RESTORING THE LOS ANGELES WATER SUPPLY SYSTEM FOLLOWING AN EARTHQUAKE

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ABSTRACT :

This paper introduces a discrete event simulation model of post-earthquake restoration for the Los Angeles Department of Water and Power (LADWP) water supply system, the largest municipal system in the United States. The real-life post-earthquake restoration process carried out by LADWP is detailed. Past approaches to modeling post-disaster lifeline restoration are reviewed, highlighting the key benefits and limitations of the discrete event simulation approach used here. The new model of LADWPs' post-earthquake water restoration process is then described briefly, including sample results from calibration to the restoration observed following the 1994 Northridge, CA earthquake. This research is part of a larger effort by the Multidisciplinary Center for Earthquake Engineering Research to measure and understand community resilience.

KEYWORDS: lifeline engineering, restoration modeling, water supply, discrete event simulation

1. INTRODUCTION

Earthquakes can cause widespread damage to water supply systems resulting in extensive service interruptions that can last for days. In the 1994 Northridge, CA earthquake, the Los Angeles Department of Water and Power (LADWP) sustained more than 70 incidents of damage to trunk lines, 1,013 incidents of damage to distribution lines, and damage to 5 water tanks (Shi 2006). Approximately 500,000 people (14% of those served by LADWP) lost water service (McReynolds and Simmons 1995). It took five days to restore water to 99% of customers and repairs continued for months, costing about $41 million (McReynolds and Simmons 1995, Lund et al. 2005). Loss of water service and water purification notices in events like the Northridge earthquake can significantly disrupt drinking supply, sanitation, hospital functioning, industrial processes, and many other aspects of daily life. Both the number of post-earthquake water outages and their durations are important in determining the final impact of an event. In this paper, we first provide some background on LADWP and describe their real-life post-earthquake water supply restoration process. We then review past approaches to modeling post-disaster lifeline restoration, and finally, introduce a new discrete event simulation model of post-earthquake restoration for the LADWP water supply system.

2. LOS ANGELES DEPARTMENT OF WATER AND POWER

2.1. LADWP Water System

Established in 1902, the Los Angeles Department of Water and Power is the country's largest municipal utility. During the 2005-2006 fiscal year, the LADWP water supply system provided water to about 680,000 customers, representing 3.9 million people in a service area of approximately 1,200 km^2 (LADWP 2007). It supplies about 2.5(10^6) m^3 of water on a typical summer day, and 1.2(10^6) m^3 on a typical winter day (Wang 2006). In 2004-2005, residential, commercial/governmental, and industrial users accounted for 72%, 25%, and 3% of the water consumption, respectively (LADWP 2007). The three main water sources for the system are the Los

Angeles Aqueducts, the Metropolitan Water District (MWD), and local groundwater wells, providing about 48%, 41%, and 11% of the total water supply, respectively, in 2004-2005 (LADWP 2007).

2.2. Important Models of the LADWP Water System

A hydraulic network model called H2ONET was developed for the LADWP water supply system and is used by LADWP engineers for planning and analysis. The 2002 version of H2ONET used in this work explicitly models 2,186 km (1,358 mi.) of pipeline, 230 regulator stations, 110 tanks and reservoirs, 151 local groundwater wells, and 73 pump stations (Wang 2006). The size of the LADWP water supply system does not allow for the explicit modeling of all pipelines within H2ONET. As a result, more than 10,000 km (6,214 mi.) of the smaller diameter pipeline are represented by 1,052 demand nodes within the model. Each demand node is considered to represent an area of distribution pipelines. H2ONET contains more than 10,000 links and approximately 9,300 nodes.

