Simulated Annealing Algorithm for Seismic Optimization of Lifeline Networks

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ABSTRACT :

Usually lifeline systems, such as water distribution network and gas supply network, cover large areas. For these systems, network topology design based on seismic reliability provides a good way to design new systems and update old ones. In this paper, a topology optimization model for lifeline networks is established. The aim of the model is to find the least-cost network topology while its seismic reliability between the sources and the terminals satisfies prescribed constraints. As above optimal problem is a typical combinatorial optimization problem, simulated annealing algorithm is presented to solve this problem. Simulated annealing algorithm takes network topology as its solution, and tries to find the optimal solution by perturbing and updating the current solution. In order to verify the efficiency and capacity of the proposed algorithm, an example network is investigated in detail. The results indicate simulated annealing algorithm is an efficient algorithm to optimize lifeline networks topology.

KEYWORDS: Lifeline systems, Simulated annealing algorithm, Topology optimization

1. Introduction

The lifeline systems, including water distribution, gas supply and power networks etc, are the arteria of modern cities. As the development of modern society, lifeline systems play more and more important roles in urban everyday life (Li, 2005). The investigations of many previous earthquakes indicated that the performances of lifeline systems under earthquake determined the property losses and casualties of cities during the disasters and the recovery of cities after the disasters. However, almost all the lifeline systems suffered serious damages during many previous strong earthquakes. The 1995 Kobe earthquake (Investigation Group of Kobe Earthquake, 1997) is a typical example. In this earthquake, the main gas supply network suffered extensive damages. The number of leaks or breaks was as high as 5190. As the result, approximate 857 thousand customers were stopped gas supply and the secondary disaster caused by fire made even higher losses. It took about three months for the gas supply network to be fully recovered.

For lifeline systems, many researchers focused on elements seismic analysis (Takada, 2000; Ai and Li, 2005) and networks seismic analysis (Li and He, 2002; Hwang and Lin, 1998). However, the ultimate goal of researches on lifeline systems should be designing the new lifeline systems and updating the old ones to improve their seismic reliability. As the lifeline systems are usually distributed as networks in a large area, the network topology optimization provides a good way for lifeline systems design and update under earthquake. For network topology optimization, many scholars assume that the lifeline systems work under common operation conditions. In 2002, Yan et al (2002) used virtual flows to calculate the least-cost topology of water distribution network. Using minimal tree method, Kang and Yuan (2001) obtained the network topology with minimal total pipelines length. When considering the optimization under earthquake, only Shinozuka (1981) investigated a simple water distribution network by simulation approach.

In this paper, an optimization procedure to calculate the least-cost topology of lifeline network systems is presented. Taking the system cost as the optimization object and the system seismic reliability as the constraint, an optimization model is established. As solving this optimization model is a typical combinatorial optimization problem, a simulated annealing algorithm(SAA) is suggested to solve this problem. SAA takes network topologies as its solutions, and tries to find the least-cost solution by perturbing and updating the current solution. Also an example network is investigated in detail to verify the capacity of the proposed algorithm.

2.Optimization Model

The seismic reliability of lifeline network is determined by its edge seismic reliability and its topology. Apparently, if each edge reliability in the network is 1 subject to seismic wave excitation, network seismic reliability is 1 too. However, it is impossible that each edge of lifeline networks remains unbroken under earthquake. Also in many cases, network topologies are more important to network seismic reliability. As an extreme case, a network with 100 edges connected in series is considered. Even if all edges reliabilities are 0.95, the network reliability is only 0.006. In another extreme situation, if all edges reliabilities are 0.05, the network reliability with 100 edges in parallel can reach 0.994. In practice, the strategies to improve edge reliability include using ductile pipeline materials and adopting larger diameter pipelines and so on. But this method is not suitable for existing lifeline networks because some pipelines have to be discarded before they are out of service. So modifying the network topology by adding several edges to or removing several edges from the network is a feasible way to improve the network seismic reliability.

From above analysis, the network topology can be set as a variable in the optimization model. So to fulfill the optimization object is actually to find the least-cost network topology which satisfies prescribed nodal seismic reliability constraints. And the optimization model can be mathematically formulated in the following general form

$$\begin{aligned} &\text{minimize} \quad C(G^*) = \sum \gamma_j \cdot c_j \\ &\text{subject to} \quad P_k \geq P_0 \quad k = 1, 2, ..., n \\ &\quad G^* \quad \text{is a subgraph of } G \end{aligned} \tag{2.1}$$

where G represents a network and is usually generated empirically, G^* is a solution of the model, γ_j takes value of 1 if edge *j* exists in G^* and 0 inversely, P_k represents the seismic reliability between sources and terminal *k* and can be calculated using RDA (Li and He, 2002), P_0 represents the reliability constraint, c_j represents the edge *j* cost and can be evaluated easily in an actual lifeline network.

