The 14th World Conference on Earthquake Engineering
October 12-17, 2008, Beijing, China

FAR FIELD SOLUTION OF SH-WAVE BY CIRCULAR INCLUSION AND LINEAR CRACK

HongLiang Li[1] , GuoHui Wu[2]

[1, 2] *Associate Professor, Department of Engineering Mechanics, Harbin Engineering University, Harbin. China*
Email: leehl@sina.com

ABSTRACT :

Circular inclusion is used widely in structure engineering. In this paper, the method of Green's function is used to investigate the problem of far field solution of circular inclusion and linear crack impacted by incident SH-wave. Firstly, a Green's function is constructed for the problem, which is a fundamental solution of displacement field for an elastic space possessing a circular inclusion while bearing out-of-plane harmonic line source force at any point; Secondly, in terms of the solution of SH-wave's scattering by an elastic space with a circular inclusion, anti-plane stresses which are the same in quantity but opposite in direction to those mentioned before, are loaded at the region where the linear crack is in existent actually, we called this process "crack-division"; Finally, the expressions of the displacement and stresses are given when the circular inclusion and linear crack exist at the same time. Then, when the special Green's function has been constructed and close field solution has been illustrated, the far field of scattered wave is studied. The displacement mode of scattered wave at far field and scattering cross-section are given. Numerical results are illustrated and the influence of wave number, incident angles of SH-wave, and the combination of different media parameters are discussed. The results can be applied in the study of fracture, and undamaged frame crack detection.

KEYWORDS: crack, circular inclusion, Green's Function, SH-wave scattering, displacement mode of scattered wave at far field, scattered cross-section

1. INTRODUCTION

Circular inclusion exists widely in natural media, engineering materials and structures, and defects are usually found around the inclusion. When a composite material with circular inclusion and cracks is impacted by the dynamic load, on the one hand, the scattering field produced by the circular inclusion and cracks determines the dynamic stress concentration factor around the circular inclusion, and therefore determines whether the material is damaged or not; on the other hand, the scattering field also presents many characteristic parameters of the inclusion and cracks such as defect composition, location and shape, so the research on the scattering far-field is important to the geological prospects, seismological investigation, non-destruction evaluation and the other fields. In the ocean acoustics, the scattering far-field of the acoustic wave is also used in the under-water survey, object distinguishing and so on. In theory, the scattering solution of elastic waves is one of the basic topics of reverse problems on elastic wave. On the basis of literature, few paper concentrates on the scattering far-field solution of SH-wave by a circular inclusion and a linear crack around the inclusion. In the paper a new model and a new method are presented in order to investigate deeply on this kind problem.

At present, to obtain the theoretical solution of the problem concerned in this thesis is of great interest and certainly it has some difficulties. The development of computational mechanics has provided many methods to solve the problem, but a theoretical answer is still expected in order to investigate the characteristics of the circular inclusion and crack. The paper uses the Green's function to study the scattering far-field of elastic wave by a circular inclusion and a linear crack. The Green's function should be a fundamental solution of displacement field for an elastic space possessing a circular inclusion while bearing out-of-plane harmonic line source force at any point. In terms of the solution of SH-wave's scattering by an elastic space with a circular inclusion, anti-plane stresses which are the same in quantity but opposite in direction to those mentioned before, are loaded at the region where the linear crack is in existent actually, we called this process "crack-division". Then, the expressions of the displacement and stresses are given when the circular inclusion and linear crack exist at the same time. Then, when the special Green's function has been constructed and close field solution has been illustrated, the far field of scattered wave is studied. The displacement mode of scattered wave at far field and scattering cross-section are given. At last, an example is given and its numerical results are discussed.

2. MODEL AND GOVERNING EQUATION

The model is shown as Fig.1, an elastic space containing a circular inclusion and a linear crack around the inclusion. In this paper, the anti-plane shear SH wave model is studied. The displacement in the elastic space is expressed as $W_1(x,y,t)$,The displacement in the inclusion is expressed as $W_2(x,y,t)$. The governing equation of W_i can be written in the polar coordinate system as:

$$\frac{\partial^2 W_1}{\partial r^2}+\frac{1}{r}\frac{\partial W_1}{\partial r}+\frac{1}{r^2}\frac{\partial^2 W_1}{\partial \theta^2}+k_1^2 W_1=0 \tag{2.1}$$

$$\frac{\partial^2 W_2}{\partial r^2}+\frac{1}{r}\frac{\partial W_1}{\partial r}+\frac{1}{r^2}\frac{\partial^2 W_2}{\partial \theta^2}+k_2^2 W_2=0 \tag{2.2}$$

where $k_i=\dfrac{\omega}{C_{Si}}, C_{Si}=\sqrt{\mu_i/\rho_i}$, ω is the circular frequency of the displacement $W_i(x,y,t)$, C_{si} stands for the shear wave velocity, ρ_i and μ_i are the mass density and the shear modulus of elasticity respectively.