A software program called *Graphical Iterative Response Analysis of Flow Following Earthquakes* (GIRAFFE) was developed at Cornell University to estimate earthquake performance of water supply networks (Shi 2006, Wang 2006). Developed as part of the MCEER-LADWP partnership, GIRAFFE estimates damage and functionality for heavily damaged water systems. Standard hydraulic analysis models like H2ONET no longer apply when a system is heavily damaged. In GIRAFFE, first, the hydraulic network being analyzed is defined. As in H2ONET, the LADWP water system trunk lines are represented explicitly as lines in the network model. Distribution lines are represented as demand nodes on the trunk network. Second, the system is modified to simulate the occurrence of earthquake-caused damage. Damage to trunk lines is represented as distinct breaks and leaks. Damage to distribution lines is represented by increasing the demand at the demand node that represents the damaged distribution lines. This reflects the fact that distribution pipes with breaks and leaks in them will draw more water than normal from the trunk line network because water will spill into the ground rather than just serve customers. Next, GIRAFFE performs a hydraulic analysis on the modified system using the engine from EPANET, a free, standard hydraulic analysis program (Rossman 2000). In this step, GIRAFFE first checks the connectivity of the modified system and removes any components that are isolated from water sources. It then runs a normal hydraulic analysis. If any nodes are found to have negative pressure, they are removed from the system and the analysis is rerun. This step is repeated until there are no nodes with negative pressure. Those results are the final ones. The output from GIRAFFE includes the flow and/or pressure at each system component (e.g., pipe, junction, pump). For each demand node, it indicates whether it is satisfied or not (i.e., whether the trunk network can get water to that node). It also produces the system serviceability index (SSI), which is defined as the ratio of the total satisfied demand at demand nodes after an earthquake to total required demand at demand nodes after an earthquake.

2.3. LADWP Water Organization

The LADWP water supply organization is divided into four main divisions: Water Distribution (WD), Water Quality and Operations (WQ&O), Water Engineering and Technical Services (WETS), and Water Resources (WR). Each of these divisions has its own set of crews and procedures to be followed in the event of a large earthquake. The Water Distribution (WD) division is responsible for the installation and maintenance of water distribution facilities, which includes trunk lines, distribution lines, meters, fire hydrants, regulators, valves, appurtenances, and other related items. During the restoration process following an earthquake, WD is responsible for the inspection and repair of its facilities. The Water Quality and Operations (WQ&O) division divides its responsibilities between two main sections. The Operations and Maintenance (O&M) Section oversees the operation and maintenance of the filter plants, pump stations, regulator stations, tanks, reservoirs, and ground wells. During the restoration process, this section is in charge of inspecting and repairing these same facilities. The Water Quality Compliance Section oversees the quality of the water distributed throughout the system. During the restoration process, this section will assist managers in determining whether to issue water purification notices. Water Engineering and Technical Services (WETS) is a technical division that focuses on the design of facilities for the water system, including trunk (but not distribution) lines. During the post-earthquake restoration process, the division's primary duty is to assess the safety of dams and reservoirs in the LADWP service area and to provide technical assistance to the other divisions. The Water Resources division is responsible for the facilities that deliver water to the LADWP system, e.g., the aqueducts. Since its facilities are not within the service area, it was not considered in development of the restoration model.

2.4. Post-Earthquake Restoration Process

The goals that guide the LADWP restoration process are: (1) to restore water service to the most people as quickly as possible, with special consideration given to hospitals, fire fighting needs, and life threatening and other high priority situations; (2) to have a water purification notice for as short a time as possible; (3) to not interrupt water to an area after it has been restored; and (4) to not reinstitute a water purification notice in an area after it has been lifted. The description presented in Section 2.4 was developed based on extensive interviews and conversations with many LADWP water personnel, the LADWP Emergency Response Plan for each major division in the organization, and experiences in the San Fernando and Northridge earthquakes. Tabucchi and Davidson (2008) presents the real-life process in more detail.

2.4.1. Restoration Crews

Different types of crews are involved in post-earthquake restoration, each with different responsibilities, skills, and modes of operation. During the restoration process, the Water Distribution division mobilizes inspectors and repair crews. Following an earthquake all employees report to their assigned district yards. Each inspector is dispatched to examine pipelines and find leaks in areas identified by the Trouble Board associated with his district yard. They then report their findings back to the Trouble Board. They may also be assigned to operate valves within the system to aid in rerouting water and/or isolating damage. Construction crews that do pipe installation and repair in normal times do repair in a post-earthquake situation. Two-person crews undertake smaller repair projects; five-person repair crews undertake the larger repair projects. Repair and construction crews are responsible for repairing pipe damage and restoring service to customers. Repair crews from other companies that come to help are used for small projects, such as repairing leaks in distribution lines.

