A STUDY ON SLOSHING FREQUENCIES OF FLUID-TANK SYSTEM

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ABSTRACT:

Liquid storage tanks constitute an important component of life line systems such as water distribution system, petroleum plants etc. Seismic design of liquid storage tanks requires knowledge of sloshing frequency of liquid and hydrodynamic pressure on the wall. This in turn, requires proper analysis of fluid-tank interaction under seismic excitation. In the design codes, mechanical analogs of tank-fluid system are commonly used to obtain the sloshing frequency, hydrodynamic pressure and design seismic forces. Such mechanical analogs are and developed for simple geometries, like circular and rectangular tanks. However, for tanks of other shapes and for tanks with internal obstructions, there are a very few studies on the mechanical analogs. In the present paper, experimental and numerical study is taken up to obtain the sloshing frequency of liquid contained in tanks of other shapes and tanks with internal obstructions. The experimental study is done on laboratory models of tanks, which are excited using an Electro-Magnetic Shake Table. The numerical study is done with the help of finite element model of tank-fluid system using ANSYS software. A comparison of experimental and numerical results is given.

KEYWORDS: Fluid-tank interaction, Sloshing frequency, tanks with obstruction, tanks of other shapes, ANSYS

1. INTRODUCTION

Liquid storage tanks are commonly used in water distribution systems, and in industries for storing toxic liquids. Importance of seismic analysis of liquid storage tanks does not need any over emphasis. During the past earthquakes, tanks have suffered varying degree of damages. Typical damages include: Buckling of ground supported slender tanks (Malhotra, 1997), rupture of steel tank shell at the location of joints with pipes, collapse of supporting tower of elevated tanks (Manos and clough, 1983, Rai, 2002), cracks in the ground supported RC tanks, etc. Damages to tanks in the Chilean earthquake of 1960 led to the development of simple mechanical analogue of tank-liquid system by Housner (1963). This simple mechanical analogue of tank-liquid system forms the basis for evaluating hydrodynamic pressure in most of the design codes across the globe. Subsequently, the effect of shell flexibility on the hydrodynamic pressure and fluid- structure interaction is included (Haroun et al, 1982 and Vetelsos, 1984). These studies have led to the development of mechanical analogues for flexible tanks, which are also being used in the design codes (NZSEE, 1986). With the availability of fast computing machines and finite element techniques, more rigorous analysis of tank-liquid system has been performed (Liu et al, 1982). These rigorous analyses have validated mechanical analogue of tank-liquid system. There are some experimental studies also on dynamic response of tank-liquid system (Haroun et al, 1982).

During lateral base excitation seismic ground acceleration causes hydrodynamic pressure on the tank wall which depends on the geometry of tank, height of liquid, properties of liquid and fluid-tank interaction. Proper estimation of hydrodynamic pressure requires a rigorous fluid-structure interaction analysis. In the mechanical analogue of tank-liquid system, the liquid is divided in two parts as, impulsive liquid and convective liquid. The impulsive liquid moves along with the tank wall, as it is rigidly connected and the convective and sloshing liquid moves relative to tank wall as it under goes sloshing motion. This mechanical model is quantified in terms of impulsive mass, convective mass, and flexibility of convective liquid. Housner (1963) developed the expressions for these parameters of mechanical analogue for circular and rectangular tanks. The hydrodynamic seismic forces are obtained using the frequency of sloshing and impulsive modes. Generally the first sloshing and impulsive modes are of interest. The mechanical analogue developed by Housner, gives good estimation of hydrodynamic forces for circular and rectangular tanks. However, for tanks of other shapes and for tanks with obstruction inside the liquid, the knowledge about sloshing frequency and hydrodynamic pressure is not adequate and there are not many studies on these tanks. Tanks of other shapes and tanks with obstruction, give rise to quite complicated problem in fluid structure interaction, which is not amenable to analytical solution. Some researchers have attempted numerical study for tanks of other shapes (Joshi, 2000 and Damatty, 2000). Similarly, for tanks with internal obstructions, there are some analytical and numerical studies (Aslam, 1979, Choun, 1996 and Pal, 1998). There are some experimental studies on the effect of obstructions on the sloshing frequency (Kimura et al. 1995, Armenio 1996, and Reed et al. 1998).