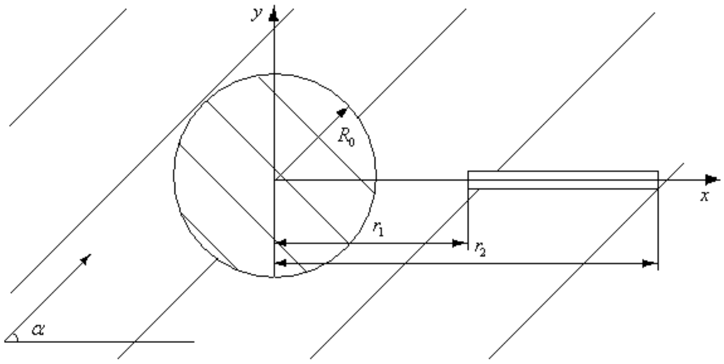


Figure 1 Model of Problem

3. GREEN'S FUNCTION

The Green's function used in this paper is regarded as the displacement response to the elastic space containing a circular inclusion impacted by anti-plane harmonic linear source force at any point. The dependence of the displacement function G_i on time t is $e^{-i\omega t}$.In the polar coordinate system, the governing equation of G_i can be written as:

$$\frac{\partial^2 G_1}{\partial r^2}+\frac{1}{r}\frac{\partial G_1}{\partial r}+\frac{1}{r^2}\frac{\partial^2 G_1}{\partial \theta^2}+k_1^2 G_1=\delta(\vec{r}-\vec{r}_0), \frac{\partial^2 G_2}{\partial r^2}+\frac{1}{r}\frac{\partial G_2}{\partial r}+\frac{1}{r^2}\frac{\partial^2 G_2}{\partial \theta^2}+k_2^2 G_2=0 \tag{3.1}$$

$\vec{r}_0$ stands for the position of the linear source force in polar coordinates. The boundary conditions can be expressed as below:

$$G_1\big|_{r=R_0} = G_2\big|_{r=R_0}, \tau_{rz1}\big|_{r=R_0} = \tau_{rz2}\big|_{r=R_0} \tag{3.2}$$

The basic solution which satisfies the control equation (3.1) and the boundary conditions (3.2) should include two parts of motion: the disturbance of anti-plane linear source force and the scattering wave incited by the circular inclusion. The wave displacement of the complete elastic space due to the line source load $\delta(\vec{r}-\vec{r}_0)$ on the arbitrary position can be given:

$$G^{(i)} = \frac{i}{4\mu} H_0^{(1)}(k\,|\,\vec{r}-\vec{r}_0\,|) \tag{3.3}$$

Where $H_0^{(1)}(*)$ is the first kind of Hankel function and zero-order. The scattering wave in the elastic space and in the circular inclusion can be written as:

$$G^{(s)} = \sum_{m=0}^{\infty} A_m H_m^{(1)}(k_1 r)\cos[m(\theta-\theta_0)], G^{(in)} = \sum_{m=0}^{\infty} B_m J_m(k_2 r)\cos[m(\theta-\theta_0)] \tag{3.4}$$

where A_m, B_m are unknown coefficients.

Therefore, $G_1 = G^{(i)} + G^{(s)}, G_2 = G^{(in)}$ .According to the boundary conditions, we can obtain A_m, B_m. So, the wave field G_1 of this problem can be obtained.

4. EXPRESSION OF DISPLACEMENT AND STRESS FOR THE MODEL

The stress on the crack around the inclusion produced by incident SH-wave and the scattering wave incited by the circular inclusion can be obtained. A pair of opposite forces is applied to the crack; therefore the resultant force on the crack is zero, which can be thought as crack.. The above constructing process is called crack-division technique which can be used to obtain the expression of displacement and stress for the model. The detail can be discussed as follows.

