# NONLINEAR-INELASTIC SEISMIC PERFORMANCE OF WATER STORAGE TANKS

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**ABSTRACT :**

This paper evaluates the seismic response of ground-level cylindrical steel water storage tanks, which were designed under the American Water Works Association standards for steel tanks. Three tank sizes were selected to represent different height to radius ratios. All tanks are anchored on concrete ring-wall foundations. Nine earthquake time history records were used to develop response spectrums as input for design level ground motion. A combined fluid-structure finite element model was developed for each of the tank geometries. The computer models were run using linear elastic and nonlinear inelastic model property assumptions. Results of this study show the substantial variation of values in base shear and overturning moment depending on the seismic inputs and modeling assumptions used. The ratios of base shear between linear elastic and nonlinear properties ranged from 1.0 to 3.1, and overturning moment values ranged from 1.0 to 3.0, which are mostly smaller or equal to the design standard reduction R-factored value of 3.0 for anchored tanks.

KEYWORDS: Liquid, storage, tanks, seismic, water, nonlinear, finite element

## 1. INTRODUCTION

Water storage tanks are important components to the continued operation of community water distribution systems in the event of earthquakes. Current knowledge about the performance and seismic response of liquid storage tanks is extensive, but many of the analytical and theoretical results used in current design code approaches are based on a number of simplifying conditions, including small deformation and linear elastic material assumptions. Many water storage tanks consist of thin wall, cylindrical steel shells of constant or varying wall thickness, a base plate which may rest on a concrete foundation, and roof members to support a plate or structural roof system. In the U.S., structures are normally designed to state or local design codes, most of which are based on the International Building Code (IBC). The most current version of the IBC was published in 2006 and includes design approaches for water storage tanks, and incorporates other standards of practice by reference. For water storage tanks the American Water Works Association (AWWA) Standard D100 for welded steel tanks (AWWA 2005) and D103 for bolted steel tanks (AWWA 1997) was used for design. Experiences in past earthquakes have shown tanks to be relatively stable but with various levels of structural damage to both anchored and unanchored ground level water or oil storage tanks under strong seismic events (ASCE 1991).

This paper focuses on three different sizes of ground level steel water storage tanks, which were designed and constructed under the AWWA standards in effect at the time of construction and constructed in different parts of Alaska. All three tanks are anchored cylindrical steel tanks on a concrete ringwall foundation. Tank sizes and dimensions were selected to represent different height to radius ratios, including a broad tank, with a height to diameter ratio of 0.25, a middle range tank with a ratio of 0.75, and a tall standpipe with a ratio of 3.75. Figure 1 shows two of the three tanks and the associated finite element model. Nine time history records were selected as inputs. These time history records were converted to response spectra and used as input into a finite element model for each of the three tanks. The objective of this study was to evaluate the response modification coefficient (R) as used in current design and demonstrate how the variation impacts the results for base shear, overturning moment, and convective wave heights in linear elastic and nonlinear inelastic analyses.

A combined fluid-structure finite element computer model using the computer program ANSYS was developed for each tank configuration with the up to nine seismic events as inputs. Computer models were run using linear elastic and nonlinear material and geometric property assumptions. Results were then compared to the AWWA standard calculations. Results of this study show how the assumptions and material properties selected have a large effect on the modeling outputs.