Since the primary concern of the restoration model is the time at which customers have their service restored, only the Operations and Maintenance Section of the WQ&O Division is considered (not Water Quality Compliance). The O&M Section includes Water Utility Operators (WUO) and Water Utility Workers (WUW). Following an earthquake, the WUOs and WUWs assess the damage and functionality of the facilities, and determine what is needed to repair them. The Water Utility Operators inspect the 80+ pump stations and about 110 tanks and reservoirs (which are often located near the pump stations). They will also inspect any trunk lines connected to the pump stations. After the pump stations, tanks, and reservoirs have been inspected, the WUOs will be assigned to inspect the 60 to 80 ground wells in use at the time. The Water Utility Workers inspect the 350+ regulator stations (about 30 to 40 regulator stations per crew). Repair and construction crews from WQ&O repair damage to the pump and regulator stations, tanks and reservoirs, and wells.

There are two main types of crews originating from the WETS division that are involved in the restoration process: Damage Assessment Teams (DATs) and Reservoir Inspection Teams (RITs). The DATs serve as technical support for the other divisions in assessing damage to facilities, recommending repairs, and documenting damage. DATs are called in to inspect building damage, for example. RITs are responsible for assessing the damage and safety of the reservoirs and dams in the LADWP system. They conduct more detailed technical evaluations than the initial evaluations conducted by WUOs.

For all types of crews, it is expected that not all crews will report as they are supposed to immediately after an earthquake, but most of those initially not there will gradually report over the following day or two. For all WD and WQ&O crews, the first post-earthquake shift, which will begin immediately after the earthquake, is likely to be especially long, as it was following the Northridge earthquake. After the first shift, they will adjust into a schedule of two 12-hour shifts, with approximately ⅔ of all personnel on the day shift and ⅓ on the night shift.

2.4.2. Restoration Tasks

The LADWP post-earthquake restoration process can be divided into 4 main phases: inspection, rerouting around trunk line damage, isolating distribution line damage, and repair. During inspection, which begins immediately following an earthquake, facilities and pipelines are examined to determine the level of damage and degree of functionality, if there are any safety concerns, and what needs to be done to isolate the damage and repair it. The goals of the rerouting and isolating phases are to minimize water loss, minimize the number of customers without service, and maximize the water available for fire fighting. This is accomplished by opening and closing valves and adjusting settings at regulator and pump stations so as to minimize flow to damaged areas and redirect water to customers through different paths. (Minor rerouting around a damage location as part of a repair is considered part of the repair process). Rerouting and damage isolation occur concurrently with inspection and repair. In the repair phase, damage is repaired so that the facility or pipeline is functional. Repairs may be temporary or permanent. Temporary repairs are assumed to last days or weeks (i.e., beyond when the earthquake event is considered over), but ultimately to require more extensive work. For purposes of modeling the post-earthquake restoration process, temporary repairs are considered to be in effect permanent. For each facility or pipeline, the repair phase can begin immediately after inspection is completed. In water supply systems, unlike other lifelines, one can consider three different types of restoration curves, related to: (1) restoration of non-potable water service, (2) restoration of potable water service, and (3) permanent repair of all components in the system. These three levels of restoration may occur at different times. This study focuses on the first type of restoration.