Firstly, we consider the incidence of SH-wave on the infinite linear-elastic space containing a circular inclusion. The incident displacement field $W^{(i)}$ harmonic to time can be written as follows:

$$W^{(i)} = W_0 \sum_{n=0}^{\infty} \varepsilon_n i^n \cos[n(\theta-\alpha)]\cdot J_n(kr) \tag{4.1}$$

where α is the incident angle. $n=0, \varepsilon_n = 1; n \ge 1, \varepsilon_n = 2$.

The scattering wave in the elastic space and in the circular inclusion can be written as:

$$W^{(s)} = \sum_{n=0}^{\infty} A_n H_n^{(1)}(k_1 r)\cos[n(\theta-\alpha)], W^{(in)} = \sum_{n=0}^{\infty} B_n J_n(k_2 r)\cos[n(\theta-\alpha)] \tag{4.2}$$

By using the boundary conditions, we can obtain A_n, B_n. The displacement field $W^{(t)}$ can be given as:

$$W^{(t)} = W^{(i)} + W^{(s)} \tag{4.3}$$

Then, we consider the scattering problem of incident SH-wave when the circular inclusion and crack exist at the same time. According to incident field and scattering field in the elastic space containing only a inclusion, the crack-division technique is used to construct the model of SH-wave scattering by an elastic space containing a circular inclusion and a linear crack. The constructing process is that: the space is separated along the crack and a pair of anti-plane opposite forces with the multitude $-\tau_{\theta z}^{(t)}$ are applied to up and down section of the region where crack will appear, therefore the resultant force on up (or down) section of the region is zero, which can be thought as crack . The above constructed Green's function indicates that the basic displacement solution can be obtained wherever the anti-plane linear source force's position it is. Consequently, we can obtain the total displacement field and stress field under the interaction of the circular inclusion and the crack for incident SH-wave.

The force $-\tau_{\theta z}^{(t)}|_{\vec{r}=\vec{r}_0}$ is applied on the crack and the tectonic additional displacement field can be obtained:

$$-\tau_{\theta z}^{(t)}|_{\vec{r}=\vec{r}_0} \times G_1(r, r_0, \theta, \theta_0) \tag{4.4}$$

Integrating along the line of crack, we can obtain:

$$-\int_{\vec{r}_1}^{\vec{r}_2} \tau_{\theta z}^{(t)}|_{\vec{r}=\vec{r}_0} \times G_1(r, r_0, \theta, \theta_0) d\vec{r}_0 \tag{4.5}$$

Hence, the total displacement field can be written as follows:

$$W = W^{(t)} - \int_{\vec{r}_1}^{\vec{r}_2} \tau_{\theta z}^{(t)}|_{\vec{r}=\vec{r}_0} \times G_1(r, r_0, \theta, \theta_0) d\vec{r}_0 \tag{4.6}$$

5. THE SCATTERING DISPLACEMENT MODE AT FAR FIELD

The total scattering wave field includes the scattering wave $W^{(s)}$ produced by the circular inclusion, and $-\int_{\vec{r}_1}^{\vec{r}_2} \tau_{\theta z}^{(t)}|_{\vec{r}=\vec{r}_0} \times G_1(r, r_0, \theta, \theta_0) d\vec{r}_0$ produced by the linear crack, that is

$$W^{(zs)} = W^{(s)} - \int_{\vec{r}_1}^{\vec{r}_2} \tau_{\theta z}^{(t)}|_{\vec{r}=\vec{r}_0} \times G_1(r, r_0, \theta, \theta_0) d\vec{r}_0 \tag{5.1}$$

The scattering wave can be expressed as a series of Hankel function, and their common item $H_n^{(1)}(Kr)$ can be abstracted. Make use of the asymptotic expression of Hankel function as the independent variable is large enough:

$$H_n^{(1)}(z) = (-i)^n \sqrt{\frac{2}{\pi z}} e^{i(z-\pi/4)} \tag{5.2}$$

The scattering far-field displacement can be expressed as:

$$W^{(zs)}(r,\theta) = \sqrt{\frac{8\pi}{k_1 r}} e^{i(k_1 r - \frac{\pi}{4})} F(\theta) \tag{5.3}$$

