USE OF NETWORK ANALYSIS TO ASSESS THE VULNERABILITY OF AN ELECTRICAL SUPPLY NETWORK

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ABSTRACT :

This paper presents the results of a network analysis of the UK's electrical distribution network. A mathematical network model was developed which could simulate the network's response to the effect of natural disaster. This model was then used to calculate the topological properties of the network and simulate the effects of a disaster by randomly removing links and nodes from the model. The model was constructed using a map of the transmission system of the National Grid of the United Kingdom. The nodes in the model represented the power stations and substations and the links the transmission lines. The final model representing the network contained 55 large power stations, 169 substations and 313. Topological properties, such as the clustering coefficient, of the network were calculated. In addition to calculating network properties, an algorithm was developed which independently removed links at random from the network. 2000 simulations were conducted to assess the vulnerability of the network. The results of the study show that the UK's electrical distribution network has a clustering coefficient of 0.14, which is slightly higher than the value of 0.1 calculated by Watts P& Strogatz (1998) for the North America power grid. This suggests the UK's electrical distribution network is likely to be more robust. Also, the average number of links that can be removed before failure is estimated to be 6, implying that on average, 6 links (high voltage transmission lines) would have to fail simultaneously or shortly after one another before a community or region loses electricity supply. As a result of this analysis it is proposed that by adding several critical links to the network its robustness can be significantly increased and the vulnerability of certain nodes can be greatly reduced.

KEYWORDS:

Network analysis, disaster management, network topology, complexity.

1 INTRODUCTION

One of the greatest technological challenges of our time is to protect society and vital infrastructure from the consequences of disasters; whether they are results of a natural hazard, or induced by human activity, it is clear that disasters represent a widespread problem throughout the world. Although natural hazards have always posed a formidable threat to mankind, there is now growing evidence that as we proceed into the twenty first century, vulnerability to disaster is increasing around the world. Industrialization, urbanization, environmental mismanagement and global warming are all thought to be factors leading to this heightened vulnerability. Of particular importance to the stability of communities is the continued operation of lifelines. Lifelines tend to be interconnected and are often interdependent. It is this interdependence and interconnection that makes lifelines particularly vulnerable to disaster as any break in the system could hamper the operation of the whole system.

This paper presents the results of a study of the vulnerability of the electrical distribution network of the UK to natural disasters.

Using an approach based on the understanding of complex networks this paper endeavours to determine the robustness and vulnerability of such critical networks to random and targeted hazard. To achieve this, a case study was undertaken on a idealised model of the UK's electrical distribution network and conclusions drawn on how the design of such systems could be enhanced to improve their disaster performance.

2 NETWORK MODEL

In recent years, following the pioneering work of Watts & Strogatz (1998) and Barabasi & Albert (1999), complex graphs have been used successfully to model many technological networks. The simplicity of these network models make them particularly powerful for studying systems that are too complex to represent with conventional process based models. Albert et al. (2004) suggest that producing an analytic representation of a network's entire electromagnetic processes would be a "*daunting, if not impossible task*". Therefore in this paper we have resorted to developing a mathematical model which can be used to simulate the network's response to the effect of natural disaster. Models have been built representing the North American power grid as a network but so far no models exist of the UK's electrical distribution network. Using such a model it would be possible to analysis the network by calculating its topological properties and simulate the effects of a disaster by randomly removing links and nodes from the model. Conclusions could then be drawn of the overall robustness of the network and the most vulnerable nodes identified.

The UK's transmission system (2005/2006) consists of 181 large power stations, a 400kV and 275kV transmission system (and 132kV transmission system in Scotland) and 14 distribution systems. It includes over 11,500 circuit kilometres of 400kV overhead lines and underground cables, around 9,800 circuit kilometres at 275kV and 5,250 circuit kilometres at 132kV or lower voltage (National Grid 2005).

The model of the UK's electrical distribution network was constructed by firstly producing a network of the electrical infrastructure and secondly assembling an incident matrix of this network. The network properties were based on a map of the 2005/2006 transmission system obtained from the National Grid (see Figure 1). The nodes in the network represent the power stations and substations in the network, and the links the transmission lines connecting them. The network was simplified as much as possible, containing only large power stations (those defined as having a generation capacity of over 30 MW) and the two main 400kV and 275kV transmissions systems. The majority of the large power stations are directly connected to the transmission system; however, some are embedded within the lower voltage distribution networks and were therefore omitted from the model.