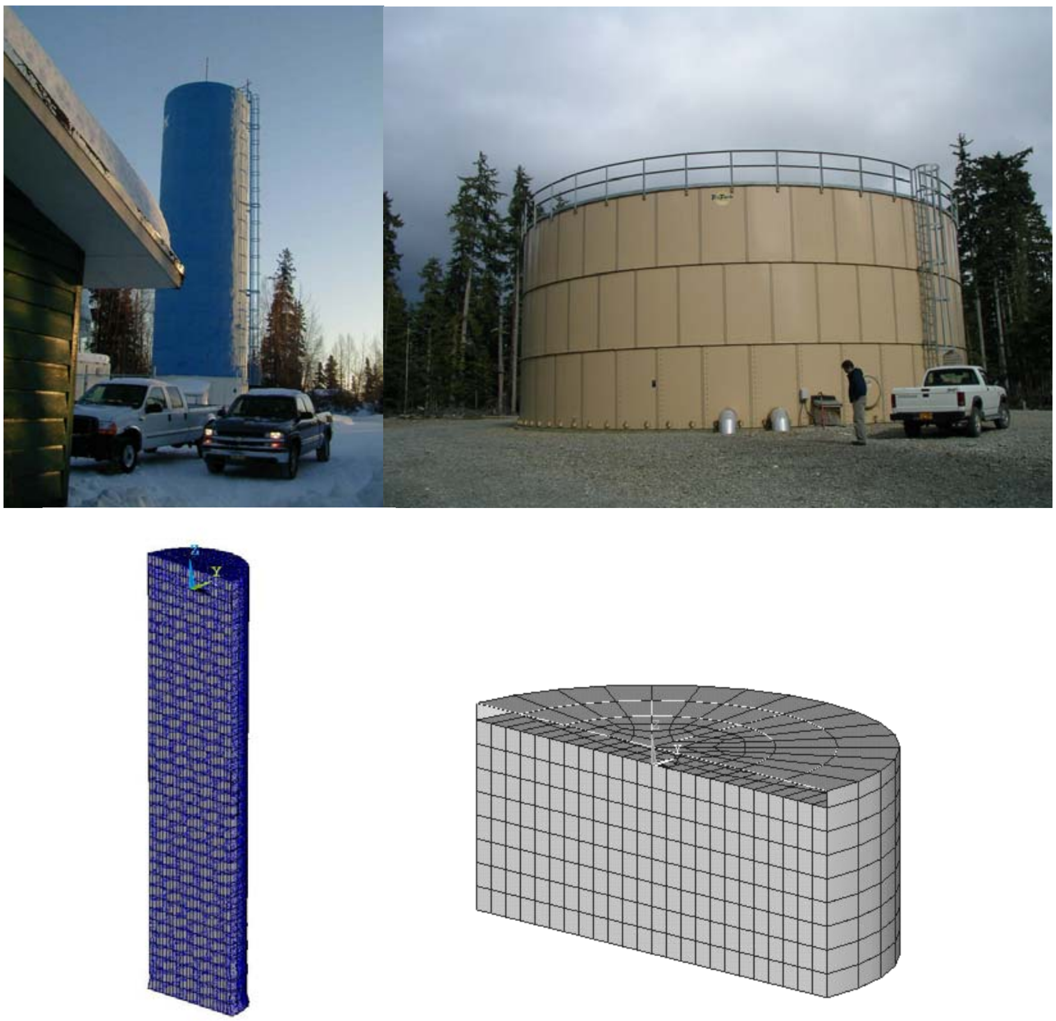

Figure 1 Water Storage Tanks Used in Analysis and Finite Element Model

## 2. BACKGROUND

Current design concepts for water storage tank design considers the response of the tank and contents based on two response modes, a high frequency amplified response to the lateral motion of the tank structure and a portion of the contained fluid that moves with the shell structure, and a low frequency response of the liquid in a sloshing mode. In the available theoretical solution, it is assumed that the liquid is incompressible, inviscid, irrotational potential flow, and that all structural and liquid motions remain within the linear range. The determination of hydrodynamic pressure is based on the solution of the Laplace equation with appropriate boundary conditions. By separation of variables, two solutions to the velocity potential and resulting pressure can be found. This is the basis of the concept of dividing the hydrodynamic response into two parts, those representing the body terms, called the impulsive components, and those representing the surface wave terms, or the convective component of pressure (Housner).

For a rigid tank, sloshing is due to a part of the liquid that moves as a surface wave, and is associated with the convective pressures. The modes are proportional to $J_1(\lambda_j, r/R)$ where $J_1$ is the first order Bessel function of the 1st kind, and $\lambda_j$ = $j^{th}$ zero root of the first derivative of the first order Bessel function of 1st kind. The first five values of $\lambda$ are 1.8412, 5.3314, 8.5363, 11.706, and 14.8631. The frequency is given by:

$$f_j = \frac{1}{2\pi}\sqrt{\lambda_j \frac{g}{R}\tanh(\lambda_j \frac{H}{R})} \tag{2.1}$$

