ANALYSIS OF PILE EQUIVALENT ANCHORAGE LENGTH FOR ELEVATED PILE CAPS UNDER LATERAL LOAD

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ABSTRACT :

Calculation model of equivalent anchorage length for elevated pile caps under lateral load was proposed based on the invariability of basic period. Pile-soil interaction was considered by elastic subgrade reaction method and using the power series method solved the curve differential equation. Variety rules of anchorage length for elevated pile caps with the changes of soil properties, diameter-spacing ratio of pile, pile-pier stiffness ratio etc. were studied. The empirical equation of pile equivalent fixed length for elevated pile caps under lateral load was given in the paper. The main factors affecting the equivalent anchorage length of pile consist of soil properties and pile-pier stiffness ratio. So the paper can provide some theoretical foundation for simplifying calculation and conceptual design of bridges.

KEYWORDS: Lateral load, Elevated pile caps, Fixing length

1. INTRODUCTION

Recently elevated pile caps have been widely applied in long-span bridges, wharfs and offshore oil platforms. Elevated pile caps can penetrate the soft stratum to reach the deeper stratum and its ability of bearing vertical loads is well. The researches in this area are relatively rich , but the study on working performance of elevated pile caps under lateral wind load is not yet perfect. So its working performance and response under horizontal load, such as wave force, seismic forces, ship impact load, brake force etc, become the concerned problem.

On the static and dynamic analysis for pile foundation, many researchers adopted winkler model to simulate soil-structure interaction and determined equivalent soil spring (Fig.1(b))[Harry G. Poulos. (1999), Rollins et al. (2005)]. Although Piles and soil can be finely simulated by the method, but the disadvantage is that element number of finite model is enormous and the method is difficult to master. Another way is directly to fix pile in a certain depth under ground or scour line and ignore the effect of soil around piles(Fig.1(c)).This method is simple, clear and easy to be applied in the practical engineering. Whether the value of pile anchorage length is reasonable has great influence on the superstructure responses. Murat[Murat Dicleli et al. (2005)]determined the pile equivalent anchorage length for a certain integral abutment by push-over. An initial value should be hypothesized and then it needs to repeat the process of trial calculation until the pile top bending moment and angle of equivalent system are equal to that of refined model. Y. Chen[Y. Chen(1997)] summarized several methods to determine the pile equivalent anchorage length and discussed the limitation of empirical formula given by AASHTO LRFD Specification. But his research is only to H shaped steel piles. The simple cubic equation to solve pile equivalent anchorage length is given in the literature[NCHRP REPORT 489 (2003)], but the empirical formula is only suitable for single pile model with the same diameter pile or column.

The paper is based on the theory that the primitive period of equivalent system is equal to the prototype and the elastic subgrade reaction method is used to consider pile-soil interaction. Analytical method to solve equivalent anchorage length for elevated pile-group caps is proposed. This method can precisely calculate the response of superstructure and reduce calculation workload. Therefore, the method can be applied for concept design of bridges.

2. DEFLECTON DIFFERENTIAL EQUATION OF PILE UNDER LATERAL LOAD

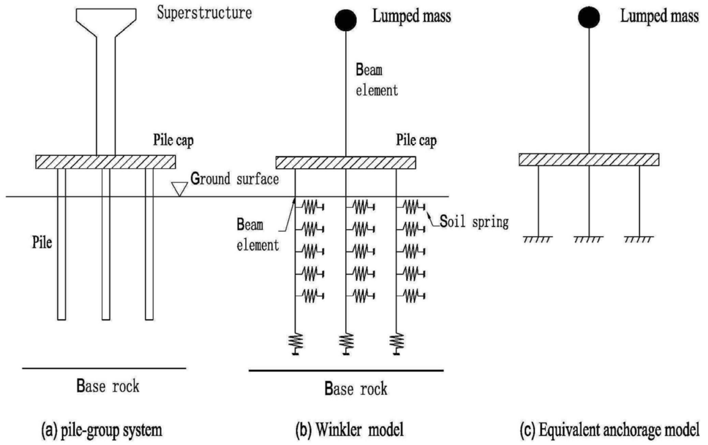


Figure 1 Pile-soil model

The pile is supposed as elastic component embedded in soil. When the pile at the depth z produces lateral displacement x, soil resistance of pile side at the depth z is q_z. When the hypothesis of M-method is adopted, the foundation coefficient increases linearly with the depth. Namely, the deflection differential equation of the pile can be given as:

