A RISK ASSESSMENT METHOD FOR TRAFFIC FUNCTION OF HIGHWAY BRIDGES WITH EARTHQUAKE DAMAGE

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ABSTRACT :

This study proposes a method of assessing loss in surplus value of traffic flow in highway network with many damaged bridges so as to estimate the optimal allocation budget for seismic retrofitting.
The deficit value of traffic flow in damaged network that means drivers' disutility takes a role in estimating one of risks. The study, employs long-term Risk-Shifting problem such as a compensation mechanism in order to indemnify the drivers' disutility, analyzes the problem between the travelers and the administrative body who have asymmetrical information about highway facilities and/or traffic conditions. This problem can be feasible to assess drivers' travel risks from designing compensation system, in which the amount of losses many drivers would suffer during a disaster can be provisionally shifted toward the administrative body as insurance premium. In the scheme, thus the administrative body can carry seismic retrofitting projects under aggregated premium on the basis of prioritized order in accordance with evaluated risks of being damaged to bridges. Therefore, the premium has an ability to assess travel risks.
In the paper, the optimal solution is proved that travelers' losses cannot be guaranteed unless additional reparation is concerned. Besides, it is confirmed that the amount of optimally additional compensation has a strong point to evaluate risks in trips as well as to estimate the optimal allocation budget for seismic preparedness on highway bridges.
The paper presents the mathematical analysis and the numerical result of applying to the highway network in Tokyo Metropolitan area.

KEYWORDS: Risk-Shifting Problem, Risk Assessment, Highway Network Flow Service, Insurance Premium

1. INTRODUCTION

We suppose the reasons why all but projects of highway infrastructure themselves are recently faced with severe social criticism in Japan are that there exists an underlying structure of opinionated public opinion against the general inherence to depreciate marginal cost of production within infrastructure. It is, also leading to the skepticism with narrow range of views for those projects, leaving the controversies unsettled; whether social capital stock of those have been still satisfied.
New public investment for infrastructural construction projects - in rural area, for instance, have to discreetly implement with accurate and fair assessment on the basis of critical studies based on a long term viewpoint of analysis of costs vs. benefits. In the Metropolitan area such as Tokyo, on the other side, the truth of the matter is that both to research the best method for infrastructural maintenance and to conduct practical utilization of social capital stock for other needs have to be required, since new public investment is unexpected to remain strong or progress in future.
However, it is now also fact that the needs for improvement in the reliability of infrastructures on and after an earthquake have been highly grown since the Hyogo-ken Nambu Earthquake of January 17, 1995. On a purely road traffic matter, it goes without saying that to draft countermeasures against deterioration of traffic services is one of the most vital and urgent task for the regional disaster damage prevention plan.
It needs a method to contribute for evaluating an ability of elasticity to withstand quake not to deteriorate the quality of traffic service, on the occasion of disaster from the safety-oriented point of view, while there are firmly rooted practices to take countermeasures employing seismic retrofitting for bridges, owing to one of factors influencing the reliability to traffic service.
Tokyo Metropolitan Government has, in the behalf of holding for seismic shocks, fixed the order of priority for bridges on highways, utilized for disaster relief, taken seismic retrofitting projects using its prioritized order (2004-2).
However, it is not clear whether those projects have been carried out according to an evaluation of maintaining reliability within traffic service.
Social infrastructure must be defined as one of assets to make a profit and offer convenience to road users. Thus there exits a mention to be deserved that assets evaluation of network-type infrastructure such as transportation facilities should be given consideration to spillover effects, external effects, and risk of losses from earthquake damage. Tsukai et al.(2002-2) offered a model of production function simultaneously capable of measuring both direct and indirect spillover effects, then showed that highway infrastructure improvement projects in one region are able to contribute to increase productivity in other region's. This study offered valuable knowledge to urge the fundamentals of measuring flow value within transportation facilities. Ito and Wada (2002-1) also, estimated life-cycle cost of highway bridges incorporating earthquake loss, therefore urged the importance of considering transportation network flow analysis. On the other hand, Nakamura and Hoshiya (2004-1) applied the Discounted Cash Flow method to measure assets value of highway bridges, combining cost of seismic retrofitting projects estimated from risk of losses by quake damage as well as earthquake insurance cost.
The study holds the essence of a matter that loss value by earthquake risk is needed to take the same framework of assessing whole value of infrastructure assets into account.
Based on a viewpoint of having reckoned up the amount of losses caused by physically damaged as well as deteriorated service value on an expected damage, this study offers an assessment method, for risk of losses in network flow value, which would be deteriorated by seismic damage of highway bridges, and then to accurately measure its effectiveness of retrofitting projects.

