0 \\ 1 & x > 0 \end{cases} \quad (2-9)$$

$$H(\Phi) = H(\|F_p\| - Q_D) \approx \frac{\|F_p\|^\eta}{Q_D^\eta} \quad (2-10)$$

Where  $\eta \geq 0$ ,  $H(\dot{\Phi})$  is defined as

$$H(\dot{\Phi}) = H(F_p^T F_p) = \frac{1 + \text{sgn}(F_p^T \dot{u})}{2} \quad (2-11)$$

The rate of plastic force is approximated with

$$\dot{F}_p = (K_1 - K_2)\dot{u} - \frac{\|F_p\|^{\eta-2}}{Q_D} \cdot (K_1 - K_2)(F_p^T \cdot \dot{u}) \cdot \frac{1 + \text{sgn}(F_p^T \dot{u})}{2} F_p \quad (2-12)$$

By defining a dimensionless plastic variable  $Z$ , such that  $F_p = Q_d Z$  and uni-directional yielding displacement

$$u^Y = Q_D / (K_1 - K_2) \quad (2-13)$$

Equation (2-12) becomes

$$\dot{Z}u^Y = \dot{u} - \frac{1}{2}\|Z\|^{\eta-2} [1 + \text{sgn}(Z^T \dot{u})] (ZZ^T) \dot{u} \quad (2-14)$$

Casciati model can be rewritten in a more general form

$$\dot{Z}u^Y = A\dot{u} - \|Z\|^{\eta-2} [\gamma + \beta \text{sgn}(Z^T \dot{u})] (ZZ^T) \dot{u} \quad (2-15)$$

where  $A$ ,  $\gamma$ ,  $\beta$  are coefficients that control the shape of the hysteretic loop of Bouc-Wen model. By choosing  $A = 1$  and  $\gamma = \beta = 0.5$ , the model is the smoothed form of the rate-independent plasticity model with a circular interaction surface. The coefficient,  $\lambda$ , and  $\beta$ , govern the unloading force-deformation relation. The parameter

$\eta$  governs the transition from the elastic regime to the plastic regime.

## 2.2. Modeling of the Isolated Bridge System

The assumptions are made for nonlinear seismic response analysis of multi-span continuous deck supported by isolation bearings as follow:

(1) The bridge superstructure and piers are assumed to remain in the elastic state during the earthquake excitation, and the abutments of the bridge are assumed to be rigid. The bridge piers are assumed to be rigidly fixed at the foundation level and without considering the SSI effects.

(2) The superstructure and substructure of the bridge are modeled as a lumped mass system divided into a number of small discrete segments. Each adjacent segment is connected by a node, and at each node two degrees of freedom are considered. The masses of each segment are assumed to be distributed between the two adjacent nodes in the form of point masses.

(3) The force-deformation behavior of isolation bearings is regard as bi-directional Bouc-Wen hysteretic model under two horizontal directional excitation.

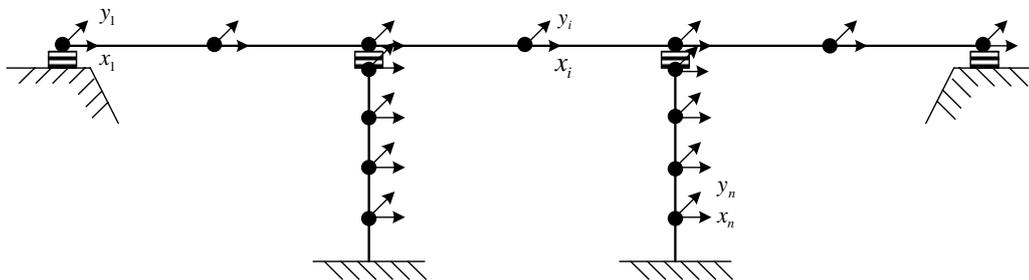


Figure1 Multi-degrees-of-freedom mathematical model of isolated bridge system

According to the above assumptions, the mathematical model of the isolated bridge system is shown in Figure1.