$$EI\frac{d^4x}{dz^4}+m(z)b_1zx=0 \tag{2.1}$$

where x is the horizontal displacement of the pile; EI is rigidity of the pile; b_1 is the width of the pile; z is the depth of the pile; $m(z)$ is the function which reflects the trend of the soil level foundation coefficient increasing with the depth. The solution of Eqn. 2.1 can be described as following power series:

$$x=\sum_{i=0}^{\infty}a_iz^i \tag{2.2}$$

where a_i is undetermined coefficient.

Getting the first to forth derivative of above formula we substitute them into Eqn. 2.1. According to analytic theory of differential equation, the power series representation of the pile displacement, rotation, moment and shear force can be solved[HU Ren-li(1987)].

3. EQUIVALENT ANCHORAGE LENGTH OF THE PILE FOR ELEVATED PILE CAP

3.1. Basic Principles

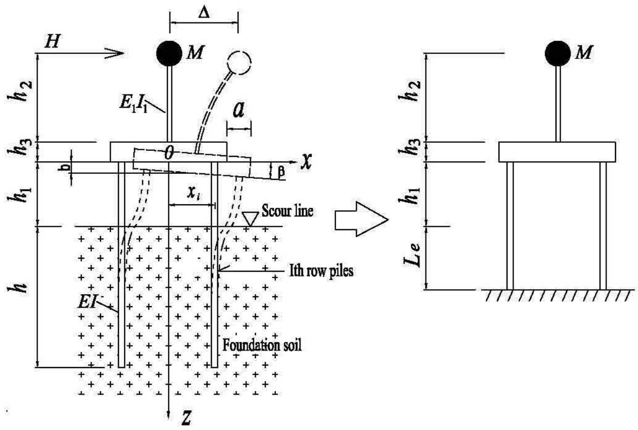
For the regulation system of beam bridges with elevated pile-group caps, the mass of the superstructure can be concentrated on the pier top. Then the whole bridge could be simplified as a single pier system(such as Fig. 2(a)). If ignoring the response of piles under the scour line, the pile can be fixed at a certain depth under the scour line and the system would be further simplified as an equivalent system as Fig. 2 (b). The Pile equivalent anchorage length Le can be calculated based on the assumption that the basic period of equivalent system is equal to that of single pier system.

To solve the basic period of the single pier system, elastic subgrade reaction method is adopted to consider the pile-soil interaction. Flexibility method is used to solve the structure flexibility matrix, and displacement of pier crown is solved by power series method. In terms of the boundary conditions of pile tips have great impact on the calculated results, the calculation model would be divided into two types: one is the end of pile being supported on the soil or rock layer (model 1), the other is the end of pile being embedded in the rock (model 2).

3.2. The End of Pile Is Supported on Soil or Rock

3.2.1 The basic period of single pier system

Fig.2 shows the deformation of the single pier system with the end of piles being supported on the soil or rock. The stiffness of column and each single pile is assumed as E_1I_1, EI respectively; the length of piles below and above the mudline is h, h_1; the height of pier column is h_2;the abutment thickness is h_3;horizontal force on pier crown is H;the displacement of pier crown is Δ; the stacking mass of pier crown is M; b, a and β are used to represent vertical and horizontal displacement of pile cap, angle of pile round coordinate origin respectively. To the single particle system two degrees of freedom (horizontal and vertical) should be considered and flexibility method is used to solve the structure flexibility matrix. Then natural frequency of the system can be calculated.