The present study focuses attention on the sloshing frequency of liquid contained in tanks of shapes other than uniform circular and rectangular shapes and tanks with internal obstructions. This study has two parts; in the first part, which is experimental work, small size models of tanks of other shapes and tanks with internal obstructions are excited harmonically on a small size shake table. The shaking frequency at which the sloshing is maximum, taken as the sloshing frequency of the liquid. In the second part, which is numerical in nature, the sloshing frequency is obtained using ANSYS software. The tank and the liquid are modeled using suitable finite elements. From the modal analysis, the sloshing frequency of liquid is obtained. A comparison of the numerical and experimental result is presented.

2. EXPERIMENTAL WORK

The set up for the experimental work comprises of an electro-magnetic shake table along with a digital amplifier. The shake table is driven by a signal conditioner which is attached to a computer (Figure 1). The frequency and amplitude of excitation are controlled with the help of a computer. Small scale models of tanks are of glass, which is a transparent material. The liquid filled tank is mounted on the shake table and shake table is excited harmonically at a particular frequency. The sloshing motion of liquid is observed, then, the frequency of excitation is changed and again sloshing motion of liquid is observed. The amplitude of excitation is kept constant for all the frequencies of excitations. If the excitation frequency is very close to the sloshing frequency of liquid, then the liquid sloshes with large amplitude for the same amplitude of excitation. The excitation frequency at which the liquid sloshes with large amplitude is taken as the sloshing frequency of liquid. It is noted that at frequencies on lower and higher side of sloshing frequency, the amplitude of sloshing is less. Using this procedure; the first sloshing frequency of liquid is obtained.

Figure 1 Set up for experimental work

3. FINITE ELEMENT ANALYSIS

The sloshing frequency of the liquid is obtained from the finite element analysis of the fluid-tank system. ANSYS software is used for the finite element modeling, wherein, the container wall and the base is modeled using the four nodded shell elements (SHELL 63) with six degrees of freedom at each node. The fluid is modeled using eight nodded brick type fluid element (FLUID80) with three degrees of freedom at each node. At the interface, the fluid elements and shell elements have separate nodes. At the wall and fluid interface, the fluid nodes and wall nodes are coupled in normal direction. This implies that in the normal direction, fluid and wall are constrained to move together. However, in the tangential and vertical direction, the fluid can move relative to the wall. Likewise, at the base, the fluid elements are coupled in the vertical direction only. The FLUID 80 element supports lumped mass matrix only and the reduced method is used for the modal analysis. The reduced method works with only the master degrees of freedom. The accuracy of the result depends on the number and locations of the master degree of freedom. The total number of master degrees of freedom shall be at least twice the number of modes of interest. The finite element analysis is used to obtain the first sloshing frequency of the laboratory models of tanks which were used in the experimental analysis and the experimental and numerical results are compared.

4. DETAILS OF LABORATORY MODELS OF TANKS

There are three sets of laboratory models of tanks made of glass; the first set comprises of three tanks, one each of circular, square and rectangular shape. The details are: Circular tank of 170 mm diameter and 230 mm height (Model 1); square tank of plan size 270mm x 270mm and 300 mm height (Model 2); rectangular tank of plan size 270 mm x 135 mm and 300 mm height (Model 3). For these shapes of tanks the analytical closed-form expressions for the sloshing frequency are available and they can be used for comparison with the experimental and finite element results. The second set comprises of tanks of other shapes, wherein, one tank has circular conical shape and two tanks are truncated pyramid of square shape. One of these truncated pyramid has smaller dimension at the base and larger dimensions at the top, whereas second one has larger dimension at the base and smaller at the top (Table 1). The third set is of a square tank with centrally placed square internal obstruction. The size of the obstruction is varied and its effect on sloshing frequency is studied. The detail of tank with obstructions is given in Table 2. The liquid considered is water, for which, bulk modulus and mass density is taken as 2.18×10^9 N/m^2 and 1000 Kg/m^3 respectively. For the glass, the elastic modulus, mass density and passions ratio are taken as 1.173×10^9 N/m^2, 955 Kg/m^3 and 0.35 respectively.