The links in the Figure 1 represent high-voltage transmission lines or pylons that connect the power station to the substations. Although the UK's electrical distribution system is split into two main transmission systems, a 400kV system and a 275kV system, for modeling purposes all links (transmission lines) were assumed to have the same properties.

The model considered the UK's electrical distribution network to be a single connected component (i.e. a path exists from every node to every other node (Wilson 1996)). This means that any substation can receive electricity from any power station anywhere in the network. This is an idealization of the real distribution system as in practice there will be many other constraints on the system.

The final network, representing the electrical distribution system, contained 224 nodes (representing 55 large power stations and 169 substations) and 313 links (representing the high voltage power lines.

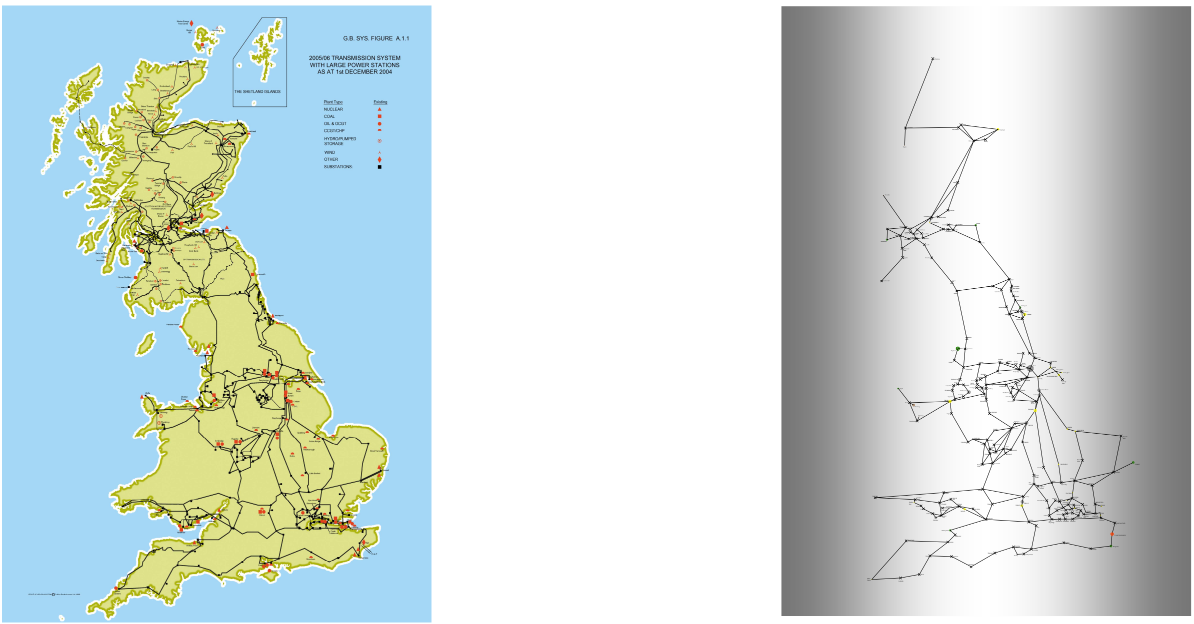
The power stations included in the model have a total generation capacity of 69099MW, approximately 90% of the total generation capacity of the actual system. The majority of the generation capacity omitted from the model is produced from wind farms and hydroelectric power stations, mostly in Scotland, which are not directly connected to the high voltage distribution system and with output capacities less than 30MW. Several gas power stations were also omitted as they were embedded within the lower voltage distribution networks which were not incorporated in the model.

As this work only considers large scale disaster (rather than failure due to poor maintenance for example) any two power stations located at the same site were combined. For example Sizewell A nuclear Magnox plant and Sizewell B nuclear power station were represented by one node having the combined output capacity of both plants. For the purpose of assessing the network's robustness and vulnerability it was assumed that any potential disaster, such as an earthquake, severe storm or industrial explosion, would affect both power stations on any same site.