where $R$ is the tank radius, $H$ is the liquid height above the base, and $g$ is the acceleration of gravity. The general frequency of the fluid/tank system for anchored tanks has been the subject of numerous studies (Clough, Haroun, and Veletsos). Several different methods have been used. The combined system frequency can be expressed in the form:

$$f_{0m} = \frac{C_m}{2\pi H}\sqrt{\frac{Es}{\rho_l}} \tag{2.2}$$

where $Es$ is Young's modulus and $\rho l$ is the density of contained liquid. The dimensionless frequency coefficient $C_m$ depends on the tank proportions of height, radius, wall thickness, Poisson ratio, and relative mass density of contents and tank shell material. Tabular values of the frequency coefficients are available in the literature (Veletsos) or graphically (Haroun). The natural frequency of the fluid/tank system depends on the assumed deformed shape.

The design of water tanks for seismic considerations uses a simplified formula for base shear by the following formula in AWWA (AWWA 2005).

$$Vf = \sqrt{[Ai(Ws + Wr + Wf + Wi)]^2 + [AcWc]^2} \tag{2.3}$$

$Vf$ is design shear at the top of the foundation due to the horizontal design acceleration, $Ai$ is the impulsive design acceleration as a decimal of g, $W_s$ is the total weight of tank shell and significant appurtenances; $W_r$ is the total weight of the tank roof; $W_f$ is total weight of the tank bottom; $W_i$ is the weight of effective mass of tank contents that moves in unison with the tank shell (the effective impulsive weight), $Ac$ is the convective design acceleration as a decimal of g, $Wc$ is the weight of effective mass of the first mode sloshing of the tank contents (effective convective weight). The base shear calculation is used for anchorage and connection design or sliding check.

The overturning moment applied to the bottom of the tank shell due to seismic forces is determined in accordance with the following formula.

$$Ms = \sqrt{[Ai(WsXs + WrHt + WiXm)]^2 + [AcWcXc]^2} \tag{2.4}$$

$Ms$ is the design overturning moment at the bottom of the shell caused by the horizontal design acceleration, $X_s$ is the height from the bottom of the tank shell to center of gravity of the shell; $H_t$ is total height of the tank shell; $Xi$ is height from the bottom of the tank shell to the centroid of lateral seismic impulsive force applied to $W_i$; and $Xc$ is the height from the bottom of the tank shell to the centroid of lateral seismic force applied to the effective convective weight $W_c$. The base moment is used to determine the uplift forces and the compressive forces acting on the tank shell near the base.

The impulsive design acceleration ($Ai$) is based on the design spectral acceleration, 5 percent damped for the natural period of the tank shell-fluid system and is calculated as shown in Equation (2.5).

$$Ai = \frac{S_{ai} I_E}{1.4 R_i} \tag{2.5}$$

*Sai* is the design response spectrum acceleration for the impulsive component, $I_E$ is the seismic importance factor which is based on a “Seismic Use Group”, with values ranging from 1.50 for tanks that are deemed essential, 1.25 for tanks that a deemed important, and 1.0 for all others. *Ri* is the response modification factor for the impulsive component, which is based on the type of structure. For ground-supported, flat-bottom anchored tanks, *Ri* is 3.0.

The convective design acceleration (*Ac*) is based on design spectral response acceleration for the convective component, 0.5 percent damped at the first mode of sloshing wave period, and is calculated as shown in Equation (2.6).

$$Ac = \frac{S_{ac} I_E}{1.4 R_c} \tag{2.6}$$

*Sac* is the design response acceleration for the convective component, 0.5 percent damped, at the first mode sloshing wave period and is a function of a damping scaling factor to convert the spectrum from 5 percent to 0.5 percent damping (a value of 1.5), the design earthquake spectral response acceleration ($S_{D1}$), and the first mode sloshing wave period (*Tc*). The value of *Sac* is related to a regional-dependent transition period for longer period ground motion. *Rc* is the response modification factor for the convective component based on the type of structure; for both anchored and unanchored tanks, this value is 1.5.