5. RESULTS

The first sloshing frequency for all the models of the tanks described above is obtained experimentally as well as using finite element analysis. The results are presented for each of the three sets of tanks.

5.1 Circular, Square and Rectangular Tanks

For the circular and rectangular tanks, the first sloshing frequency is given by Eqn. 5.1 and Eqn. 5.2 (Housner, 1963):

For circular tanks
$$f_c = \frac{1}{2\pi}\sqrt{\frac{3.68\,g\tanh\left(3.68\,\frac{h}{D}\right)}{D}} \tag{5.1}$$

For rectangular tanks
$$f_c = \frac{1}{2\pi}\sqrt{\frac{3.16\,g\tanh\left(3.16\,\frac{h}{L}\right)}{L}} \tag{5.2}$$

Where, f_c = Sloshing frequency (Hz), h = height of water in tank (m), L = length of rectangular tank along direction of excitation (m), D= diameter of circular tank (m), g = acceleration due to gravity (m/s^2)

Table 1 Details of tanks of other shapes

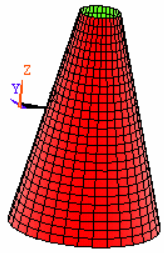
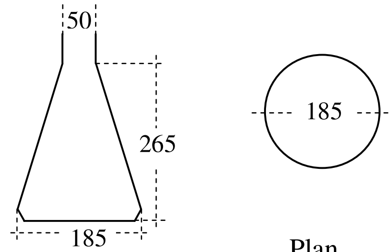
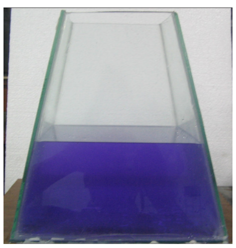
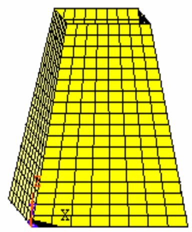
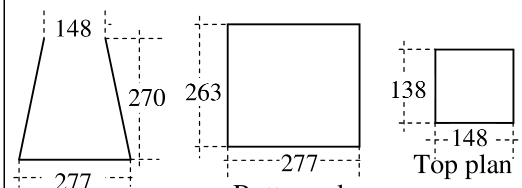
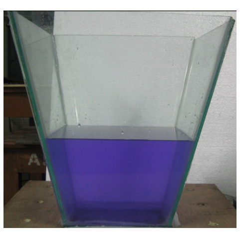
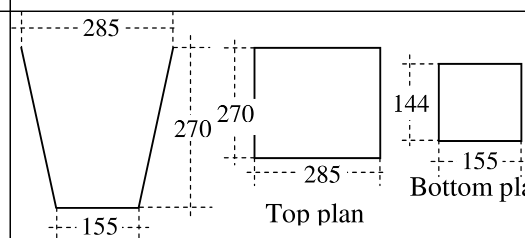
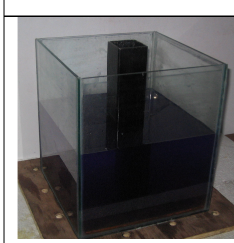
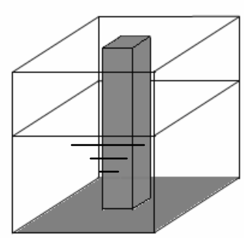
Experimental Model	ANSYS Models	Geometry (all dimensions in mm)
 Model 4		 Elevation Plan
 Model 5		 Elevation Bottom plan Top plan
 Model 6	ANSYS model not available	 Elevation Top plan Bottom plan