In urban areas, particularly in the centre of London and Glasgow where the network is highly complicated, the model was further simplified by combining substations that were within very close proximity of each other.

The power stations and substations were then connected by links representing power lines. All links are represented by single straight lines and are assumed to carry the same voltage. Again, the physical network generally has two parallel transmissions lines following the same route and these were combined in the model.

Once the network was developed, it was transformed into an incidence matrix, where rows and columns represented the power stations and substations and the links (transmission lines) were represented by ones within the matrix.



a) UK electrical distribution network, National Grid (2005) b) Network Model

Figure 1 UK Electrical Distribution

3 RESULTS

Once the matrix was assembled, the topological properties of the network could be calculated.

3.1 Average Node Degree

One of the simplest topological properties of a network is its average node degree <k>. The degree k of a node is defined as the number of links connected to it. Therefore the average node degree is the sum of the degrees in the network divided by the total number of nodes. For this network, the average node degree was 2.78

As expected, the average node degree <k> is very similar to the value of 2.67 calculated by Watts & Strogatz (1998) in their study of the North America power grid.

3.2 Degree Distribution

The second important topological property of a network is its degree distribution as this is an indication of how the elements of a network are connected. Figure 2 shows the degree distribution for the UK's electrical transmission system. As expected the distribution follows an exponential:

$P(K) \approx \exp(-0.55k)$

Where P(K) is the probability that a node (substation) will have k links (transmission lines).

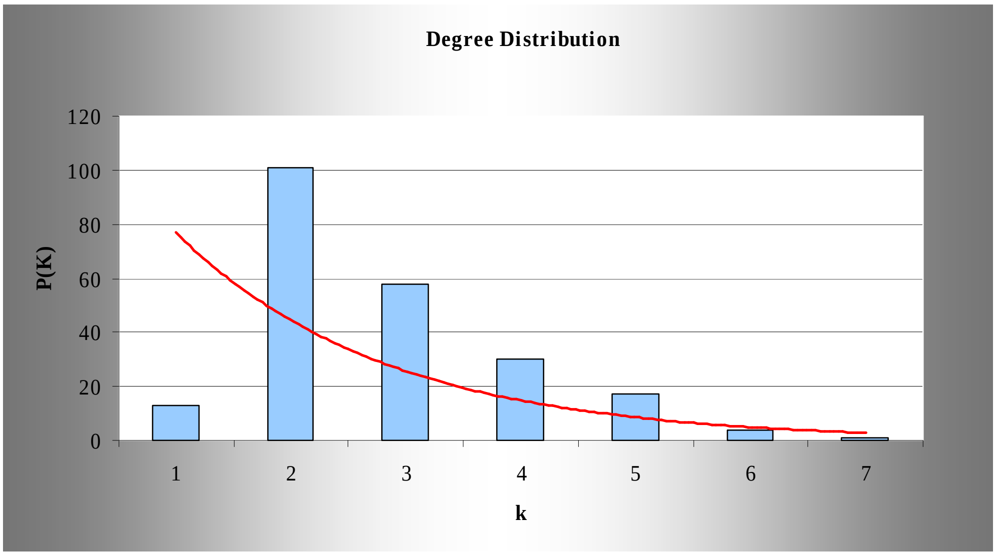


Figure 2 The Degree Distribution for the UK's Electrical Transmission System.

This exponential function form agrees with results obtained by Albert et al. (2004) for the North America power grid and also with Crucitti et al. (2004) who studied the Italian power grid. The network is classified as a single-scale network because, as predicted, all the nodes have roughly the same degree.

3.3 Clustering Coefficient

A slightly more complicated property of a network is its clustering coefficient C. Originally defined by Watts & Strogatz (1998) the clustering coefficient measures the fraction of triples in the network that have their third edge filled to complete the triangle. Although the concept was initially developed for use in social networks it has been widely accepted by many as a measurement of clustering or transitivity within many other types of network. It is relevant to this study as network transitivity (the presence of triangles in the network) is closely related to its robustness.