## 3. EQUATIONS OF MOTION

The equations of motion of the isolated bridge model are expressed in the following matrix form under two horizontal components of earthquake ground motion:

$$[M]\{\ddot{z}\} + [C]\{\dot{z}\} + [K]\{z\} + [D]\{F\} = -[M][r]\{\ddot{z}_g\} \quad (3-1)$$

$$\{z\} = \{x_1, x_2, x_3, \dots, x_N, y_1, y_2, y_3, \dots, y_{1N}\}^T \quad (3-2)$$

$$\{\ddot{z}_g\} = \begin{Bmatrix} \ddot{x}_g \\ \ddot{y}_g \end{Bmatrix} \quad (3-3)$$

where  $[M]$ ,  $[C]$ ,  $[K]$  represent the mass, damping, and stiffness matrices, respectively, of the bridge structure of order  $2N \times 2N$ ;  $\{\ddot{z}\}$ ,  $\{\dot{z}\}$ ,  $\{z\}$  represent the structural acceleration, structural velocity, and structural displacement vectors, respectively;  $[D]$  is location matrix for the restoring of the;  $\{F\}$  is vector containing the restoring force of the LRB;  $[r]$  is the influence coefficient matrix;  $\{\ddot{z}_g\}$  is the earthquake ground acceleration vector;  $\ddot{x}_g, \ddot{y}_g$  represent the earthquake ground acceleration in the longitudinal and transverse directions, respectively; and  $x_i, y_i$  is displacements of the  $i$ th node of the bridge in the longitudinal and transverse directions, respectively.

The equation of motion shown in Eq. (16) was integrated numerically using Newmark's step-by-step integration procedure (Newmark 1959). An iterative procedure is required at each time step because the assumed force-displacement relationship for the seismic isolators is nonlinear (bilinear). A modified Newton-Raphson procedure is used to determine the restoring force at each time step during the solution procedure. This solution procedure is implicit and unconditionally stable due to the choice of integration parameters,  $\gamma = 1/2$  and  $\beta = 1/4$  (Newmark 1959). The deformation and force results can be obtained from bi-directional time-history response analysis of isolated bridge using numerical procedure Matlab.

#### 4. OUTLINE OF SHAKING TABLE TESTS OF ISOLATED BRIDGE MODEL

A bridge model consisting of a two-span continuous steel girder deck supported by LRB has been constructed for the shaking table test, as shown in Figure 2. The bridge model is isolated by the LRB installed on the top of each pier. The substructure of bridge model consists of two rigid abutments and a circular column. The total span length and deck width of the prototype bridge are equal to 60 m and 9 m, respectively. The total pier height is equal to 10 m, including the cap beam.

Considering the shaking table capacity, a scaling factor of 1/10 is determined for the bridge model. Since the bridge deck is expected to exhibit rigid-body motion under horizontal excitations, the mass similarity is the major concern for the deck model. The plan dimensions of the deck model are determined to be 3 m in length and 0.9m in width. Concrete blocks are placed on the rigid steel girder to result in a total weight of 90 kN for the deck model.

To preclude stiffness degradation due to possible concrete cracks, concrete-filled portal frame steel columns are used and designed based on stiffness similarity for the pier models. The thickness and exterior diameter of the steel pipe are determined to be 8 mm and 120 cm, respectively, from a scaled equivalent transformed section. Also, the steel cap beams are jacketed with steel plates to prevent cracks. Correspondences similitude ratio of the bridge model to the prototype are shown in Table 1.

The LR bearings are foursquare and constructed with a 16mm diameter central lead core, and the length of these foursquare bearings is 100mm. The shear modulus of the elastomer used these experimental bearings is  $0.8\text{N/mm}^2$ . The bearing is composed of 9 layers of 3mm thick rubber and 8 layers of 1.5mm thick steel shims with an outer (bonded) length of foursquare of 90mm. The total rubber thickness in this bearing is 27mm, and the first shape factor,  $S_1$ , is 8.5. The top and bottom steel end plated are 79mm thick.