## 3. FINITE ELEMENT MODEL AND ANALYSIS METHODS

The Finite Element Analysis (FEA) models developed for this study consider the tank roof system to be represented by shell and beam elements, which are placed in the radial and circular directions. The tank wall is modeled by shell elements. The contents are represented as three-dimensional contained fluid elements which are not attached to the shell elements at the wall boundary, but have separate coincident nodes that are coupled only in the direction normal to the interface. The relative movements in the tangential and vertical directions are allowed to occur. The fluid element nodes at the base are allowed to move horizontally, while the shell wall is fixed around the perimeter base.

The fluid element selected is used to model fluids contained within tanks having no net flow. The fluid element is particularly well suited for calculating hydrostatic pressures and fluid/solid interactions. Fluid elements are rectangular (brick shaped) whenever possible, as results are known to be of lower quality for some cases using non-rectangular shapes. The bulk modulus *K*, 2,068 MPa, is taken to be characteristic of the water rather than infinite (incompressible), since it is used for shear stability as well as for compressibility effects.

Because of the system symmetry with only one horizontal component of ground motion concerned, one half of the tank is modeled. The tank has a radius *R*, total height *H*, constant wall thickness $t_s$, constant base thickness $t_b$, and is filled with water of density $\rho_l$ to a depth *h*. The tank is covered with a roof and supported by framing elements and a center column. Material properties include density of the steel, $\rho_s$, Young’s modulus of elasticity, $E_s$ and tangent modulus $E_T$ for nonlinear analysis. Dynamic input is aligned in the horizontal direction. The ground acceleration time history occurs at the base of the tank/foundation interface. Geometric and material values for the three tank models are summarized in Table 1.

Table 1 Tank and Material Properties

| Parameter | Tank A – Broad | Tank B- Medium | Tank C - Tall | Units |
|---|---|---|---|---|
| Height, *H* | 8.44 | 8.44 | 23.86 | m |

| | | | | |
|---|---|---|---|---|
| Liquid Depth, $H_t$ | 7.44 | 7.44 | 22.86 | m |
| Radius, $R$ | 9.06 | 4.98 | 3.05 | m |
| Wall Thickness, $t_s$ | 4.76 | 4.76 | 9.52 | mm |
| Roof Thickness, $t_r$ | 4.76 | 4.76 | 9.52 | mm |
| Young's Modulus, $E$ | 200,000 | 200,000 | 200,000 | MPa |
| Yield Point, $\sigma_Y$ | 250 | 250 | 250 | MPa |
| Tangent Modulus, $E_T$ | 20,000 | 20,000 | 20,000 | MPa |
| Poisson Ratio, $v$ | 0.3 | 0.3 | 0.3 | |
| Steel Density, $\rho_s$ | 7.83 | 7.83 | 7.83 | kg /m$^3$ |
| Liquid Density, $\rho_l$ | 1.00 | 1.00 | 1.00 | kg /m$^3$ |

In addition to linear elastic model assumptions, a large deformation and a bi-linear stress-strain curve was assumed for the tank shell. Structural steel exhibits a linear stress-strain relationship up to the yield point (250 MPa in this study) beyond which the relationship becomes plastic and nonlinear. This has been represented by a bilinear kinematic hardening model such that the Bauschinger effect is included. The resulting behavior is non-conservative and path dependent. The sequence of applying the loads and the resulting plastic response affects the final solution.

The FEA model was previously validated by comparing the model natural frequency of the convective and impulsive components with the theoretically derived values for the three tank geometries considered (Liu 2004).

## 4. SEISMIC GROUND INPUT

The time history ground inputs were obtained from published records of eight of the nine earthquakes. The 1964 Alaska earthquake was a synthetic record generated using a model representation of the area and the probable frequency range and soil conditions considered typical for at grade water storage tanks. These were selected to give a variety of frequency contents and durations. Design Response Spectra (DRS) were developed using procedures in the AWWA standard for Anchorage, Alaska, and adjusted for site-class effects. The short and long period site coefficients used a Site Class C, with the resulting values for *Fa* of 1.2 and *Fv* of 1.6. A scaling factor (*U*) of 2/3 was used to develop the design earthquake response acceleration.

To develop the design level earthquake inputs into the model, a reasonable approach is to adjust or scale the acceleration response spectrum value to the same level of the DRS value at the fundamental natural frequency of the fluid/tank system. The three tanks have separate fundamental periods in a range of 0.097 sec. to 0.392 sec. As such for each of the time history inputs, a separate scaling factor was used, as summarized in Table 2.