where

$$F(\theta)=\frac{1}{2\pi}\{W_0\sum_{n=0}^{\infty}(-i)^n A_n\cos\left[n(\theta-\alpha_0)\right]-\int_{\bar{r}_1}^{\bar{r}_2}\tau_{\theta z}^{(t)}\,|_{\bar{r}=\bar{r}_0}\bullet\frac{i}{4\mu}\sum_{n=0}^{\infty}\varepsilon_n(\cos n\theta)[J_n(kr_0)+\frac{A_1}{A_2}]dr_0\}$$
$$=\frac{1}{2\pi}\{\ W_0\sum_{n=0}^{\infty}(-i)^n A_n\cos\left[n(\theta-\alpha_0)\right]-\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\int_{\bar{r}_1}^{\bar{r}_2}\frac{\mu W_0}{r_0}\varepsilon_m i^m(\sin m\alpha_0)m[J_m(kr_0)-\frac{A_{11}}{A_{12}}H_m^{(1)}(kr_0)]\times$$
$$\frac{i}{4\mu}(-i)^n\varepsilon_n(\cos n\theta)[J_n(kr_0)+\frac{A_1}{A_2}]dr_0\ \} \tag{5.4}$$

6.THE SCATTERING CROSS SECTION (SCS)

The time average energy flow of the wave over one period T can be defined as:

$$\begin{aligned}\mathrm{Ave}(\dot{E})&=-\frac{1}{4}i\omega\iint_A n_i\left(\sigma_{ij}\bar{u}_j-\bar{\sigma}_{ij}u_j\right)dA\\&=\frac{\omega}{2}\iint_A\mathrm{Im}\left[n_i\sigma_{ij}\bar{u}_j\right]dA\end{aligned} \tag{6.1}$$

Im($\bullet$) is the imaginary part of a complex function. The above-mentioned formula can be used to calculate the time average energy flow of the elastic wave.

The time average energy flow passing through the surface $r=R$ (which is axis Z direction is a unit long) is:

$$\begin{aligned}\mathrm{Ave}(\dot{E})&=\frac{\omega}{2}\int_0^{2\pi}\mu\,\mathrm{Im}(\frac{\partial W^{(zs)}}{\partial r}\overline{W^{(zs)}})Rd\theta\\&=\frac{\omega\mu}{2}\mathrm{Im}\int_0^{2\pi}\frac{\partial W^{(zs)}}{\partial r}\overline{W^{(zs)}}Rd\theta\end{aligned} \tag{6.2}$$

Substitute (5.1) into formula(6.2), and make use of

$$H_n^{(1)'}(kr)H_m^{(2)}(kr)\approx\frac{2}{\pi r}e^{i\pi(m-n)/2},\quad r\to\infty \tag{6.3}$$

we can obtain the result at far distance R:

$$\mathrm{Ave}(\dot{E})=\frac{\omega\mu}{\pi}W_0^2\,\mathrm{Im}(E) \tag{6.4}$$

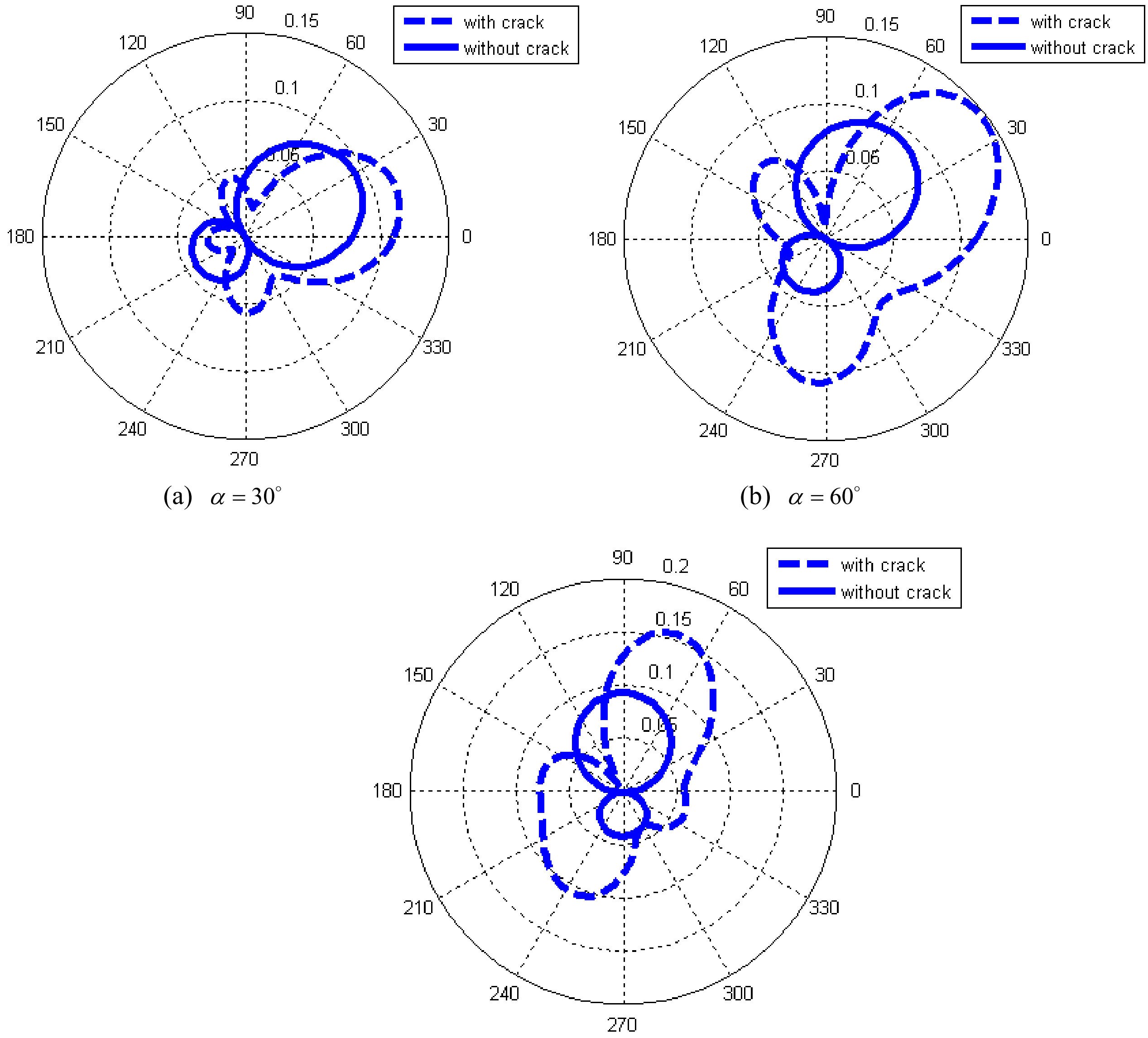
The scattering cross section is the ratio of the total energy of far field scattering wave to the time average energy flow per unit area of the incident wave. For the plane incident SH wave, the time average of energy flow per unit area is:

$$\mathrm{Ave}(\dot{e})=\frac{\mathrm{Ave}(\dot{E})}{A}=\frac{1}{2}\mu_1K_1\omega W_0^2=\frac{1}{2}\sigma_0\omega W_0 \tag{6.5}$$

and let γ expresses the ratio of these two energy, we can obtain the following result: $\gamma=\frac{2}{\pi k}\mathrm{Im}(E)$,where, γ is the scattering cross section.

7.EXAMPLE

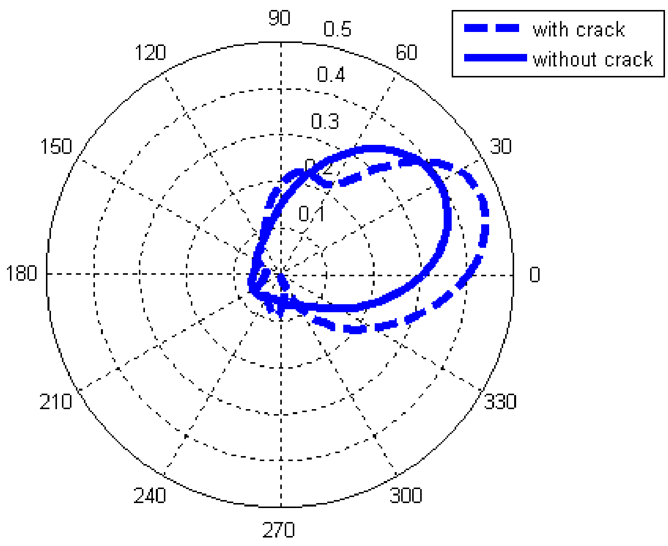
In this paper, we pay attention to a representative kind of models, which is shown as Figure 1. The radius of the circular inclusion is 1, and the length of the crack is 2. In Fig.2 and Fig.3, the distance between the inner tip of the crack and the center of the inclusion is 2. Fig.2 and Fig.3 show that since there is a linear crack compared with the displacement mode of far field produced by the circular inclusion scattering wave, the displacement mode of far field produced by the circular inclusion and crack scattering wave is changed a lot. When the incident wave is vertical to the crack there is the most change. In Fig.4, it shows the influence of μ_1/μ_2 to the Displacement Mode when $k=1$ $\alpha=90^{\circ}$. It can be found that the more different the material of elastic space with the material of inclusion is, the bigger infulence the crack have. In Fig.5, the change of the scattering cross section going together with the change of the incident wave number is given. It can be found that when there is a linear crack low frequency sympathetic vibration come into being.