The isolated bridge model system is tested for the three real earthquake excitations. The test wave is obtained and used in experiment by compressing original real earthquake wave. Namely, the compression ratio of the test wave to the original real earthquake wave is 1/3.16, and the amplitude of acceleration is modified to 0.2g, 0.4g and 0.6g. In experiment, 1Dx, 2Dxy, 3Dxyz represent longitudinal, horizontal bidirectional (longitudinal and transverse), and multi-directional (horizontal and vertical) earthquake wave input of bridge model, respectively.

These tests have been carried out on the shaking table that manufactured MTS company in Earthquake Engineering and Test Research Center of Guangzhou University in China.



Figure2 The 1/10 isolated bridge model

Table1 Similitude ratio of the bridge model to the prototype

Items	Similitude ratio	Items	Similitude ratio
length	1/10	Acceleration	1.0
Stress	1.0	Displacement	1/10
Time	1/3.16	Velocity	1/3.16
Density	25/2	Weight	1/80

## 5. COMPARATIVE STUDY OF TEST AND NUMERICAL CALCULATION RESULTS

### 5.1. Deck Acceleration Response

The deck acceleration time-history curves are shown in Fig.3 under the 0.6g 2Dxy El Centro record. Experiment results agreed expectably well with the results obtained from theoretically analytical results of acceleration of deck roughly. Fig.4 shows the deck acceleration time-history curves under the 0.6g 3Dxyz El Centro record. From Figure3-4, it is also observed that experiment results agreed expectably well with the results obtained from theoretically analytical results of the deck displacement. The difference of peak deck displacement for analytical results is 20 percent in comparison to experimental results roughly.

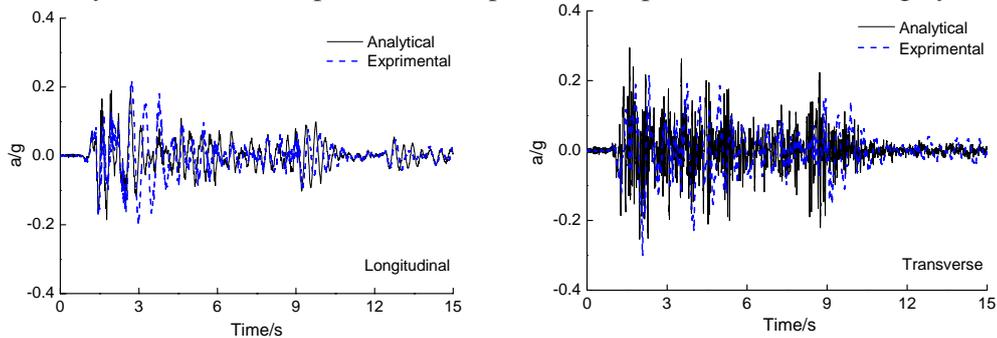


Figure3 Deck acceleration time-history curve under the 0.6g 2Dxy El Centro record