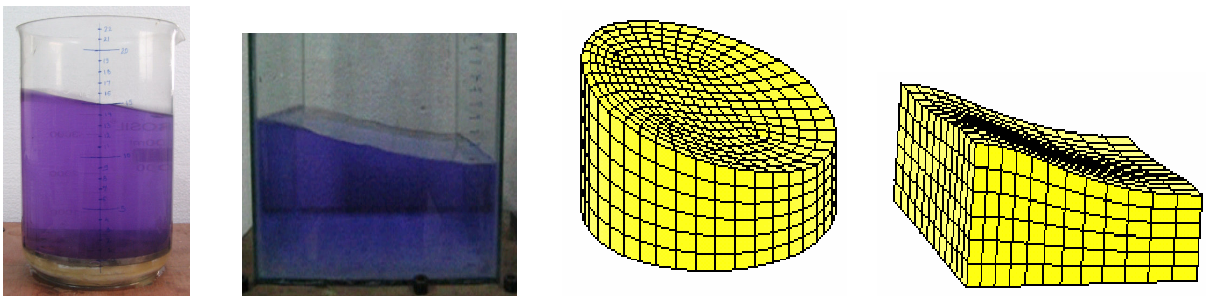
Table 2 Details of tank with obstructions

Model	Tank Size	Size of obstruction (mm)	Diagram of model with centrally placed obstruction
 Model 7	270 mm x 270 mm with 300 mm height	i) 12×12 ii) 26×26 iii) 37×37 iv) 46×46 v) 60×60	

The results on the first sloshing frequency of circular and square tank (Model 1, and Model 2) for different liquid height are shown in Table 3. Here, experimental and ANSYS results are given along with the results obtained using the closed form expression (Housner, 1963). The results of rectangular tank in both the direction are given in Table 4. The first sloshing mode of liquid from experiment and ANSYS is shown in Figure 2

Table 3 Fundamental sloshing frequencies of liquid for circular and rectangular tank

Depth of liquid (mm)	Fundamental sloshing frequency (Hz)					
	Circular tank, Model 1			Square tank, Model 2		
	Experimental	Housner (1963)	ANSYS	Experimental	Housner (1963)	ANSYS
50	2.07	2.066	1.99	1.28	1.231	1.23
100	2.30	2.289	2.18	1.52	1.542	1.53
150	2.33	2.316	2.22	1.63	1.649	1.62
200	2.33	2.319	2.22	1.67	1.684	1.66



(i) Circular (ii) Square (i) Circular (ii) Square

(a) Experimental (b) ANSYS

Figure 2 First sloshing mode of liquid in circular and square tank

Table 4 Comparison of fundamental sloshing frequencies of liquid in rectangular tank (Model 3)

Depth of liquid (mm)	Fundamental sloshing frequency (Hz)					
	Along longer direction			Along shorter direction		
	Experimental	Housner (1963)	ANSYS	Experimental	Housner (1963)	ANSYS
50	1.26	1.237	1.23	2.17	2.189	2.15
100	1.50	1.548	1.53	2.33	2.390	2.33
150	1.60	1.655	1.62	2.38	2.410	2.35
200	1.64	1.690	1.66	2.40	2.411	2.35
250	1.70	1.700	1.68	2.40	2.411	2.35

5.2 Tanks of other shapes

For tanks of other shapes, there are no closed form analytical expressions for the sloshing frequency. However, in the design codes like Eurocode 8 (2000), IITK-GSDMA Guidelines (2005), an approximate approach is followed. In this approach, the circular conical tank is replaced by an equivalent circular tank of diameter equal to diameter at the liquid surface by keeping same volume of water. Same approach is followed for truncated pyramid type of rectangular tanks. For model 6, i.e. tank which has larger plan dimension at top and smaller plan dimension at bottom, the ANSYS results could not be obtained. This is due to the requirement in ANSYS that fluid elements shall be of shapes very close to square. The results for first sloshing frequency of liquid in these tanks along longer dimension are shown in Table 5. Experimental and ANSYS results are compared with the approximate approach suggested above. The first sloshing mode of liquid in experimental and ANSYS models are shown in Figure 3.