3. POST-DISASTER LIFELINE RESTORATION MODELING METHODS

Available post-disaster restoration lifeline models can be grouped into six main approaches: (1) empirical curve fitting, (2) deterministic resource constraint, (3) Markov, (4) statistical regression, (5) optimization, and (6) simulation. See Çağnan (2005) and Liu (2006) for more thorough reviews. In the empirical curve fitting approach, data obtained from previous events and/or expert opinion are employed to fit restoration curves, and it is assumed that those curves represent future restorations. In the deterministic resource constraint approach, the actual restoration process is modeled, but in a simplified way, typically using a set of simple equations and rules. Some studies have modeled the restoration process of individual or groups of lifelines by assuming they follow a discrete-state, discrete-transition Markov process. Liu et al. (2007) offer the only example of a statistical approach to restoration modeling, applying it to electric power systems in hurricanes and ice storms. While all the other approaches focus on descriptively modeling the current restoration process, optimization aims to determine the "best" way to conduct a restoration process in terms of, for example, how to prioritize repairs and how many of each type of restoration crew to have (Xu et al. 2007 reviews studies using this approach).

Monte Carlo simulation has been used in a simplified way to estimate post-storm electric power restoration. A simplified version of the storm restoration process is simulated using estimated failure rates and mean times to repair and switch. Newsom (1977) presents early work on post-earthquake electric power restoration using discrete event simulation, but interestingly, no other studies could be found that use or even mention that approach until almost 30 years later. Çağnan and Davidson (2007) and Çağnan et al. (2006) present a discrete event simulation model of the post-earthquake restoration process for the Los Angeles Department of Water and Power electric power system.

Discrete event simulation offers many benefits for restoration modeling compared to alternative methods. The water supply system and restoration process are represented in great detail with few simplifications. Some of the simplifications adopted in other methods may lead to large errors. For example, in most previous studies using the other methods, rerouting around and isolating damage, two parts of the process that can significantly affect restoration times, are neglected. The utility company's decision variables (e.g., number of repair crews, repair prioritization rules) are included explicitly, allowing exploration of their effects on the speed of the restoration. Restoration times are estimated separately for each region within the service area, and uncertainty in the process is modeled explicitly. This approach to modeling post-disaster lifeline restoration is new, and while both previous applications of discrete event simulation were to electric power systems, water supply systems introduce additional challenges when developing a discrete event simulation model of the restoration process. Among the most important differences when addressing water supply instead of electric power, are: (1) many more components of the system are damaged, (2) the ability to reroute around or isolate damage is important and more difficult to capture, (3) restoration decisions depend on serviceability, so damage and restoration models must be coupled, and (4) the restoration process lasts longer and thus, modifications to the plan are made repeatedly. The key limitations of discrete event simulation are that it can be quite time-consuming to develop a discrete event simulation model and the model itself is system-specific. However, through sensitivity analysis one can use a discrete event simulation model to identify the most influential features of the restoration process and draw more general conclusions.

4. POST-EARTHQUAKE LADWP WATER SUPPLY SYSTEM RESTORATION MODEL

In the new LADWP water restoration simulation model, the physical components of the system (e.g., a piece of trunk line, pump station) are included as entities. A key attribute of each entity is its status, which indicates how far along it is in the restoration process. There are two key types of resources in the model, crews and materials. As in real-life, several different types of crew are defined (e.g., water distribution inspectors), each with a set of tasks it can complete and rules of behavior. Each crew also has a status attribute that can take on values that include waiting to go on-duty, traveling, working, idle (not currently needed), or off-duty. At each time step (time to the next event; can be less than 1 minute to 30 minutes in length), events occur, causing the status of entities and crews involved to be updated. Events that take place within the simulation include inspecting entities, rerouting around trunk line damage, isolating distribution damage, repairing pipe breaks and leaks, and traveling. To determine which specific events will take place in each time step, events are prioritized according to rules that reflect LADWP's real-life restoration priorities. All event durations are modeled as triangularly-distributed random variables. As time progresses, the simulation mimics the restoration process quite literally until all customers have water service restored and no more events need to take place.