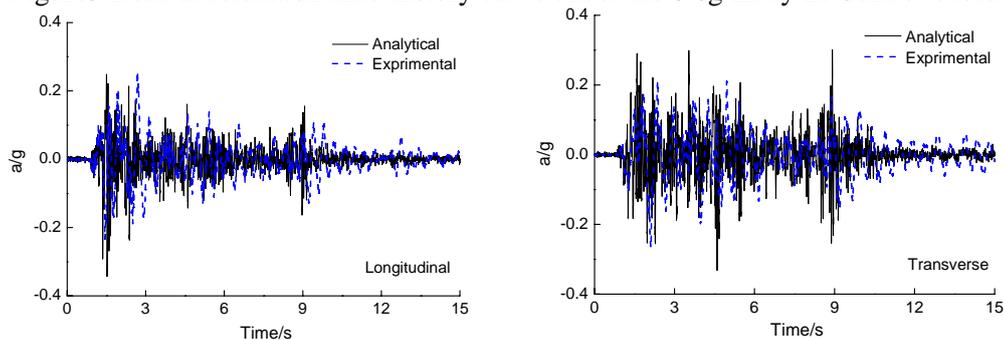


Figure 4 Deck acceleration time-history curve under the 0.6g 3Dxyz El Centro record

### 5.2. Deck Displacement Response

The deck displacement time-history curves are shown in Figure 5 under the 0.4g 2Dxy Kobe record. Fig.6 shows the deck acceleration time-history curves under the 0.4g 3Dxyz Kobe record. From Figs.5-6, it is also observed that experiment results agreed expectably well with the results obtained from theoretically analytical results of the deck displacement. The difference of peak deck displacement for analytical results is 20 percent in comparison to experimental results roughly.

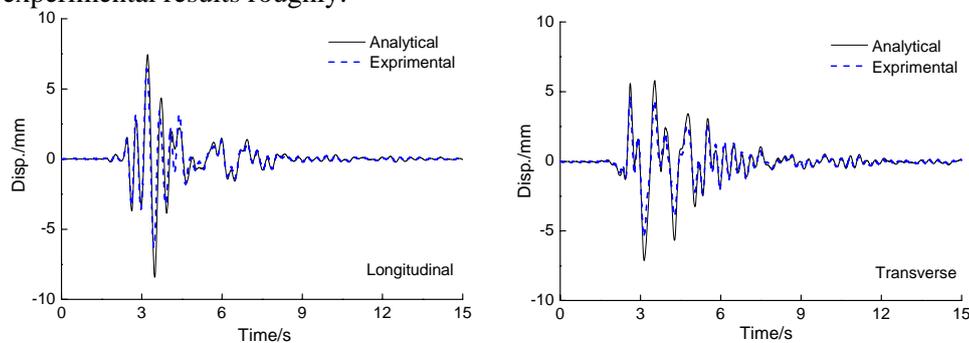


Figure 5 Deck displacement time history curve under the 0.4g 2Dxy Kobe record

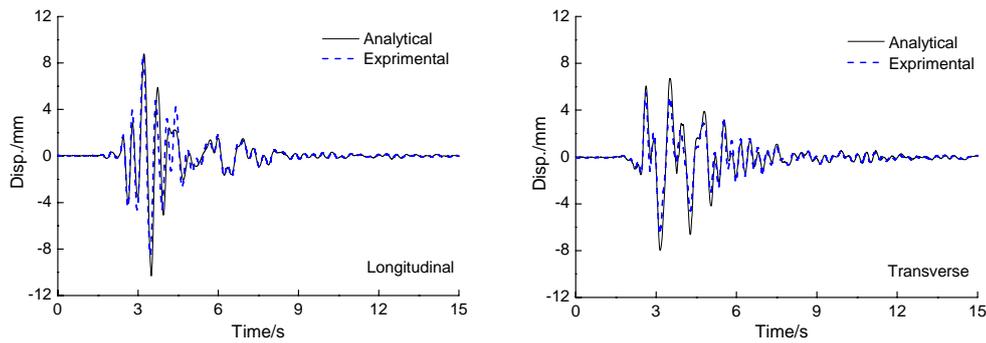


Figure 6 Deck displacement time history curve under the 0.4g 3Dxyz Kobe record

### 5.3. Vertical Force Response of LRB

The tensile force of LRB rarely arises in design region, therefore the studies on the tensile force of isolator is few. But the shear properties are significantly influenced by bearing tension state (HAN 2006). Figure 7 shows the vertical force of LRB time-history curve under the 0.6g El Centro record. From Figure 7, it is also observed that the tensile force of LRB will come into being if the peak acceleration quantities are bigger, especially, the vertical component of earthquake ground motion is bigger.