2. DAMAGE TO HIGHWAY BRIDGES AND RISK-SHARING PROBLEM

2.1. Risk-Shifting Agreement

Now let us examine a situation in which some highway bridges suffer physical damage then are out of service. Many highway users, generally speaking, would face risks of getting prolonged travel time and trip delay for expected schedule time. It is reasonable for representative traveler to hold request that his or her own disutility be alleviate or set off provided that he or she can have risk-averse type utility function to travel. Risk-Sharing scheme is one of the system designs aiming at alleviating user's disutility or shifting their risk to others. The Risk-Pooling, is scheme aiming at alleviating risk as a whole, implements agreement/contract, under which all users have to share an averaged risk of pooled risk every user himself provides an individual one's. On the other hand, Risk-Shifting scheme consists of agreement/contract, under which lower risk-averse type users and risk-neutral users take over the risks risk-averse users can transfer (1979-3). The study treats the latter problem.

2.2. Long-Term Risk-Shifting Agreement and Asymmetrical Information

Let a road user define a contracting party or policyholder faced with risk, and let an administrative body define an insurer characterized as risk-neutral. Basic framework of example defines the agreement, in which the administrative body has an obligation to take over in some kind of manner risk of varying in travel time caused by traffic confusion originated from damage to highway bridges in the wake of an earthquake; namely disutility road user would face.

2.2.1 Mathematical Formulation

Here we are considering the same form of problem such as an insurance contract that an insurer warrants loss of policyholder's. There is one point to be paid attention to examine above agreement that an insurer (an administrative body) hardly grasps about policyholder's (road user's) true profit or loss (for instance, expected travel time) from production opportunities to behave (trips), since that bit of information cannot but become quit private property under the present circumstances. Therefore, analysis for risk within travel time fluctuation contains asymmetrical information problem in above framework.

Now let policyholder's expected outcome about profit denote y_1 and y_2, $(0 < y_1 < y_2)$, subjective incident probability of theirs denote π_1 and π_2, $(\pi_1 + \pi_2 = 1,\ \pi_i > 0, i = 1,2)$. Let policyholder, be risk-averse, have strictly monotonous increasing utility function $U(y)$: $R^+ \to R, (U'(\cdot) > 0, U''(\cdot) < 0)$ such as von-Neumann Morgenstern type one. In addition, let form of an agreement between the parties be two-term contract, the discount rate during the agreement be disregarded and trip opportunities for policyholder be independent as to time. Long-term Risk-Shifting agreement problem under asymmetrical information is formulated in formulas (2.1)-(2.4) as mathematical optimization problem.

$$Max._{\{X_I, X_{IJ}\}} W = \sum_{i\in I} \pi_i \left[U(x_i) + \sum_{j\in J} \pi_j U(x_{ij}) \right] \tag{2.1}$$

$$s.t.\quad U(x_i) + \sum_{j\in J} \pi_j U(x_{ij}) \geq U(y_i - y_k + x_k) + \sum_{j\in J} \pi_j U(x_{kj}),\ (i \neq k) \tag{2.2}$$

$$U(x_{ij}) \geq U(y_j - y_k + x_{ik}),\quad (j \neq k) \tag{2.3}$$

$$\sum_{i\in I} \pi_i \left[(y_i - x_i) + \sum_{j\in J} \pi_j (y_j - x_{ij}) \right] \geq 0 \tag{2.4}$$

In above formulas x_i and x_{ij} means the revenue policyholder can gain when he or she declares one's own experience state i on the first phase and the other when he or she declares one's own experience turned-to-be state j on the second phase following state i, respectively. Formula (2.1) and (2.2) prescribe the condition of compatibility for incentive to policyholder (hereinafter IC condition) and formula (2.4) is entry condition for an insurer. As it has to be needed to consider a false report from policyholder in case that only policyholder can hold his or her own information perfectly such as agreement under asymmetrical information framework (equivalent to the cases that an administrative body scarcely ever observed true losses of travelers'), the IC condition can incorporate false report for that purpose. The entry condition is that deliberates upon putting restrictions on an insurer's loss to be no more than zero based on IC condition. Policyholder's expected profit yields $\sum_{i\in I} \pi_i y_i$ as well.

Now counting on false report from policyholder and putting maximizing condition into practice, condition (2.4) can be transformed into the formula by changing some variables (1979-2) as formula (2.5). This formula is, the entry condition for an insurer expecting policyholder's false report in advance, substantially able to arrive as the same condition as condition (2.4). Therefore, the optimal solution can be restricted to the truly experienced state reported from a policyholder nonetheless an insurer reckons false report (1995).

$$\sum_{i \in I} \pi_i \left[\left(y_i - x_i^* \right) + \sum_{j \in J} \pi_j \left(y_j - x_{ij}^* \right) \right] \geq 0 \tag{2.5}$$