At a given time *t*, many decisions about how to prioritize pending events (e.g., which damaged pipe to repair first) depend on both which pipes are damaged and where customers are and are not getting water service at that time. As a result, it is important to know how both the damage and functionality within the system evolve over time. The system damage state is described in terms of trunk breaks and leaks and demand node normalized demands (post-earthquake demand divided by pre-earthquake demand). The system functionality is described in terms of serviceability. For each demand node, serviceability is a binary indicator of whether or not it is being satisfied. For the system (or a region within the system), serviceability is the ratio of total demand available (satisfied) after an earthquake to that required. It is zero if the post-earthquake demand is not satisfied and one if it is satisfied.

For the restoration model to base prioritization decisions at time *t* on current serviceability as well as damage, it had to be coupled with the earthquake performance model, GIRAFFE. For one damage realization, based on the input damage and serviceability, the restoration model repairs some breaks and leaks in trunks and distribution lines during the next set of time steps. The updated damage state of the system is then input back into GIRAFFE, which is run to determine the system serviceability associated with that new damage state. The revised system damage state and serviceability are then input back into the restoration model, which uses that information to decide which additional damage locations to repair during the next set of time steps. The process continues until all damage is restored and all customers have water service (i.e., system serviceability equals one). Each time *t* at which GIRAFFE is called provides one estimate of system serviceability that can be plotted to create a serviceability-versus-time restoration curve. See Tabucchi and Davidson (2008) for more detail about the model.

The restoration model requires a few types of input: (1) system definition, (2) initial system damage and serviceability, (3) definitions of key locations and areas (e.g., earthquake epicenter, district yards), and (4) various user-specified parameter values (e.g., threshold of distribution damage that indicates if the demand node will be isolated, time period between runs of GIRAFFE). The system definition, which is taken from H2ONET, describes the components of the hydraulic network being analyzed, i.e., the LADWP water system. It includes locations and key attributes for each entity (e.g., trunk line location, size, and capacity). Multiple realizations of the initial post-earthquake damage and serviceability of the system can be obtained from an initial run of GIRAFFE. Each realization of the damage includes the numbers of breaks and leaks on each length of trunk line and the post-earthquake demand at each demand node. For each realization of damage, GIRAFFE also provides a corresponding description of serviceability that indicates whether or not each demand node is being served (i.e., whether water is getting through the trunk network to the distribution pipes represented by that node).

The restoration model is run for all GIRAFFE damage realizations and results can be obtained for each earthquake-damage state combination separately, or by combining the results for all damage state realizations, a single set of results can be obtained for one earthquake. The model collects the following key types of output:

a. **Restoration curves**. Curves showing serviceability versus time for the whole system and specific subregions, including 90% confidence intervals that capture uncertainty in the restoration process. Several scalar values derived from these curves are also provided, including the serviceability at any time *t* (percentage of demand met), average time each customer is without water, and the time require to restore the system and each service area to 90%, 98%, and 100% serviceability.
b. **Spatial distribution of restoration**. Serviceability and demand node restoration times can be mapped to show the spatial evolution of the restoration process.
c. **Crew usage**. Total time idle, traveling, and working for each type of crew, by reporting location.
d. **Material usage**. Number of materials used during each 12-hour period, by district yard and material type.

5. MODEL CALIBRATION TO 1994 NORTHRIDGE EARTHQUAKE

As a recent, well-documented earthquake affecting the LADWP service area, the January 17, 1994 Northridge, California earthquake (M_w6.7) provided data for calibrating the post-earthquake restoration model. The earthquake caused significant damage to the LADWP water system, mostly concentrated in its northern and central areas. There were 70 and 1,013 repairs made to LADWP trunk and distribution lines, respectively (Jeon and O'Rourke 2005). As a result of the damage, 114,000 service connections, or roughly 450,000 people, lost water service. Twenty-five percent of those connections were restored after one day, 65% after 3 days, 94% after 5 days, and virtually all after 7 days. Since the preliminary calibration results presented in Tabucchi and Davidson (2008), GIRAFFE has been extensively debugged, and importantly, the ability of tank water levels to rise and fall during the course of the restoration has been incorporated.