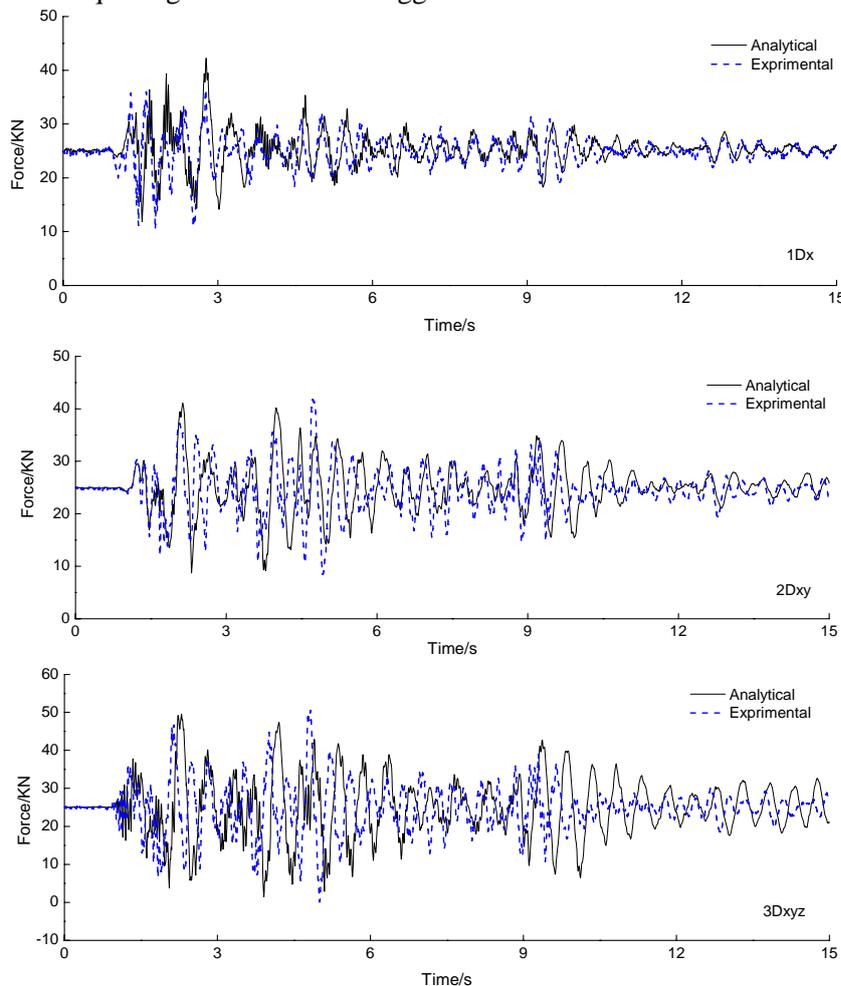


Figure 7 Vertical force time history curve of LRB under the 0.6g El Centro record

### 5.4. Isolated Layer Displacement Response

The deformation of isolated structure focused on isolated layer which absorbed and dissipated the most energy. As a result, the superstructure is protected effectively. Figure 8 shows the displacement of isolated layer

time-history curve under the 0.6g Kobe record. There are three similar characteristic for analytical results in comparison to experimental results. Firstly, is the peak displacement is equal and simultaneity roughly for analytical and experimental results. Secondly, the displacement response of the isolated layer is significantly influenced by the type of earthquake ground motion selected. Lastly, Experiment results of isolated layer agreed expectably well with the results obtained from theoretically analytical results.