2.3. Example of Numerical Operation

Here a quite simple example of numerical simulation for the long-term Risk-Shifting agreement (2.1)-(2.4) is given as below. Let the example be assumed that there is a highway bridge being impassable. The purpose of its numerical example is to examine fluctuation of representative policyholder's profit according to his or her travel time varying with changing state. Policyholder's subjective incident probabilities and expected profits before and after an event are shown in Table-1. The scenario established in Table-1 is that policyholder's expected profit is 99.9 with its likelihood of 0.999 before an event (lucky state) and 1.0 with 0.001 after an event (hard luck state), respectively. The expected profit between the phases yields 98.902. As policyholder's strictly monotonous increasing and concave utility function is given as $U(x) = -\exp(-\alpha x), (\alpha > 0)$, the result of the operation yields as in Table-2 when $\alpha = 1.0$. The value of the objective function, of course, arrives at 0.0.
According to the trial calculation, it is obvious that the strategy for an insurer fixes the best-reasonable one in case of stable state if the insurer allocates the insurance benefit equivalent to policyholder's expected profit at the end of respective phases. On the other hand, under a case in which the phase shifts from hard luck to luck, the best-reasonable strategy is to pay insurance benefit equivalent to policyholder's expected profit at the first phase and next to pay premium more profitable than his or her expected profit at the following phase. Also it becomes vice versa under the case in which the state changes toward the worse.
It is clear that the total profit over the period under the stable state comes to the summation of an expected profit a policyholder would anticipate on the respective phases, meanwhile that of the turn-to-getting-better state rises more than policyholder's expected profit and vice versa under the turn-to-getting-worse state, shown as Table-2 in row 3.
Therefore it can be pointed out that there exists the reason to be imperative and crucial for an administrative body to secure road users' risks of being decreased in profit during a situation where road users' travel time would change catastrophically owing to disruption/detour in highway network.
It is, however, needed to paid attention that, since as obvious as described in formula (2.2) and (2.3) and implication of those formulas, policyholder's profit on an end of a phase, as of total itself, becomes $y_i - y_j + x_k^*$ and $y_j - y_k + x_{ik}^*$, respectively. Thus policyholder's net profit comes to the summation of gross profit gained from truly own experience, contract money or premium paid for warrant and insurance benefit paid back from an insurer. In addition, an insurance benefit paid back on the second phase must maximize the summation of profit on both the preceding phase (the surplus or deficit) and itself as in Table-2, column 3.

3. NEUMERICAL ANALYSIS IN REAL NETWORK

This chapter does the feasibility study, and then examines some problems to evaluate surplus value of traffic flow derived from a public enterprise, for instance, effects of retrofitting projects for highway bridges by applying the framework of the Risk-Shifting agreement, which was basically reviewed in the preceding chapter.

3.1. Requisition of Traffic Flow Model and Characteristics of Demand

Traffic demand and network flow during disaster, usually, strikingly differ from those of normal. It has the root

Table-1 Configuration of A Simple Example Case

Phase	π_i	y_i	$\pi_i y_i$
After $(i=1)$	0.001	1.000	0.001
Before $(i=2)$	0.999	99.000	98.901
Total	1.000	100.000	98.902

Table-2 Results of A Simple Example Case

Phase	x_i	x_i	$\sum x_i + x_{ij}$
$i=1,\ j=1$	98.9020	98.9020	197.8040
$i=1,\ j=2$	98.9020	112.7155	211.6175
$i=2,\ j=1$	98.9020	85.0885	183.9905
$i=2,\ j=2$	98.9020	98.9020	197.8040

in some difficulties, in advance, attributing the disaster-stricken area, where becomes the source and sink point of demand and estimating scale of disaster, which influences traffic demand volumes. It is also no exaggeration to say that travelers' behavioral criterion to travel under such circumstances would hardly be the same as daily.
As considering the requisites for knotty problem of those, estimation model ought to have the capability to stochastically represent demand changes and trip behavior. Moreover, its model should be able to logically explain elasticity between the demand and network flow performance such as the combined model of trip distribution and assignment. Hence the study employs the stochastic user equilibrium traffic assignment with elastic demand model, which can logically describe varying demand and representative traveler's path choice behavior based on discrete choice theory, i.e., Multinomial Logit model, as network flow estimation model.

3.2. Network Traffic Flow Estimation Model

The study defines that source demand (Origin) is given. Thus network traffic flow estimation model is one-sided constrained stochastic user equilibrium traffic assignment with elastic demand model, which is mathematically incorporating assignment estimation with OD trip distribution estimation into integrated model based on Multinomial Nested Logit model.
Let a trip maker's origin, destination and chosen path denote r, s and k, respectively, and let conditional utility on k with given rs pair, combinational utility on rsk and utility being independent of choosing k denote $U(rs,k)$, $U(rsk)$ and $U(rs)$, respectively. Conditional utility ;

$$U(rs,k)=U(rs)+U(rsk), \qquad \forall k \in K_{rs}, r \in R, s \in S \tag{3.1}$$

where

$$U(rs)=-C_{rs}+\xi_{rs}, \forall r \in R, s \in S \tag{3.2}$$

$$U(rsk)=-c_k^{rs}+\varepsilon_k^{rs}, \forall k \in K_{rs}, r \in R, s \in S, \tag{3.3}$$