(a) Single pier system (b) Equivalent system

Figure 2 Calculating diagram of model 1

Under a unit horizontal force, the lateral displacement and angle at the top of every single pile is respectively δ_{HH}, δ_{MH}; under a unit moment, the lateral displacement and angle at the top of every single pile is respectively δ_{HM}, δ_{MM}. The calculation formulas can be written as:

$$\delta_{HH} = \frac{l_0^3}{3EI} + \delta_{MM}^{(0)} l_0^2 + 2\delta_{MH}^{(0)} l_0 + \delta_{HH}^{(0)} \tag{3.1}$$

$$\delta_{MH} = \frac{l_0^2}{2EI} + \delta_{MM}^{(0)} l_0 + \delta_{HM}^{(0)} \tag{3.2}$$

$$\delta_{MM} = \frac{l_0}{EI} + \delta_{MM}^{(0)} \tag{3.3}$$

where $\delta_{HH}^{(0)}$, $\delta_{HM}^{(0)}$, $\delta_{MH}^{(0)}$ and $\delta_{MM}^{(0)}$ are the flexibilities of a single pile at the mudline; their physical meaning is same as explained in structural mechanics and can be calculated by Eqn. 3.4:

$$\begin{aligned}
\delta_{HH}^{(0)} &= \frac{1}{\alpha^3 EI} \times \frac{(B_3D_4 - B_4D_3) + k_h(B_2D_4 - B_4D_2)}{(A_3B_4 - A_4B_3) + k_h(A_2B_4 - A_4B_2)} \\
\delta_{MH}^{(0)} &= \frac{1}{\alpha^2 EI} \times \frac{(A_3D_4 - A_4D_3) + k_h(A_2D_4 - A_4D_2)}{(A_3B_4 - A_4B_3) + k_h(A_2B_4 - A_4B_2)} \\
\delta_{HM}^{(0)} &= \frac{1}{\alpha^2 EI} \times \frac{(B_3C_4 - B_4C_3) + k_h(B_2C_4 - B_4C_2)}{(A_3B_4 - A_4B_3) + k_h(A_2B_4 - A_4B_2)} \\
\delta_{MM}^{(0)} &= \frac{1}{\alpha EI} \times \frac{(A_3C_4 - A_4C_3) + k_h(A_2C_4 - A_4C_2)}{(A_3B_4 - A_4B_3) + k_h(A_2B_4 - A_4B_2)}
\end{aligned} \tag{3.4}$$

where A_i, B_i, C_i, D_i （i=3,4） are dimensionless parameters that can be solved by Eqn.2.1; α is deformation coefficient of foundation; k_h is the influence factor of soil resistance due to the rotation of pile surface which can be calculated by Eqn.3.5.

$$k_h = c_0 I_0 /(\alpha EI) \tag{3.5}$$

where c_0 is the foundation coefficient of pile basement soil; I , I_0 is respectively the sectional moment of inertia and bottom surface moment of inertia of the pile under ground or local scour line; E is the elastic modulus of concrete.

When the unit displacement occurs at the bottom of the pile cap, the calculation of internal force produced on pile top is as follows: If the unit displacement occurred along pile axis, axial force produced on pile top is denoted by ρ_1 ; when the unit transverse displacement occurred along perpendicular to pile-axis, shear produced on pile top is ρ_2 ; when the unit transverse displacement occurred along perpendicular to pile-axis, moment produced on pile top is ρ_3 ; when the unit angle occurred, moment produced on pile top is ρ_4 , all of these can be calculated by Eqn.3.6~3.9.

$$\rho_1 = \frac{1}{(l_0 + \xi h)/(EA) + 1/(C_0 A_0)} \tag{3.6}$$

where A is the cross sectional area of piles; E is the elastic modulus of piles; A_0 is the projected area at the bottom of the pile which diffused with $\varphi/4$ angle from the mudline to the pile' s bottom and φ is internal friction angle; C_0 is vertical elastic resistance coefficient of foundation at the plane of pile bottom; ξ is correction coefficient.