Table 2 Seismic Ground Input Listing and Time History Scaling Factor

| Time History Record | Tank A | Tank B | Tank C |
|---|---|---|---|
| 1. Alaska (1964) | 1.637 | 2.540 | 2.217 |
| 2. Denali, Alaska (2002) | 23.810 | 36.880 | 35.714 |
| 3. Nenana, Alaska (2002) | 43.478 | 76.833 | 76.923 |
| 4. El Centro, California N-S (1940) | 1.156 | 0.883 | 1.355 |
| 5.Taft, Kern Co., California (1952) | 3.846 | 4.210 | 1.976 |
| 6. Northridge, California (1994) | 0.696 | 0.572 | 0.319 |
| 7. Kobe, Japan (1995) | 0.749 | 0.858 | 0.402 |
| 8. Mexico City, Mexico (1995) | 5.376 | 5.180 | 3.953 |
| 9. Loma Prieta, California (1989) | 3.185 | 3.023 | 1.395 |

## 5. LINEAR AND NONLINEAR ANALYSIS RESULTS

A comparison of the model fundamental period for both combined tank/fluid, and convective components with the theoretical and code-based values are summarized in Table 3. Tables 4, 5, and 6 summarize the results of the model analysis for base shear, overturning moment, and maximum convective wave height for the various seismic ground inputs for the broad, middle, and tall tank geometries. The code-based calculated values are also shown for each of the tanks based on the design response pseudo-acceleration spectrum value at the calculated fundamental frequency from the aforementioned equations 2.1 through 2.4. Some of calculated R's of smaller than one (1.0) are due to the different settings in geometric assumptions in the modeling procedure; otherwise, those R's should be considered as equal to one (1.0).

Table 3 Model, Theoretical, and Code-based Fundamental Periods (seconds)

| | Broad Tank – A | Middle Tank – B | Tall Tank - C |
|---|---|---|---|
| Impulsive | | | |
| Theory | 0.144 | 0.099 | 0.370 |
| Model | 0.144 | 0.097 | 0.392 |
| AWWA D100 | 0.53 | 0.45 | 0.36 |
| Convective | | | |
| Theory | 4.678 | 3.314 | 2.582 |
| Model | 4.81 | 3.36 | |
| AWWA D100 | 4.60 | 3.31 | 2.58 |

### *5.1 Results: Broad Tank – Tank A*

Table 4 Base Shear and Overturning Moment

| | Maximum Base Shear | | | Maximum Overturning Moment | | | Maximum Convective Wave Ht | |
|---|---|---|---|---|---|---|---|---|
| Earthquake Input | Linear | Non-Linear | Calculated R | Linear | Non-Linear | Calculated R | Linear | Non-Linear |
| | kN | kN | | kN m | kN m | | cm | cm |
| 1 | 25,618 | 15,320 | 1.7 | 159,466 | 94,631 | 1.7 | 39 | 26 |
| 2 | 46,290 | 16,113 | 2.9 | 290,691 | 106,773 | 2.7 | 125 | 122 |
| 3 | 43,902 | 13,965 | 3.1 | 275,607 | 91,144 | 3.0 | 24 | 22 |
| 4 | 29,740 | 18,317 | 1.6 | 84,656 | 58,979 | 1.4 | 104 | 104 |
| 5 | 14,011 | 13,091 | 1.1 | 87,964 | 82,061 | 1.1 | 68 | 68 |
| 6 | 8,207 | 7,977 | 1.0 | 51,631 | 49,809 | 1.0 | 68 | 68 |
| 7 | 7,178 | 8,266 | 0.9 | 45,952 | 52,722 | 0.9 | 377 | 377 |
| 8 | 8,269 | 8,511 | 1.0 | 51,902 | 54,254 | 1.0 | 640 | 640 |
| 9 | 8,157 | 9,848 | 0.8 | 51,902 | 62,688 | 0.8 | 82 | 82 |
| AWWA D100[1] | 2,559 | | | 7,481 | | | 36 | |