Here k_{rs} R and S represents the set of path with given r and s, the set of origin and the set of destination, respectively. C_{rs} is a fixed cost being independent of path between the OD pair and c_k^{rs} is a travel cost on path k between the rs OD pair. ξ_{rs} and ε_k^{rs} are random error giving analysts traveler's cognitive bias of path and destination choice behavior, independently ruled by Gumbel distribution with respective parameters. Usually, every ξ_{rs} and ε_k^{rs} is ruled by independent Gumbel distribution with its location parameter η and positive scale parameter μ, i.e., $G[\eta, \mu]$. This study assumes an error $G[0, \theta]$.
The study postulates travelers' behavioral norm to be the state in which no user is firmly persuaded of one circumstance to economize their own destination and/or path choice cost by altering their own destination as well as path. Assuming that the above-mentioned criterion is adequate, user's travel choice behavior can formulate on the basis of Multinomial Nested-Logit model as destination choice to be the prior problem and path choice to be subordinate one. Giving numerical expression $P[k \mid rs]$ and $P[s \mid r]$ as a conditional probability of path choice between the rs pair and that of destination choice with origin r,

$$P[k|rs]=\frac{\exp\left(-\theta_1 c_k^{rs}\right)}{\sum_{k\in K_{rs}}\exp\left(-\theta_1 c_k^{rs}\right)}, \forall k \in K_{rs}, r \in R, s \in S \tag{3.4}$$

$$P[s|r] = \frac{\exp[-\theta_2(C_{rs}+S_{rs})]}{\sum_{s\in S}\exp[-\theta_2(C_{rs}+S_{rs})]}, \forall r\in R, s\in S \tag{3.5}$$

Here θ_1 and θ_2 is dispersion parameter ruling path choice probability and destination choice probability. S_{rs} is log-sum variable representing expected minimum cost of path choice between the ODs based on expected utility maximizing theorem ;

$$S_{rs} = -\frac{1}{\theta_1}\ln\left[\sum_{k\in K_{rs}}\exp\left(-\theta_1 c_k^{rs}\right)\right] \tag{3.6}$$

According to stochastic formulas as shown in formula (3.4) and (3.5), the k -th path flow volume between the rs pair f_k^{rs} and OD volume q_{rs} is formulated from given origin demand as formula (3.7) and (3.8).

$$f_k^{rs} = q_{rs}\ P[k|rs], \quad \forall k\in K_{rs}, r\in R, s\in S \tag{3.7}$$
$$q_{rs} = O_r\ P[s|r], \quad \forall r\in R, s\in S \tag{3.8}$$

The network flow satisfying formulas (3.4)-(3.6) as well as flow conservation conditions (3.7) and (3.8) prove to be an equivalent mathematical nonlinear optimization problem as :

$$Min.\ Z(x(f),q) = \sum_{a\in A}\int_0^{x_a} t_a(\omega)d\omega + \frac{1}{\theta_1}\sum_{r\in R}\sum_{s\in S}\sum_{k\in K_{rs}} f_k^{rs}\ln\left(\frac{f_k^{rs}}{q_{rs}}\right) + \frac{1}{\theta_2}\sum_{r\in R}\sum_{s\in S} q_{rs}\ln\left(\frac{q_{rs}}{O_r}\right) + \sum_{r\in R}\sum_{s\in S} q_{rs}C_{rs} \tag{3.9}$$

$$s.t.\quad \sum_{s\in S} q_{rs} = O_r, \quad \forall r\in R \tag{3.10}$$
$$\sum_{k\in k_{rs}} f_k^{rs} = q_{rs}, \quad \forall r\in R, s\in S \tag{3.11}$$
$$f_k^{rs} \geq 0, \qquad \forall k\in K_{rs}, r\in R, s\in S \tag{3.12}$$
$$q_{rs} \geq 0, \qquad \forall r\in R, s\in S \tag{3.13}$$

3.2.1 Solution Method

Partial Linearization Algorithm (hereinafter abbreviates as PLA) can solve the mathematical formulas (3.9)-(3.13). Transforming path flow variable of the 2nd term expressed by entropy in right hand of formula (3.9) into link flow volume variable every origin nodes, and then applying the Dial algorithm in order to avoid enumerating a large number of paths, finally carrying out unidimensional search by the Frank-Wolf method, the mathematical problem can be equivalently solved.

3.3. Network Topology and Its Constitution

The purpose of the study is to establish and verify an evaluation system for effectiveness on public investment of retrofitting projects, on the basis of fundamental characteristics of Risk-Shifting agreement, under network flow performance and travel time of travelers' fluctuating before and after damage. Thus the network being investigated is on Tokyo metropolitan area but abstracted only major highways from actual network. Let its topology of the network and locations of highway bridges be real while characteristics of every link (for instance, link capacity, free flow speed, the number of lanes and so on) be minimally modified. The network, illustrated in Figure-1, is composed of 340 nodes (including centroids, i.e. source/sink points), 1038 directed links (including dummy links) and 84 OD pairs.