$$\rho_2 = \delta_{MM} /(\delta_{HH}\delta_{MM} - \delta_{MH}^2) \tag{3.7}$$

$$\rho_3 = \delta_{MH} /(\delta_{HH}\delta_{MM} - \delta_{MH}^2) \tag{3.8}$$

$$\rho_4 = \delta_{HH} /(\delta_{HH}\delta_{MM} - \delta_{MH}^2) \tag{3.9}$$

Based on the hypothesis that the relative position among every pile top does not change after the absolutely rigid platform occurring displacement, the rotation angle of every pile top and platform is same. If the platform subjected to an external force, b , a and β is taken as the vertical displacement, horizontal displacement and the rotation angle about the coordinate origin respectively. If piles are intercepted at the bottom of the pile cap, then the pile cap can be treated as a free body and its equilibrium equations of all external force and internal force can be written as:

$$\begin{aligned} a\gamma_{ba} + b\gamma_{bb} + \beta\gamma_{b\beta} - N = 0 \\ a\gamma_{aa} + b\gamma_{ab} + \beta\gamma_{a\beta} - H = 0 \\ a\gamma_{\beta a} + b\gamma_{\beta b} + \beta\gamma_{\beta\beta} - M = 0 \end{aligned} \tag{3.10}$$

where $\gamma_{ba}, \gamma_{aa}, \gamma_{\beta a}$ is respectively the whole sum of vertical force, horizontal force and reverse moment of pile top acted on pile cap when the unit horizontal displacement occurred at the pile cap; γ_{bb}、γ_{ab}、$\gamma_{\beta b}$ is respectively the whole sum of vertical force, horizontal force and reverse moment of pile top acted on pile cap when the unit vertical displacement occurred at the pile cap; $\gamma_{b\beta}$、$\gamma_{a\beta}$、$\gamma_{\beta\beta}$ is respectively the whole sum of vertical force, horizontal force and reverse moment of pile top acted on pile cap when the unit rotation angle occurred at the pile cap; N, H and M are external loads.

If the vertical pile group is arranged symmetrically, let $b = 1$, $a = 0$, $\beta = 0$; $b = 0$, $a = 1$, $\beta = 0$; $b = 0, a = 0, \beta = 1$ and respectively substitute into Eqn.3.10.The displacements of pile cap can be calculated:

$$a = (\gamma_{\beta\beta}H - \gamma_{a\beta}M)/(\gamma_{aa}\gamma_{\beta\beta} - \gamma_{a\beta}^2) \tag{3.11}$$

$$\beta = (\gamma_{aa}M - \gamma_{a\beta}H)/(\gamma_{aa}\gamma_{\beta\beta} - \gamma_{a\beta}^2) \tag{3.12}$$

$$b = N/\gamma_{bb} \tag{3.13}$$

where $\gamma_{bb}=n\rho_1$, $\gamma_{aa}=n\rho_2$, $\gamma_{a\beta}=\gamma_{\beta a}=-n\rho_3$, $\gamma_{\beta\beta}=n\rho_4+\rho_1\sum K_i x_i^2$; n is the total number of pile foundation; x_i is the displacement from coordinate origin to every pile axis; K_i is the root number of ith row pile.

Let the flexibility matrix is $\begin{bmatrix}\delta_{11} & 0\\ 0 & \delta_{22}\end{bmatrix}$ and the self-vibration frequency of the system can be calculated by Eqn. 3.14.

$$\left(\omega^2\right)_{1,2}=\frac{1}{2}(\delta_{11}m_1+\delta_{22}m_2)\pm\sqrt{\frac{1}{4}(\delta_{11}m_1+\delta_{22}m_2)^2-\delta_{11}\delta_{22}m_1m_2} \tag{3.14}$$

where δ_{11} is horizontal displacement of particle under horizontal unit generalized force; δ_{22} is vertical displacement of particle under vertical unit generalized force; the two variables can be calculated respectively by Eqn. 3.15~3.16; m_1 and m_2 are the horizontal and vertical generalized masses.

$$\delta_{11}=a+\sin\beta(h_3+h_2)+x_H \tag{3.15}$$

where a, β is respectively the horizontal displacement and angle of pile cap bottom under the unit horizontal force acted on the pier top; x_H is the displacement under unit force when the pile bottom is fixed, which reflects the flexibility of the pile.

$$\delta_{22}=b+h_2/(E_1A) \tag{3.16}$$

where b is the vertical displacement of pile cap under the unit vertical force acted on pier top. The meanings of other parameters are the same as before.