### *5.2 Results: Middle Tank – Tank B*

Table 5 Base Shear and Overturning Moment

| | Maximum Base Shear | | | Maximum Overturning Moment | | | Maximum Convective Wave Ht | |
|---|---|---|---|---|---|---|---|---|
| Earthquake Input | Linear | Non-Linear | Calculated R | Linear | Non-Linear | Calculated R | Linear | Non-Linear |
| | kN | kN | | kN m | kN m | | cm | cm |
| 1 | 11,220 | 5,757 | 1.9 | 58,850 | 29,403 | 2.0 | 98 | 99 |
| 2 | 11,461 | 6,321 | 1.8 | 60,086 | 33,891 | 1.8 | 104 | 105 |
| 3 | 6,843 | 5,538 | 1.2 | 35,186 | 27,988 | 1.3 | 31 | 30 |
| 4 | 4,097 | 3,016 | 1.4 | 21,680 | 15,652 | 1.4 | 116 | 117 |
| 5 | 5,344 | 3,938 | 1.4 | 30,912 | 21,241 | 1.5 | 115 | 115 |
| 6 | 2,974 | 3,019 | 1.0 | 14,866 | 14,699 | 1.0 | 78 | 78 |
| 7 | 2,085 | 2,635 | 0.8 | 11,516 | 13,258 | 0.9 | 319 | 320 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 8 | 2,651 | 2,660 | 1.0 | 14,530 | 14,587 | 1.0 | 577 | 577 |
| 9 | 2,923 | 2,762 | 1.1 | 15,678 | 14,384 | 1.1 | 83 | 83 |
| AWWA D100[1] | 1,713 | | | 4,964 | | | 49 | |

### *5.3 Results: Tall Tank – Tank C*

Table 6 Base Shear and Overturning Moment

| | Maximum Base Shear | | | Maximum Overturning Moment | | | Maximum Convective Wave Ht | |
|---|---|---|---|---|---|---|---|---|
| Earthquake Input | Linear | Non-Linear | Calculated $R^2$ | Linear | Non-Linear | Calculated $R^2$ | Linear | Non-Linear |
| | kN | kN | | kN m | kN m | | cm | cm |
| 1 | 19,211 | *3 | | 556,672 | *3 | | 105 | |
| 2 | 24,659 | *3 | | 405,124 | *3 | | 212 | |
| 3 | 14,188 | *3 | | 198,480 | *3 | | 41 | |
| 4 | 5,841 | 6,743 | 0.9 | 97,061 | 81,270 | 1.2 | 153 | 149 |
| 5 | 6,470 | *3 | | 117,678 | *3 | | 50 | |
| 6 | 5,605 | 4,772 | 1.2 | 91,561 | 71,759 | 1.3 | 67 | 55 |
| 7 | 6,847 | *3 | | 104,928 | *3 | | 242 | |
| 8 | 4,462 | 4,366 | 1.0 | 66,261 | 65,175 | 1.0 | 984 | 984 |
| 9 | 5,082 | 6,259 | 0.8 | 79,261 | 93,606 | 0.8 | 51 | 76 |
| AWWA D100[1] | 3,619 | | | 39,654 | | | 56 | |

[1] Adjusted values includes for the use factor, *I*, and the reduction factor, *R*
[2] The R-values are ratios of peak base shears and overturning moment of linear to nonlinear results
[3] Deformation exceeds allowable limits in ANSYS program.

## 6. DISCUSSION AND CONCLUSIONS

An accurate prediction of the base shear and overturning moment is essential in determining the safety of tanks against shell buckling and uplift, and in determining forces for foundation and anchor bolt design. Results of this study show how the assumptions and material properties selected have a large effect on the modeling outputs. In particular, values of linear and non-linear displacement, base shear and overturning moment show substantial variation depending on the tank geometry and the design response spectrum used. The ratio of linear elastic and nonlinear properties range from 1.0 to 3.1 for base shear and overturning moment, which corresponds to the design standard reduction $R_i$ of 3.0 for anchored tanks.

The study results of the model runs using different seismic ground inputs show how the variation impacts the results for base shear, overturning moment, and convective wave heights as well as with the code-based calculated values. Depending on the earthquake event, the base shear, overturning moment, and resulting tank stresses may be substantially greater than the code-based values which would indicate that some deformation may occur under some conditions.

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