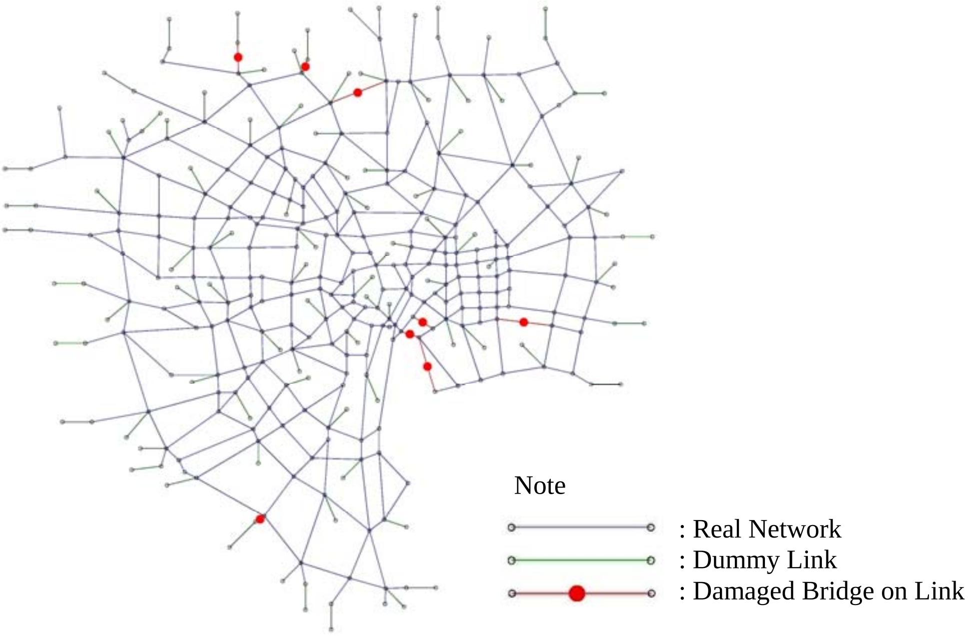

Figure-1 Network Topology and Bridges to Damage

3.4. Model Parameters Arrangements

The details of parameters for the network flow model mentioned above are shown in Table-3. Every OD pair volume will change in searching process for the optimal equilibrium point, since feasible solution of those are elastically estimated in accordance with network flow performance in the operational process. However, the initial OD pair volumes employ a traffic census data; approximately 4 million vehicles per day in control total. Every average travel time between the OD pair observed by some field survey substitutes for the fixed trip cost.

Hourly capacity of every directed link is at first commensurately calculated to the number of lane with the postulation that link hourly capacity per lane = 1,800 vehicles/hr./lane. Next, an entire day capacity of every directed link is calculated by conversion rate for entire day capacity, γ_a, as 17.9325 (1989), since the analysis operates in units of 24 hours. Directed link performance function uses modified BPR function (developed by US Bureau of Public Road) and its exogenous parameters are equals in every link (2003). Free travel time of links, $t_a(0)$, is reckoned by its distance and speed of 48, 60 and 72 km/hr, respectively, in proportion of the number of lanes on one side of street.

Probability distribution of destination and path choice generally gets decreased as variance factor $\theta \to +\infty$, i.e., control variable for distribution θ_1 and θ_2. Although parameter θ_1 especially has more effect on estimating directed link volumes without the range of $[0.1, 1.0]$ (2004-2), an elastic demand type model, however, cannot be solved straightforwardly, since θ_1 and θ_2 affects each other in estimating process. Upon this, both control variable of destination choice probability and path choice one, as shown in Table-3, are calibrated so as to increase variance.

3.5. Network and Bridge Damage Configuration by Entropy Model

The study employs a concept which intervenes in the derivation and calibration of damage configuration of bridges in network. This is the concept of entropy of a probability distribution function. Consider the set of bridges b_{gs}^d, representing the number of bridges, which is categorized by site amplification factor g, seismic intensity scale of the Metrological Agency of Japan, s and the extent of damage d. The total number of bridges is given, and is equal to B_{gs}. Clearly, it is possible to allocate, in several different ways, the same total number of bridges B_{gs} to any extent of damage d under the specific site condition and estimated seismic intensity scale, so as to attribute the given extent of damage b_{gs}^d. Generally, the number of different allotments N

Table-3 Exogenous Model Parameters Arrangements

Parameters/Control Variables	Specification	Notes
OD Pairs	84×84=7,056	Not included inner volumes
Initial OD Volumes	$\cong$4 million veh./day	Control total of Census
Links	1,038	Both sides
Nodes	340	Includes source/sink points
Link Capacity C_a	1,800 veh./hr./l.	Identical in links
Lanes	1, 2, or 3 on one side	-
Conversion Rate for Entire Day γ_a	17.9325	Identical in links
Free Flow Speed $t_a(0)$	40, 60, 72 km/hr.	-
BPR's Parameter α	2.63	Identical in links
BPR's Parameter β	5.01	Identical in links
Control Variable for Destination choice θ_2	0.10	Identical in links
Control Variable for Path Choice θ_1	10.0	Identical in links
Fixed Cost C_{rs}	Substitute Observed Average Trip Time	