3.2.2 The solution of equivalent anchorage length

The equivalent system is illustrated as Fig. 2(b). Its natural vibration frequency can be calculated by Eqn.3.14, in which the flexibility coefficient can be solved by Eqn.3.17~ 3.18.

$$\delta_{11}=\int\frac{\overline{M_1}\,\overline{M_1}}{EI}ds+\int\frac{\overline{N_1}\,\overline{N_1}}{EA}ds+\int k\frac{\overline{V_1}\,\overline{V_1}}{GI}ds \tag{3.17}$$

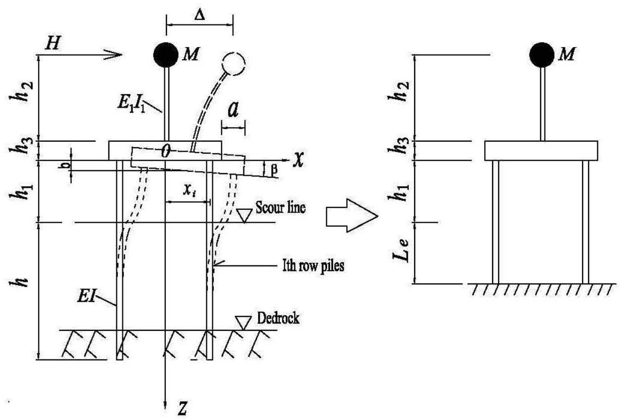
where $\overline{M_1}$, $\overline{N_1}$, $\overline{V_1}$ is respectively the moment, axial force and shear of structure when the unit horizontal force acted on the pile top; k is correction coefficient that is induced by the shear stress misdistribution on the section.

$$\delta_{22}=h_2/(E_1A_1)+(h_1+L_e)/(nEA) \tag{3.18}$$

where A, A_1 is respectively the section area of pile and column; n is the total number of pile foundation; Le is equivalent pile length; the meanings of other parameters are the same as before. Let the basic frequency of the equivalent system equal to that of the original system, the equivalent anchorage length of pile can be solved.

3.3. The End of Pile Is Embedded in Rock

Fig.3 (a) shows the deformation of a single pier system with the end of piles being embedded in rock. The meanings of parameters in Fig.3 are the same as Fig. 2



(a) single pier system (b) equivalent system

Figure 3 Calculating diagram of model 2

According to the same calculation method and formulas as model 1, the equivalent anchorage length (model 2) of pile-group can be solved. The different between the two models is that the calculation method of flexibility coefficients δ_{HH} , δ_{HM} , δ_{MH} and δ_{MM} are different. Because the displacement and angle of pile toe are zero, flexibility coefficients should be calculated by Eqn. 3.19:

$$\delta_{HH}^{(0)} = \frac{1}{\alpha^3 EI} \times \frac{(B_2D_1 - B_1D_2)}{(A_2B_1 - A_1B_2)}$$
$$\delta_{MH}^{(0)} = \frac{1}{\alpha^2 EI} \times \frac{(A_2D_1 - A_1D_2)}{(A_2B_1 - A_1B_2)}$$
$$\delta_{HM}^{(0)} = \frac{1}{\alpha^2 EI} \times \frac{(B_2C_1 - B_1C_2)}{(A_2B_1 - A_1B_2)}$$
$$\delta_{MM}^{(0)} = \frac{1}{\alpha EI} \times \frac{(A_2C_1 - A_1C_2)}{(A_2B_1 - A_1B_2)} \quad (3.19)$$

where α is deformation coefficient of foundation; A_i , B_i , C_i and D_i （i=1,2）are dimensionless parameters that can be solved by Eqn.2.1.