Above problem is a typical combinatorial optimization problem in which γ_j is the optimization variable. Though the optimization problem seems very simple as the value γ_j can only be 0 or 1, it is a very hard problem in fact. For example, considering a network with 60 edges and 30 nodes, the number of all potential networks is $2^{60} \approx 1.15 \times 10^{18}$. Considering that the network which consists edges less than 29 can't form a connected network, the number of all feasible networks can decrease to $\sum_{i=29}^{60} C_{60}^{i} \approx 7.5 \times 10^{17}$. However, use a computer which can calculate 100 networks a second, it will take about 2.38×10^{8} years to calculate all feasible networks. The computation time can't be accepted. In this paper, simulated annealing algorithm is suggested to solve above optimization model.

3.Simulated annealing algorithm for Lifeline Networks Optimization

SAA was first introduced by Kirkpatrick et.al (1983) and independently by Cerny (1985) as a problem-independent combinatorial optimization technique. SAA has been applied to a wide range of different combinatorial optimization problems, such as traveling salesman problems (Aarts et al, 1988;), large scale integration computer-aided design (Wong et al, 1988), computer communication networks design (Samuel et al, 1995) and so on.

SAA is a search procedure in which the current solution is continually compared to solutions which are obtained by carrying out a small perturbation. The perturbation result is accepted at a probability described as followings:

$$P(i \Rightarrow j) = \begin{cases} 1 & f(j) \le f(i) \\ \exp\left(\dfrac{f(i) - f(j)}{t}\right) & f(j) > f(i) \end{cases} \quad (3.1)$$

Where $f(i)$ is energy function of solution *i* and *t* represents current temperature, a control parameter which decreases gradually and approaches 0 at last.

Apparently, if the perturbation result is an improved solution, it is accepted and the current solution is updated accordingly. Otherwise, it can also be accepted at a probability described in Eq.(3.1). By accepting a worsening solution, SAA avoids being trapped too early in a local optimal solution. On the other hand, the probability of accepting a worsening perturbation solution decreases because *t* decreases gradually, which guarantees SAA will eventually converge and be less likely to move away from a global optimal solution after having approached it.

For above network topology optimization problem, the process of SAA can be described as followings.
①Produce an initial solution as the current solution;
②Determine current temperature *t* according to initial temperature *T* and cooling schedule. If current temperature is lower than the terminal temperature, stop.
③Perturb the current solution to generate a new solution. Calculate its energy function and determine the accepting probability of the new solution.
④Generate a number varied from 0 to 1 at random and compare it with above accepting probability. If the random number is smaller than accepting probability, the new solution is accepted and the current solution is updated. Otherwise, the new solution is discarded and the current solution is preserved.
⑤Judge whether the number of perturbations has reached prescribed value or not at current temperature, if yes, go to step ②, elsewise go to step ③.

Although the process of SAA is very simple, the parameters and the perturbation model must be selected carefully. If these selections are unsuitable, SAA will converge slowly and be hard to obtain a near-global optimal solution.

3.1 Networks expression

Manipulating a network with SAA requires that the network is properly expressed. Note that any solution in SAA is a subgraph of original network *G*. The simplest 0-1 binary coding is adapted. Herein an *n* bits array is used to represent a network and each bit represents an edge of original network *G*, where *n* is the number of edges in *G*. A '1' in the array means that the solution consists of a corresponding edge in *G* while a '0' means not. For example, Fig.1 is a bridge network. In this figure, the subgraph doesn't include the edge 5(dash line). Then the corresponding array of the subgraph can be written as 11110. The advantage of this expression is that the network perturbation becomes very easy.

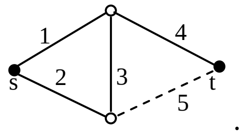

Figure 1. A bridge network

3.2 Generating initial network

Because the new solution in the SAA is obtained by perturbing the current solution, generating an initial network is the first job in SAA. To generate the initial network, an array is initialized to be '0' in all bits. That means the network contains no edge. Then each bit in the array is changed to '1' at a prescribed probability one by one. In other words, each edge in *G* is added to the network at random. In order to accelerate the convergence of SAA, the probability is determined by the seismic reliability constraint p_0 in optimization model. However, it must be noted that the initial network generated from above method may not be a feasible one in practice. For example, disconnected networks have no practical meanings. Therefore one must first judge network connectivity and modify the disconnected network by adding several edges at random. If the network is still a disconnected network after modifying, it is discarded and a new network is generated to replace it. Also, disconnected networks will be generated in the perturbation process, the same job must be done after new solution is generated.