of B_{gs} bridges which leads to a given the extent b_{gs}^d is function of the values b_{gs}^d. It is well known that the number of ways in which, out of a total of b_{gs} objects, b^1 are allotted to class 1, b^2 to class 2, b^d to class d, and so on, is equal to $N(b)=\prod_g\prod_s B_{gs}!/\prod_d b_{gs}^d!$. Since N becomes typically very large, its natural logarithm, which is called the entropy, is used instead to characterize the distribution. In general, therefore, the entropy E of an allotment of values is thus equal to $E(b)=\ln N(b)=\ln\prod_g\prod_s B_{gs}!/\prod_d b_{gs}^d!$. The importance of the entropy stems from its feasibleness to help estimate or identify damage on every bridge by only simple factor (for instance, site amplification factor, seismic intensity scale and so on) that every bridge is assumed to be categorized as the same, or similar, then the most probable allotment of N, the maximum entropy, since the logarithmic function increases monotonically with its argument.

The entropy of given allotment b^d is maximum when the allotment is uniform. Conversely, it achieves its minimum value when the allotment is concentrated in a single value, $b^d=0$ for all d s' except one. Thus this value measures the amount of information the allotment of values contains. That is, when the allotment is uniform, it contains no information about the values whatsoever. At the other end of the scale the information is the maximum, since one value has been singled out. Maximum entropy/minimum information corresponds to maximum dispersion, and vice versa. The most probable distribution is also the least information one.

In practical situation, the magnitude of the B_{gs}, measured in numbers of bridges, is in hundreds in Tokyo. In general, then Starling's approximation formula is used. So do the study.

This states that when x is large, $\ln(x!)\cong x\ln x-x=x(\ln x-1)$. With this approximation, the entropy is equal to $\ln E(b)=\sum_g\sum_s\left(B_{gs}\ln B_{gs}-\sum_d b_{gs}^d\ln b_{gs}^d\right)$. Therefore, the most likely allotment of damaged bridges in network configuration is to maximize $\ln E(b)$. As B_{gs} is given, the maximization is sufficient to carry out only on a term about b_{gs}^d.

Now let the amount of the degree about ability to withstand seismic shock every bridge has consider. Supposing that this value is assumed and/or observed to be no more than $\hat{P}$, the maximum entropy of allotment of bridges is formulated as a mathematical optimization problem :

$$Max.\ Z(b)=-\sum_g\sum_s\sum_d b_{gs}^d\ln b_{gs}^d \tag{3.14}$$

$$s.t.\quad \sum_g\sum_s\sum_d b_{gs}^d r_{gs}^d\le\hat{P},\quad \forall g\in G, s\in S, d\in D \tag{3.15}$$

$$\sum_d b_{gs}^d = B_{gs}, \qquad \forall g \in G, s \in S \tag{3.16}$$

$$r_{gs}^d \geq 0, \quad b_{gs}^d \geq 0, \qquad \forall g \in G, s \in S, d \in D \tag{3.17}$$

Here r_{gs}^d is the degree of resistance to seismic shock defined in the study as $r = y / \left[l\,(1.0 + gr)^y \right]$, where y, l, gr and a represents service year of a bridge, length of a bridge, growth rate of traffic volume and estimated on-site maximum seismic acceleration, respectively.

3.5.1 Solving Method
In order to solve above the mathematical optimization problem to attribute damage toward bridges, the study uses the PLA mentioned in *3.2.1*. Since formulas (3.14)-(3.17) have an entropy term in the similar manner to the formulas (3.9)-(3.13), this formulas can be logically transformed into the form of a numerical expression analogous to the formulas (3.9)-(3.13) from mathematical equivalency of entropy with Multinomial Logit model (1969,1977,1979-1).

3.5.2 Damage Solution
According to the operation, the 8 bridges out of 58 subjects are chosen to attribute damage; 5 bridges with totally collapsed and 3 brides with seriously damaged. These locations were still illustrated in Figure-1.

4. USERS' LOSS AND RISK-SHIFTTING AGREEMENT

4.1. Generalized Cost
User's profit assessed upon effects of highway maintenance, improvement and/or retrofitting projects is, based on the relation between the performance of traffic service and traffic demand, defined as consumer (user) surplus. It is assumed to be produced from having users' total bearing financial and schedule cost reduced. It, is generally called generalized cost, means all of cost saved. The study, contrarily, treats increased generalized cost as user's loss occurred by mainly wasting of time.
Let travelers' losses and generalized cost before and after seismic damage denote D and P^A, P^B, respectively and let OD pair demand volumes before and after damage denote Q^A and Q^A.

$$D = \sum_{r \in R} \sum_{s \in S} \frac{\left(P^A - P^B\right) \cdot \left(Q^A + Q^B\right)}{2} \tag{4.1}$$

Besides, there are three types of generalized cost achieved from the stochastic user equilibrium model; minimum cost, expected minimum cost and average cost. The average cost is easily produced by PLA method employed in the study. The average cost (generalized cost) between the origin r and destination s, P_{rs}, is formulated with denoting k for any path,

$$P_{rs} = \frac{\sum_{k \in K} f_k^{rs} \cdot c_k^{rs}}{Q_{rs}} = \frac{\sum_{ij \in L} x_{ij}^{rs} \cdot t_{ij}\left(x_{ij}\right)}{Q_{rs}} \tag{4.2}$$

Since PLA method is not needed to enumerate path volumes between the ODs, as seen in the 3rd term of formula (4.2), only link volumes every origin and destination can produce average cost.