C = 3 × number of triangles in the network/ number of connected triples of nodes

Where a "connected triple" is a single node with links running to a pair of other nodes (Newman 2003).

For the UK's electrical distribution network the clustering coefficient C, is calculated to be 0.14. This is slightly higher than the value of 0.1 calculated by Watts P& Strogatz (1998) for the North America power grid suggesting the UK's electrical distribution network is likely to be more robust.

3.4 Network Robustness Random: Link Removal

As well as calculating the networks properties, an algorithm was developed which independently removed links at random from the network until one of the nodes became completely isolated from the rest of the network. The computer algorithm was run 2000 times with the aim of assessing the number of links that could be removed on average before the network fails. The results are shown in figure 3.

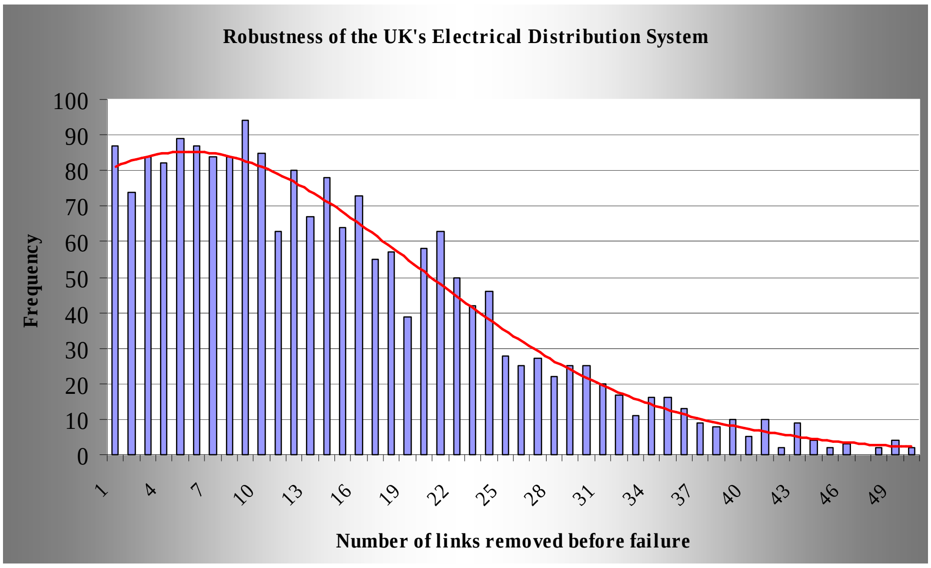


Figure 3 Histogram Representing the Minimum Number of Links That Can Be Removed Before Failure.

By modelling the distribution as a frequency curve the average number of links that can be removed before failure is estimated to be 6. According to the model this means that 6 links (high voltage transmission lines) would have to fail simultaneously or shortly after one another before a community or region loses electricity supply. The probability of six disasters happening at a similar time is very unlikely, making the network generally very robust; however, for the case of spatially correlated disasters, such as an earthquake or severe storm, this level of failure may be possible.

As a result of this analysis it is proposed that by adding several critical links to the network its robustness can be significantly increased and the vulnerability of certain nodes greatly reduced. In order to test this theory the computer algorithm that was designed to remove links at random was re-run on the modified network and the results compared. Figure 4 shows the location of the new proposed links highlighted in red. They take into account geological constraints such as river estuaries and national parks.