3.3 Energy function

After perturbating the current solution and generating a new solution, energy function is used to determine the accepting probability of the new solution. Since the optimization objection is to find the least-cost system which satisfies prescribed reliability restraint and the solution owning low energy is considered better than the solution owning high energy in SAA, the energy function of the solution can be defined as

$$f(i) = C(i) + S(i) \tag{3.2}$$

where $C(i)$ is the cost of solution *i*, $S(i)$ is the penalty factor for the solution which doesn't satisfy reliability constraint and can be written as

$$S(i) = \begin{cases} 0 & P_{\min i} \geq P_0 \\ \alpha\left[P_0 - P_{\min i}\right] + \beta P_{sumi} & P_{\min i} < P_0 \end{cases} \tag{3.3}$$

where $P_{\min i}$ is the minimum nodal reliability of the solution, $P_{sumi} = \sum_{j,\, P_0 > P_j} (P_0 - P_j)$, α and β are constants and their value are determined by the actual network. It needs to point out that penalty factor must guarantee that the energy of the solution in which nodal reliability doesn't satisfy prescribed value is higher than the energy of the solution in which nodal reliability satisfies prescribed value.

3.4 Temperature control

Temperature control, including initial temperature *T*, cooling schedule and the number of perturbations at the same temperature, is very important in SAA.

The initial temperature should guarantee that all solutions have almost the same accepting probability, that means

$$\exp\left(\frac{-\left|f(i) - f(j)\right|}{T}\right) \approx 1 \tag{3.4}$$

Apparently, *T* should be a large value which is determined by the actual network.

In SAA, the probability to accept a worsening solution decreases slowly and the probability approaches 0 at last. In other words, the temperature must decrease slowly as the process goes on. In this paper, the cooling schedule is expressed as

$$t = \gamma^k T \qquad (3.5)$$

where γ is a number between 0 and 1 and is proposed to be 0.7 in this paper, k is the decreased number of the tempeature.

Also, the number of perturbations at the same temperature is set as a constant and can be selected according to the network scale. For a large complex network, the number of perturbations is large. Elsewise it is small.

3.5 Perturbation process

The perturbation of the solution, which is used to guarantee the capability of the algorithm to search for the optimal solution, is a key process in SAA. It operates on the current solution and produces a new solution. The new solution is accepted at a probability calculated by Eq.(3.1). Though perturbation is very important, its process is very simple and can be stated as follows:

(1) For each bit of the current solution, produce a number varied from 0 to 1 at random and compare it with the perturbation probability given in Eq.(3.7) or Eq.(3.8) below. If the random number is smaller than the perturbation probability, the bit is reversed which means the bit is modified to 0 if it was 1 and 1 if it was 0.

(2) If any bit in the current solution is reversed, a new solution is produced. If not, the current solution is preserved.

Herein the perturbation probability calculating process can be expressed as followings. First, the element importance of each edge in the current solution is calculated using following equation

$$I(j) = \begin{cases} \sum_{i=1}^{m} (P_0 - P_i) e_{ji}^{pro} & P_i < P_0 \\ \sum_{i=1}^{N} e_{ji}^{pro} & P_{\min i} \ge P_0 \end{cases} \qquad (3.6)$$

where e_{ji}^{pro} is the element investment importance of edge j (Liu, 2007), m is the number of the nodes which don't satisfy the prescribed reliability constraint and N represents the number of all nodes in the network.

On the basis of element importance, the perturbation probability of one bit changing from 0 to 1 can be determined by

$$P_{01}(j) = P_{\max 01} + \frac{P_{\max 01} - P_{\min 01}}{I_{\max} - I_{\min}} (I(j) - I_{\max}) \qquad (3.7)$$

where $P_{\max 01}$, $P_{\min 01}$ represent the maximum and minimum probability of the edge changing from 0 to 1 and take the value of 0.9 and 0.5 respectively. $I_{\max}, I_{\min}$ represent the maximum and minimum element importance of the solution.

Similarly, the perturbation probability of one bit changing from 1 to 0 can be determined by

$$P_{10}(j) = P_{\min 10} + \frac{P_{\min 10} - P_{\max 10}}{I_{\max} - I_{\min}} (I(j) - I_{\max}) \qquad (3.8)$$

where $P_{\max 10}$, $P_{\min 10}$ represent the maximum and minimum probability of the edge changing from 1 to 0 and take the value of 0.5 and 0.1 respectively.

Using above equations, the perturbation probability from 0 to 1 is higher for the edge owing higher element importance while lower for the edge owing lower element importance. On the contrary, the perturbation probability from 1 to 0 is higher for edge owing lower element importance while higher for the edge owing higher element importance. Apparently, the element importance determines the searching direction in a certain extent and is helpful to speed up the optimization process.

3.6 Stopping criteria

When current temperature is smaller than the terminal temperature t_f, the algorithm stops and the result is obtained. As the probability to accept a worsening solution approaches 0 at last, the terminal temperature should be set as a small positive value.