4.2. Analyzed Network Performance
4.2.1 Damage scenarios
Let the study have 2 seismic damage scenarios. The damage scenario of case-1 has 5 bridges collapsed by

Table-4 Results of Users' Losses

Operation Case	Iteration	Travelers' Losses (veh.hr.)	
		Daily	Damaged
Case-1 (Daily vs. All Damaged)	17	4.0464	1.6272×10^{7}
Case-2 (Daily vs. 1 damaged)	10	4.0464	1.2338×10^{6}

100.0% and 3 bridges collapsed by 50.0%. On the other hand, damage scenario of case-2 has only 1 bridge collapsed totally. This bridge is evaluated the most likely one to collapsed by the damage entropy model solved in chapter 3, since the bridge is located on the largest site amplification factor in the study area and estimated to suffer an earthquake registering 6-plus on the Japanese seismic scale against the most severest seismic fault.

4.2.2 Network Performance

Network flow under the damage scenarios are estimated as illustrated in Figure-2 and 3. Undoubtedly, the damage case-1 becomes more severe state of congestion than the case-2. It is proved from comparison between the cases that traffic volumes on some comparatively non-congested links increase. Besides, the tendency is remarkable on the highway in downtown area. Therefore, it is clearly understood that a traffic flow pattern can change without difficulty and trip makers are likely to flow toward less congested highway while they keeps away from damage and/or congested highways.

4.2.3 Travelers' Losses

Travelers' losses carried out on the operational cases based on the exogenous model parameters described in previous chapter are shown in Table-4. "Daily" seen in Table-4 is the dummy case that is prepared for comparative study, with increase of generalized cost calibrated to be precisely 1.0^{-5}% under the same OD volumes. The number of iterations to converge for the equilibrium point of objective function is shown too.

According to the results of operation, it is quite obvious in comparing case-1 with case-2 that although travelers' losses seem to have a trend toward aggravation in waste of trip time in accordance with the extent of damage, beyond expectation, the amount of travelers' losses in waste of time does not necessarily improve in proportion to the number of damaged bridges. This result is surmised that different pattern of fluctuation in generalized costs as well as OD demands occur between the respective OD pairs on the operational cases with different damage scenarios. Therefore, it is corroboratively demonstrated that only to estimate/evaluate variation in traffic volume of one directed link with damaged bridge can never assess travelers' surplus or losses since traffic flow pattern in network is apt to readily change with even minor seismic damage.

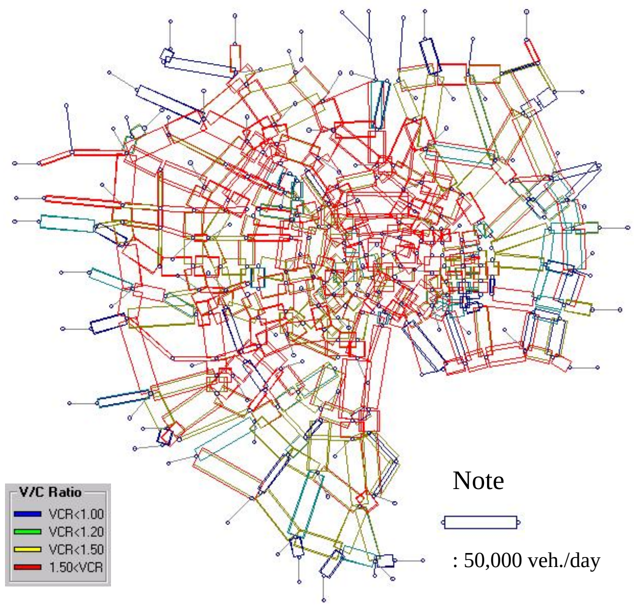

Figure-2 Network Performance (8 Damaged Bridges)

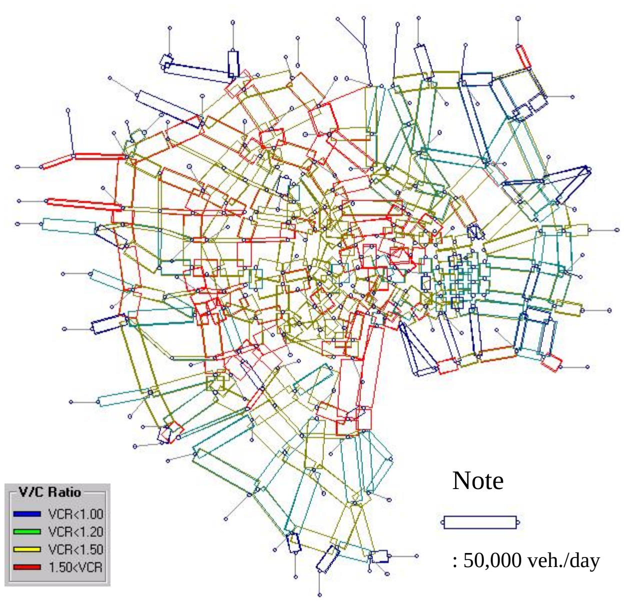

Figure-3 Network Performance (1 Damaged Bridge)