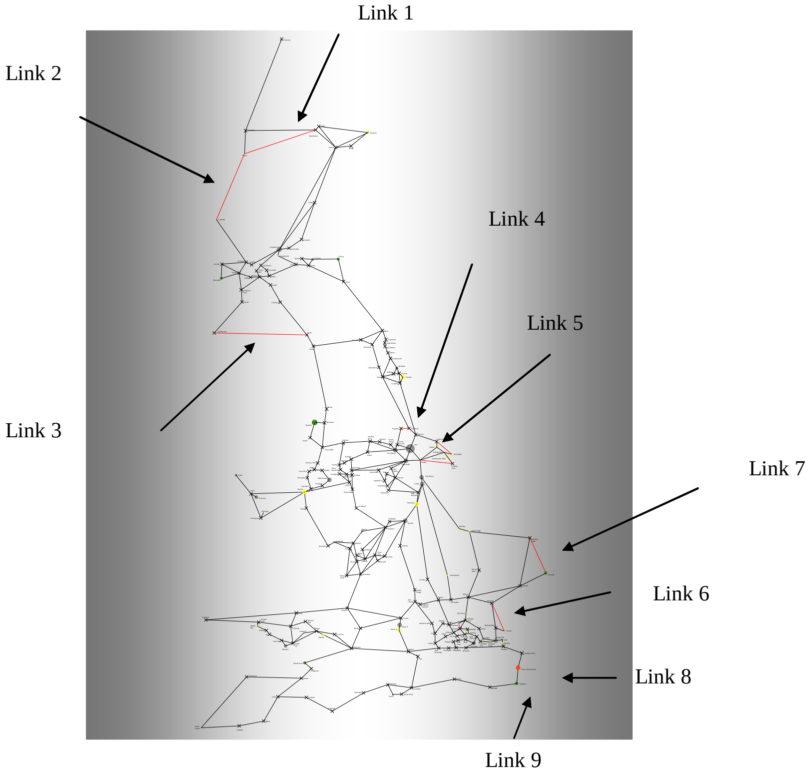


Figure 4 Modified Transmission Network.

In total nine new links were added onto the network, connecting, where considered feasible, the nodes identified as being vulnerable. Links 1-3 proposed in Scotland could also connect a series of hydro-electric power stations up to the high voltage network. As with the original network the computer algorithm was run 2000 times and a frequency curve plotted. Figure 5 shows how the network is now approximately 3 times more robust to random hazard than previously.

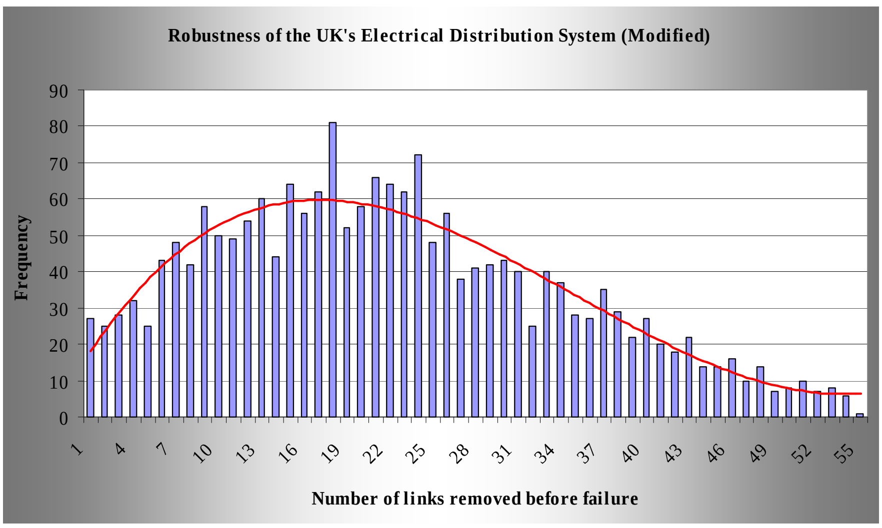


Figure 5 Histogram Representing the Minimum Number of removed Links Before Failure (Modified Network)

Table 1 Network Properties

Network	Nodes N	Links m	Average Degree <k>	Clustering Coefficient C	Links removed before failure	Nodes removed before failure
UK's Electrical Distribution System	224	313	2.79	0.14	6	6
UK's Electrical Distribution System (modified)	224	322	2.88	0.15	18	6
North America power grid after Watts and Strogatz (1998)	4941	6594	2.67	0.10	-	-

3.5 Random Node Removal: Random Node Removal

A second important measure of a network's robustness is its node-connectivity. This is defined as the minimum number of nodes (or vertices) that can be removed before the network fails. In the case of this model a failure is defined as when there is insufficient electricity generation to meet demand. Although the national demand of electricity varies considerably depending on the time of year, the peak demand in the year 2005/2006 is estimated to be 62600MW according to the National Grid's seven year statement (2005). Ignoring any losses the surplus capacity in the model is therefore calculated to be:

Surplus Capacity (Model) = 69099 – 62600 = 6499MW.