4. Case studies

For gas network, the cost of the pipeline can be evaluated by (Wang, 2005)

$$c_j = \left(-144.36 + 4313.3 d_j\right) \cdot l_j \qquad (4.1)$$

where c_j is the cost of the *j*th pipeline and in the unit of Yuan(RMB), d_j (m) is the *j*th pipeline diameter and l_j (m) is the *j*th pipeline length.

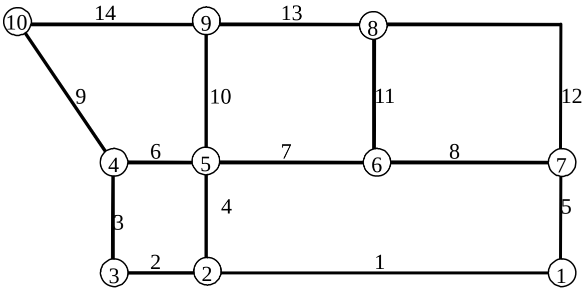

Figure 2. A gas network

Table 4.1 Pipeline characteristics of the network

NO.	Length(m)	Diameter (mm)	NO.	Length(m)	Diameter (mm)
1	1000	300	8	1000	400
2	6500	250	9	4800	200
3	4000	200	10	4000	200
4	4000	300	11	4000	250
5	4000	250	12	5000	300
6	6500	250	13	6000	200
7	6000	300	14	8500	200

Fig.2 is a simple gas network with 10 nodes. In the network, node 1 is the source and its reliability is considered as 1.0. Herein according to the location of node, network *G* is assumed to consist of 14 pipelines and the seismic reliability of each pipeline is 0.9. The length and diameter of each pipelines is shown in table 4.1 and the total cost is ￥59,197,800 ($7,688,025).

According to the charasteristic of this nework, the initial temperature T takes the value of 60000 and the terminal temperature t_f takes the value of 0.001. Also, in order to consider the effect of the reliability constraint p_0, p_0 is set different values of 0.7, 0.8 and 0.9 respectively. The results are calculated for different reliability constraints using SAA and shown in Fig.3-Fig.5. The total costs and the lowest nodal reliabilities of the results are shown in table 4.2. The results show that the total cost will increase as the reliability constraint increases. In fact, above three topologies are just the optimal topologies subject to different reliability constraints. So SAA provides a good tool to solve the optimization model.

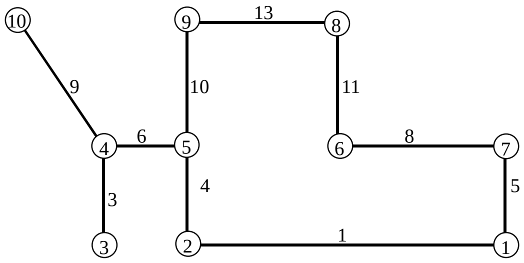

Figure. 3 The optimal network when p_0 is 0.7

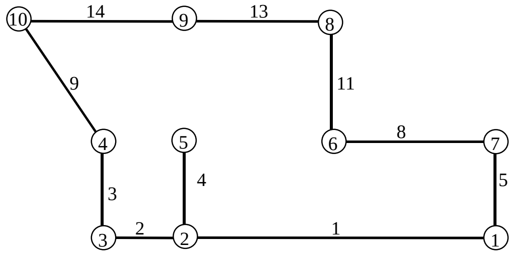

Figure. 4 The optimal network when p_0 is 0.8

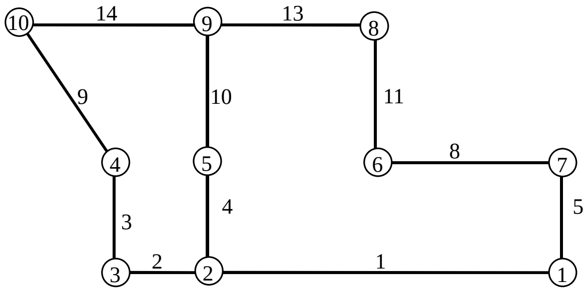

Figure. 5 The optimal network when p_0 is 0.9

Table 4.2 Optimization results

Reliability Constraint p_0	The lowest nodal reliability	Cost
0.7	0.747	￥34,370,000 ($4,463,636)
0.8	0.849	￥37,610,000 ($4,884,415)
0.9	0.920	￥40,480,000 ($5,257,143)

5.Conclusion

In this paper, a topology optimization model is presented to calculate the least-cost network which satisfies seismic reliability constraints. Also, SAA, an effect algorithm for combinatorial optimization problem, is used to solve above optimization model. In the case study, a network with 10 nodes is investigated and different results are calculated for different seismic reliability constraint. The results show that SAA can effectively give the optimal networks and the cost of network increases as the reliability constraint increases.

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