Figure 1 shows the results of 10 runs of the restoration model assuming the observed Northridge earthquake as the input damage scenario. As expected, there is variability across the individual runs, but they are all reasonable realizations of what might have happened, and they show the correct trends. Further, the average restoration curve and confidence intervals follow the actual Northridge restoration curve quite closely. Comparison of the spatial evolution of the restoration and other outputs were also used in calibration of the model, but those results are not shown here due to space limitations.

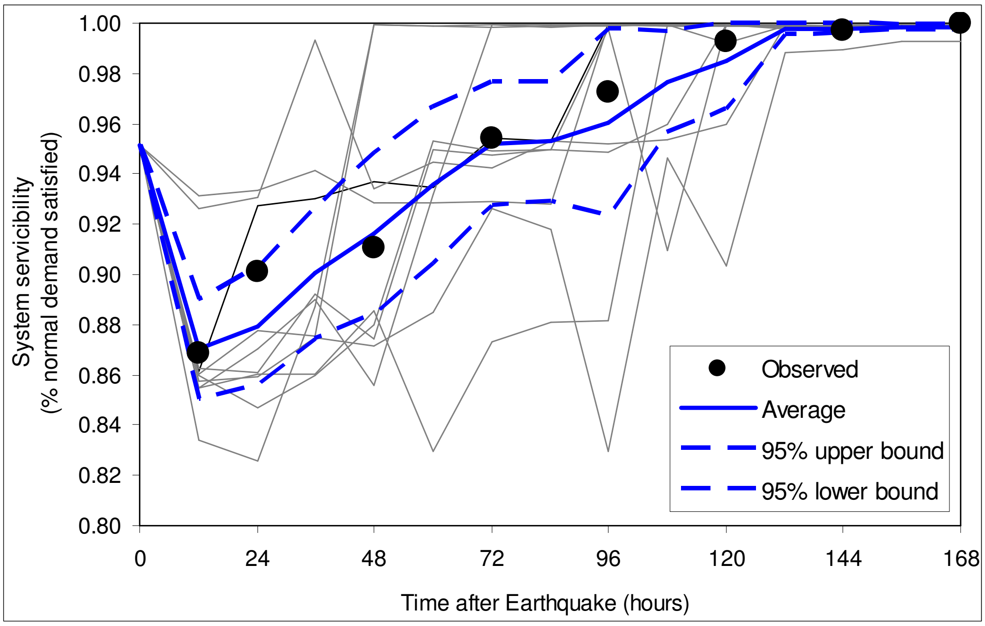

Figure 1 Actual 1994 Northridge earthquake restoration curve and 10 simulation model curves from calibration.

6. MODEL USES AND CONCLUSIONS

The restoration model can be applied to estimate the restoration for any real or hypothetical earthquake, and therefore can be useful in a few different ways. It can be used for earthquake loss estimation and resilience assessment as part of long-term planning, or even immediately following an earthquake to give an early estimate of restoration time before damage assessments are available from the field. Because so many of the utility company's decision variables are represented explicitly in the model (e.g., numbers and types of crews, task prioritization rules), by varying those parameters and seeing their effect on final restoration times, the model can be an aid in evaluating the effectiveness of possible restoration improvement activities. In general, varying the many model parameters and studying their effects can improve understanding of the restoration process and its key determinants. Finally, the model's output may be useful in supporting post-earthquake fire modeling, since water availability is an important factor in post-earthquake fire spread.

Compared to previous restoration models, this new model includes very limited simplifications of the water system and the restoration process, which should lead to a more accurate, highly detailed representation of the restoration process. It interacts closely with the damage and system functionality model for the Los Angeles water system, GIRAFFE, and thus makes use of the most realistic available methods for estimating post-earthquake damage and system functionality. The restoration model produces multiple forms of output, including system and subregion restoration curves with uncertainty estimates, spatial distribution of restoration, and information on crew and material usage. As the first application of discrete event simulation to restoration of water supply systems in particular, the model is novel in representing thousands of entities (compared to tens in electric power) and accommodating a corresponding increase in complexity of the system and the restoration process; coupling the restoration model with a damage and functionality estimation model; and incorporating rerouting and damage isolation explicitly.

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