A computer algorithm was created to randomly remove nodes (power stations or substations) from the network. The algorithm considered the output of each power station by subtracting this output capacity from the total capacity of the network until the surplus fell below zero. The algorithm was run 250 times giving the results shown in Figure 6.

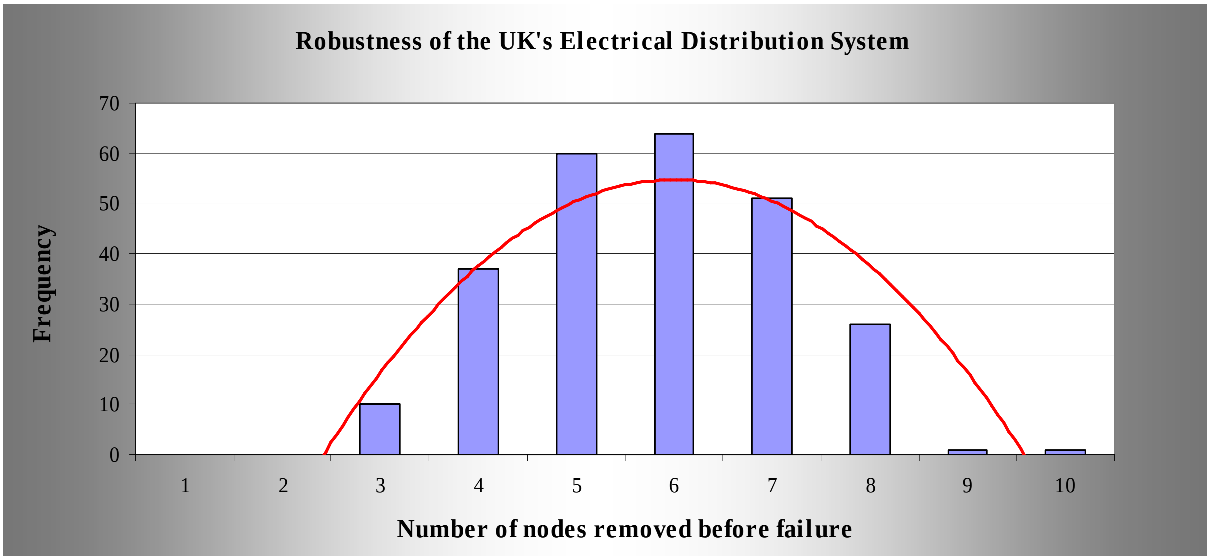


Figure 6 Histogram Representing the Minimum Number of Nodes That Can Be Removed Before Failure.

The results show that, on average, six power stations would have to be isolated from the network before a major electricity shortage occurs. The link-connectivity of the network can be seen to be 3 (i.e. minimum number of nodes that must be removed before failure) but the probability of this occurring, according to this analysis, is 0.04. When considering targeted attacks on the network however it is apparent that the removal of several power stations has a much greater effect on the system.

4 Conclusions

The network model presented in this paper is too simple to make definite claims about the real behavior of the UK's electrical supply network; however the analysis has shown that the robustness and vulnerability of lifeline networks is strongly dependent upon their topology. For the network model of the UK's electrical distribution system it has been demonstrated that by increasing the clustering coefficient C, a measure of network transitivity, by 0.01, the robustness of the network to random failure can be increased by a factor of 3. The UK's electrical distribution network is considerably smaller than the North America power grid, but was found to have very similar topological properties. Both networks are generally very robust to random hazard but, as highlighted in this study, there are often nodes situated around the outer perimeter of the network that are poorly connected and potentially vulnerable. The analysis clearly showed that pendant nodes have a great influence on the network's robustness to random hazard; significantly reducing its performance. With regards to node removal (the failure of power stations) the robustness of the UK network seems to be inherent to its topology which could not be easily addressed without significant investment. What is apparent though from this study is that having a network reliant on a series of hubs (i.e. large power stations) makes the network much more vulnerable than having numerous smaller power stations integrated throughout the network.

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