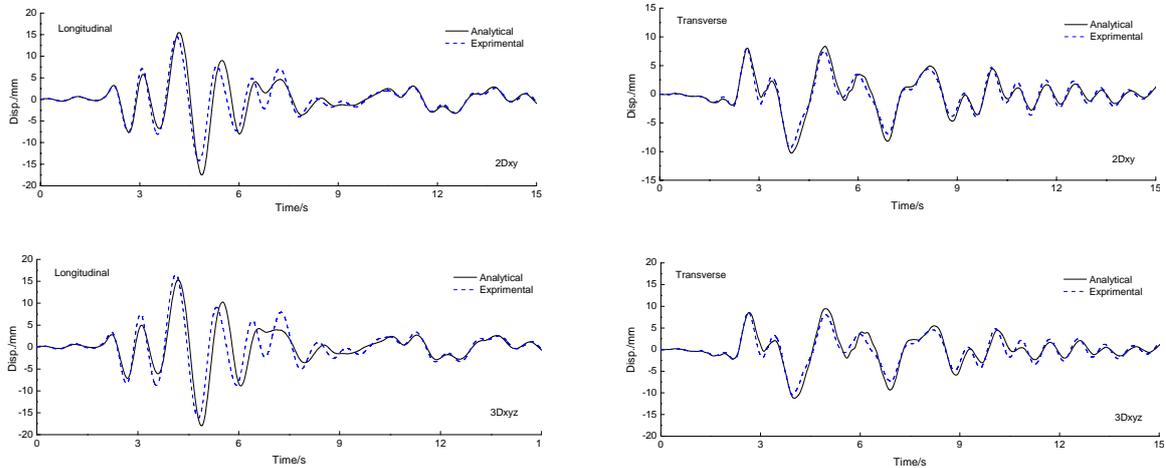
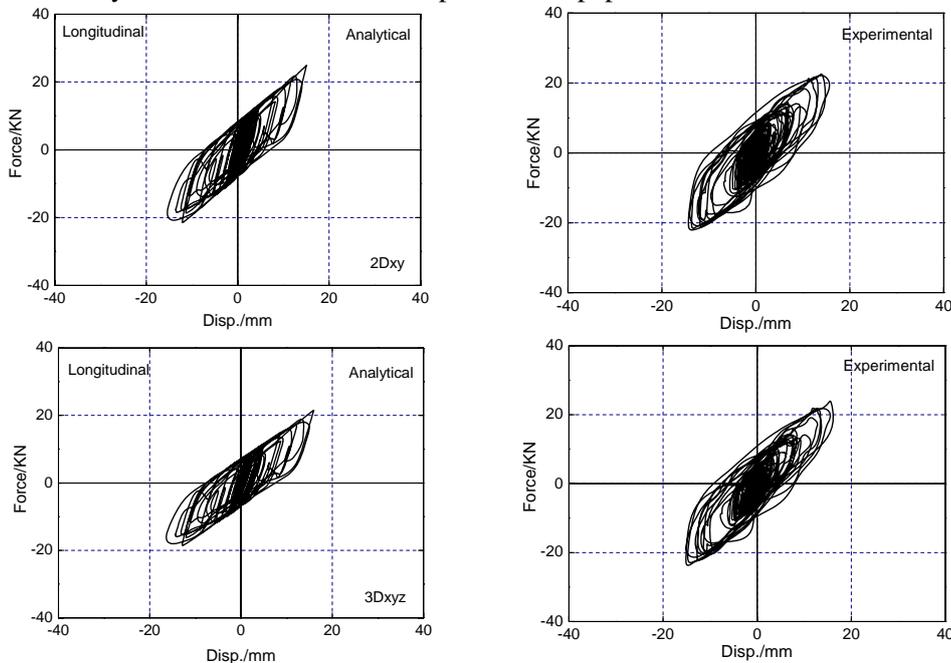


Figure 8 Displacement of isolated layer time history curve of LRB under the 0.6g Kobe record

### 5.5. Force-Deformation Relationship of LRB

The area of the hysteretic loop of the LRB implies energy dissipation capacity of isolated layer, and restoring force model depends upon the curve of the force-deformation behavior of the LRB. The force-deformation behavior of LRB is plotted in Figure 9 under the 0.6g Chi-Chi earthquake wave input. The hysteretic curve shape is similar and has bilinear behavior both analytical results and experimental results. The difference of peak bearing force and deformation for analytical results is 10 percent in comparison to experimental results roughly. So Bouc-Wen hysteretic model of LRB adopted in this paper is reasonable.