Table 5 Comparison of first sloshing frequency of liquid in circular conical tank

Depth of liquid (mm)	Fundamental sloshing frequency (Hz)								
	Model 4			Model 5			Model 6		
	Expt.	Appr.*	ANSYS	Expt.	Appr.**	ANSYS	Expt.	Appr.**	ANSYS
50	2.10	2.01	1.98	1.4	1.340	1.27	1.70	1.707	***
100	2.60	2.50	2.41	1.7	1.762	1.62	1.75	1.840	***
150	2.80	2.80	2.63	1.9	1.918	1.77	1.69	1.814	***
200	3.30	3.22	2.98	2.1	2.028	1.82	1.64	1.757	***
250	4.00	3.98	3.53	2.2	2.152	1.94	1.60	1.696	***

*Equivalent circular tank, ** Equivalent rectangular tank, *** ANSYS model is not available

Model 4 Model 5 Model 6 Model 4 Model 5

(a) Experimental (b) ANSYS

Figure 3 First sloshing mode of liquid in tanks of other shapes

5.3 Tank with obstruction

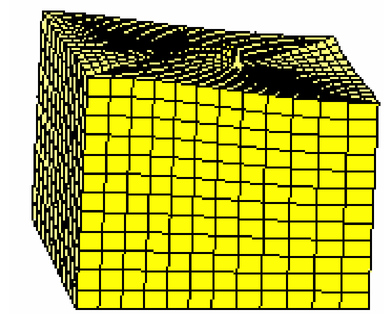
For square tank with centrally placed square obstruction, there are no analytical results on the frequency. The experimental and ANSYS results on fundamental sloshing frequency for the square tank with obstruction are shown in Table 6. The mode shapes are shown in Figure 4.

Table 6 Fundamental sloshing frequency for square tank with obstruction

Obstruction size (mm)	Fundamental sloshing frequencies of liquid (Hz)									
	H = 50 mm		H =100 mm		H =150 mm		H =200mm		H =250mm	
	Expt.	ANSYS	Expt.	ANSYS	Expt.	ANSYS	Expt.	ANSYS	Expt.	ANSYS
No obstruction	1.26	1.23	1.50	1.53	1.60	1.62	1.64	1.66	1.70	1.69
12 x 12	**	1.17	1.49	1.46	1.59	1.55	1.63	1.58	1.70	1.59
26 x 26	**	1.12	1.48	1.41	1.59	1.51	1.63	1.54	1.70	1.55
37 x 37	**	1.08	1.45	1.38	1.57	1.48	1.61	1.51	1.67	1.52
46 x 46	**	1.06	1.44	1.35	1.56	1.46	1.59	1.48	1.64	1.49
60 x 60	**	1.02	1.39	1.31	1.50	1.42	1.54	1.45	1.59	1.53

Expt. – Experimental and **-Mode could not be seen experimentally.

(a) Experimental



(b) ANSYS

Figure 4 First sloshing mode of liquid in square tank with obstruction

6. Discussion and Conclusions

Sloshing frequency is an important parameter in the seismic analysis of tank-liquid system. For the regular tank geometries, such as circular, square and rectangular, the analytical expressions for sloshing frequency are quit well known. However for the tanks of other shapes and also for tanks with internal obstruction, details of tank liquid interaction, hydrodynamic pressure and sloshing frequency are not well studied. In this present study, for circular, square and rectangular tank models, the experiential and ANSYS results on first sloshing frequency matched well with analytical solution.

From the experimental and numerical results, for the tanks of other shapes, it is seen that the approximate approach used in the design codes for the conical and truncated pyramid type of geometry is reasonable. However, for the model 6, i.e., rectangular plan with increasing dimensions at the top, the experimental results are on lower side than those obtained using approximate approach (Table 5). For the tanks with centrally placed obstruction, it is seen that as the size of the obstruction increases, the first sloshing frequency decreases. This happens for all liquid height (Table 6). This implies that sloshing stiffness reduces due to presence of obstruction.

The present study needs further extension for obtaining the hydrodynamic pressure on wall for the tanks of other than uniform circular and rectangular shapes and the thanks with obstructions.

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