4 THE SENSITIVITY ANALYSIS OF PARAMETERS

Taking a pile foundation with elevated pile cap（Fig.4）as an example, the sensitivity analysis of parameters is conducted for the equivalent anchorage length of piles. The geometry of the foundation is as follows: pile-group is consisted of 2×6 hole-drilling pouring piles; the outside diameter of each pile is 1.0m; the length of the pier is 4.5m; the concentrated mass of pier top is 5000 tons; the thickness of the pile cap is 2.0m; the elastic modulus of concrete(E) is 1.93e10Pa. The meanings of other parameters (Fig.4 and Tab. 4.1) are as follows: Sp is the pile spacing; D is the diameter of pile; h is the buried depth of pile foundation; h_1 is the protruded length of pile foundation; h_2 is the height of pier; I_1 is the moment of inertia of cross sections of the pier; m is the parameter which reflects the variation trend of the soil level foundation coefficient with the increase of depth. To analyze the variation trends of the equivalent anchorage length L_{e1} and L_{e2} of the foundation in longitudinal and transverse direction with m , the dimensionless parameter D/Sp (the ratio of pile diameter and pier spacing), dimensionless parameter h/h_1（the ratio of buried depth of pile foundation and protruded length of pile foundation),dimensionless parameter x_H/a（the ratio of anti-push rigidity of pile group and that of pier), three cases are designed to study the parametric sensitivity of these parameters. The selected values of each variable parameter in these three cases are listed in Tab. 4.1.

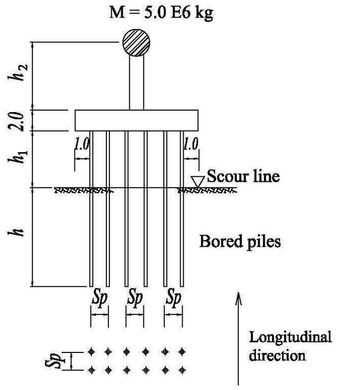


Figure 4 The layout of the elevated pile cap (Unit :m)

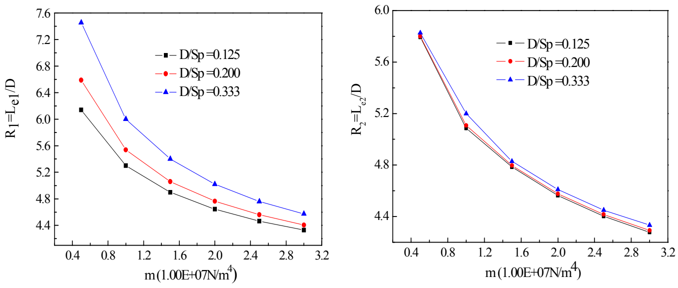
Table 4.1 The selected values of each variable parameter

parameter / Item	m (N/m^4)	D/Sp	h/h_1	h_2 (m)	I_1 (m^4)
Case 1	5.00E6～3.00E7	0.125～0.33	6.62	4.50	0.785
Case 2	5.00E6～3.00E7	0.200	1.66～6.62	4.50	0.785
Case 3	5.00E6～3.00E7	0.125～0.33	6.62	4.50	0.624～0.870

In Fig.5～Fig.7, the dimensionless parameters R_1、R_2 (the ration of the equivalent anchorage length of pile and pile diameter in longitudinal and transverse direction) are selected as vertical coordinate, trend curves of R_1、R_2 with the variation of other parameters are plotted. Fig.5 represents the model change law of R_1、R_2 with the m and D/Sp under Case 1; Fig.6 represents the model change law of R_1、R_2 with the m and h/h_1 under Case 2; Fig.7 represents the model change law of R_1、R_2 with the x_H/a and D/Sp under Case 3. In this example, the number of piles in longitudinal direction is less than those in transverse direction and under the same case R_1 is greater than R_2. But the foundation soil is harder, the difference is less. When m is larger than $3.0E7N/m^4$, R_1 is almost equal to R_2. When the number of piles is large, both R_1 and R_2 are insensitive to the change of D/Sp (Fig.5, Fig.7). It is clear that the value of m is one of the main factors that influence the equivalent anchorage length of piles. The analysis results show that the equivalent anchorage length of piles is 3～5 times of pile diameter if the foundation soil is hard plastic cohesive soil, hard cohesive soil , medium sand or gravel sand. Parameter a is the horizontal displacement of the pile cap bottom when unit horizontal force acted on the top of pier and it reflects the flexibility of the pile group. Parameter x_H is the flexibility of pier column. Parameter x_H/a is the ratio of anti-push rigidity of the pile group and that of the pier. The influence of pile spacing, pile diameter, buried depth of pile foundation, the value of m, height of pier column and size of pier column can be reflected in parameter x_H/a. Fig.7 shows that R_1 and R_2 decease monotonously with the increase of pile-group's stiffness proportion in the whole system and the trend curve can be fitted with a negative exponential function. According to regression analysis, the relation between the equivalent anchorage length Le of pile and parameter x_H/a satisfies power function:

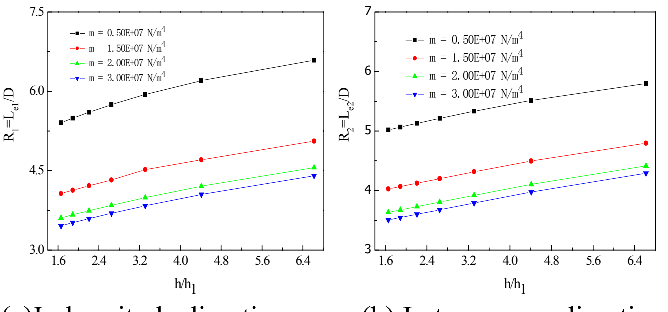
$$L_e/D = A\left(a/x_H\right)^B \tag{4.1}$$

where A is regression coefficient and B is index of correlation; D is pile diameter. In the example the value range of A is [2.866, 3.053] and the value range of B is [0.514, 0.629].



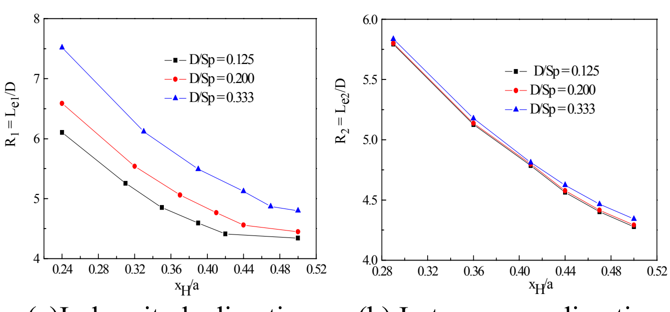
(a)In longitude direction (b) In transverse direction

Figure 5 Trend curves of R with m , D/Sp



(a)In longitude direction (b) In transverse direction

Figure 6 Trend curves of R with h/h_1 , m



(a)In longitude direction (b) In transverse direction

Figure 7 Trend curves of R with x_H/a , D/Sp

5. CONCLUSIONS

An equivalent anchorage length model of piles is proposed for pile foundations with elevated pile caps under lateral loads. The equivalent anchorage length of piles is determined by keeping the fundamental period of the pile foundations unchangeable. The single particle calculation model is applied to determine the equivalent pile anchorage length for pile foundations with elevated pile caps in this paper. Based on the m-method of linear elastic subgrade reaction method, the analytic calculation method is given. The parametric sensitivity analysis results show that the property of soils and pile-pier stiffness ratio are the main influencing factors of equivalent pile anchorage length. For pile foundations with elevated pile caps, the number of piles in longitude direction and transverse direction is always not equal. Under the same case in the direction of fewer piles, L_e is larger than the other direction. But if the foundation soil is hard, the difference can be ignored. When the number of piles is small, L_e would increases with the pile diameter-pile spacing ration (D/Sp). When the number of pile is large, L_e is insensitive to the change of D/Sp . An empirical formula, which is obtained by regression analysis based on the results of parametric sensitivity analysis, is proposed to calculate the equivalent pile anchorage length for elevated pile caps.

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