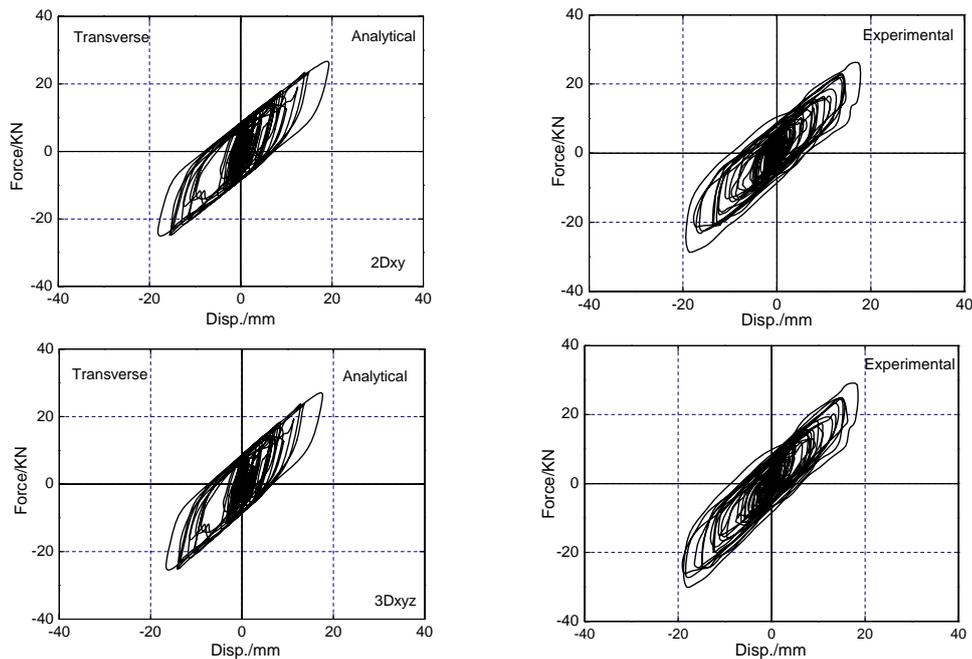


Figure 9 Force-deformation of LRB under the 0.6g Chi-Chi record

## 6. CONCLUSIONS

A nonlinear seismic response method of continuous multi-span isolated bridges with LRB was presented under bi-directional horizontal earthquake excitation base on bi-directional coupled Bouc-Wen hysteretic model. The shaking table tests of continuous girder isolated bridges model with LRB under multi-directional earthquake excitation were carried out and the analytical results were contrasted to experimental results. From trends of results of the present study following conclusions may be drawn:

(1) Experiment results agreed well with the results obtained from theoretically analytical results of displacement and acceleration of model bridge deck roughly. It is verified that computation method of seismically isolated bridge given this paper is right and effective when analyzing nonlinear earthquake response of continuous girder isolated bridges with LRB.

(2) The tensile force of LRB will come into being if the peak acceleration quantities are bigger, especially, the vertical components of earthquake ground motion is bigger. So the tensile force of LRB should be considered in design isolator if the vertical component of earthquake ground motion is bigger.

(3) The difference of peak bearing force and deformation for analytical results is 10 percent in comparison to experimental results roughly. So bi-directional Bouc-Wen hysteretic model adopted in this paper of LRB is reasonable and the interaction between the restoring forces of LRB in two horizontal directions is duly considered in the response analysis.

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