Table-5 The Results of Risk-Shifting Agreement against Damaged Highway Network

Operation Cases	Phase	x_i	x_{ij}	$\sum x_i + x_{ij}$
		(10 billion yen/day)		
Case-1-1 8 bridges damaged $\pi_1 = 0.0001$ $\pi_2 = 0.9999$	$i=1,\ j=1$	135.9536	135.9536	271.9071
	$i=1,\ j=2$	135.9536	154.3740	290.3276
	$i=2,\ j=1$	135.9536	117.5331	253.4866
	$i=2,\ j=2$	135.9536	135.9536	271.9071
Case-1-2 8 bridges damaged $\pi_1 = 0.3000$ $\pi_2 = 0.7000$	$i=1,\ j=1$	119.5571	119.5571	239.1141
	$i=1,\ j=2$	119.5571	121.2517	240.8087
	$i=2,\ j=1$	119.5571	117.8625	237.4195
	$i=2,\ j=2$	119.5571	119.5571	239.1141
Case-2-1 1 bridge damaged $\pi_1 = 0.0001$ $\pi_2 = 0.9999$	$i=1,\ j=1$	135.9586	135.9586	271.9172
	$i=1,\ j=2$	135.9586	154.3791	290.3377
	$i=2,\ j=1$	135.9586	117.5381	253.4967
	$i=2,\ j=2$	135.9586	135.9586	271.9172
Case-2-2 1 bridge damaged $\pi_1 = 0.3000$ $\pi_2 = 0.7000$	$i=1,\ j=1$	134.7153	134.7153	269.4306
	$i=1,\ j=2$	134.7153	136.4099	271.1252
	$i=2,\ j=1$	134.7153	133.0207	267.7361
	$i=2,\ j=2$	134.7153	134.7153	269.4306

Note: i or j =2 implicates the phase before disaster.

4.3. Risk-Shifting Agreement on the Real Network

The Travelers' losses in trip time calculated in the previous section apply into the Risk-Shifting Agreement. The study postulates that every traveler can usually earn same profit equivalent to all expenses incurred during his or her daily trip. Thus it can be defined as traveler's generalized cost. Thus, let travelers' expected profit before disaster be the amount of travelers' generalized cost in daily situation, and those of post-disaster be t travelers' losses in generalized cost. Monetary value of time in traveler's loss is derived from conversion rate of ¥56.0/vehicle/minute (approximately $0.52/veh./min.).

The study examines the optimal solution of Risk-Shifting agreement between the administrative body and the travelers under the scenario of the moderate damaged case as well as the severe damaged case with travelers' subjective incident probabilities of production opportunities set to 0.300 and 0.0001, respectively. On the assumption of disruption and/or malfunction of traffic service, the mathematical operation for the agreement treats 4 cases, of 2 network damaged cases by bridges multiplied by 2 subjective incident probabilities of production opportunities. The results are shown in Table-5.

5. CONCLUSION

As reviewed and analyzed in the study, it is confirmed that to evaluate damage of highway bridges, and to mitigate travelers' disutility and losses by the framework of Risk-Shifting Agreement can become one of useful methods. As mentioned in chapter 2, the study defines π_i as traveler's subjective incident probability for his or her production opportunities. As applying π_i to probability of disruption by damaged highway bridges, the optimal necessary reparation (or deposit money) for all travelers can be yielded. The proposing method for assessing travelers' losses under disaster has the ability to alternatively estimate the amount of monetary compensation indispensable to indemnify losses for users' to directly assess losses by disaster all travelers would suffer. Since a system possessing a capability to allocate the optimal necessary reparation to travelers has not yet established, the proposing method can utilize its effectiveness upon one of the evaluation system for countermeasures to avoid running into malfunction of traffic network and an assessment system for retrofitting projects against hazardous highway bridges.

6. PROOF

6.1. Optimal Condition for Solution to Risk-Shifting Agreement

Since traveler's utility function is strictly and monotonous concave, objective function becomes strictly concave function, and then its feasible space holds on the concave set. Thus the optimal solution, comes to global solution, also holds uniqueness. Therefore optimal condition for solution finally meets second order condition (2006). It is also confirmed from first order condition for optimality that policyholder's profit paid back at the first phase becomes indifference with any situation, and consequently, the amount of insurance benefit paid back to policyholder equals to twofold of policyholder's expected benefit (2006).

6.1.1 Solution Method

As it is clear from the proof of the optimal condition (2006), the problem cannot be solved analytically. Thus applying one of general optimizing methods or heuristic approach is beneficial to solve Risk-Shifting problem.

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