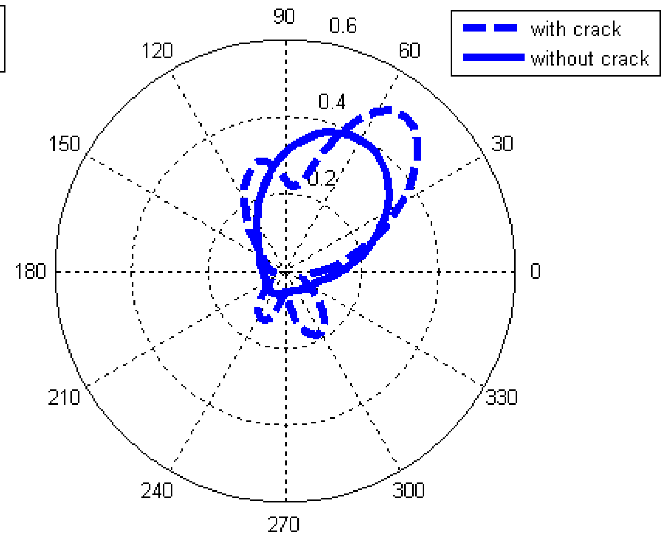
(a) $\alpha=30^{\circ}$ (b) $\alpha=60^{\circ}$

(c) $\alpha=90^{\circ}$

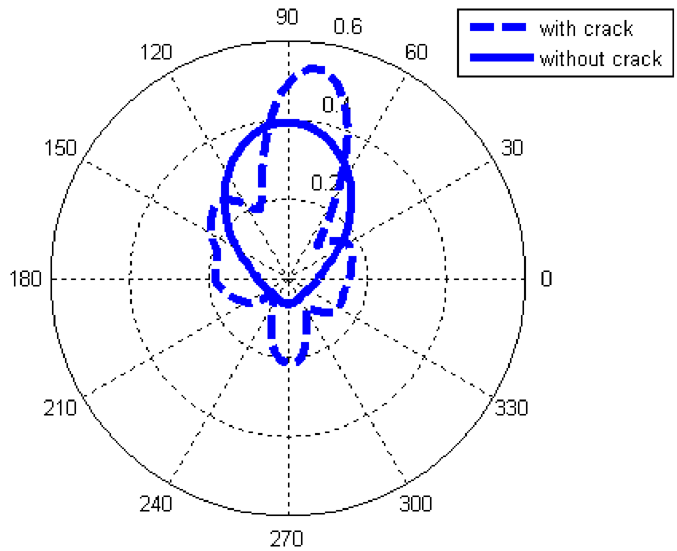
Figure 2 Influence of Crack to the Displacement Mode when $k=1$ $\mu_1/\mu_2=2$



(a) $\alpha = 30^{\circ}$

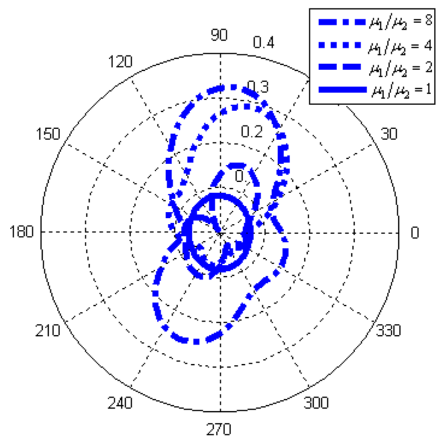


(b) $\alpha = 60^{\circ}$

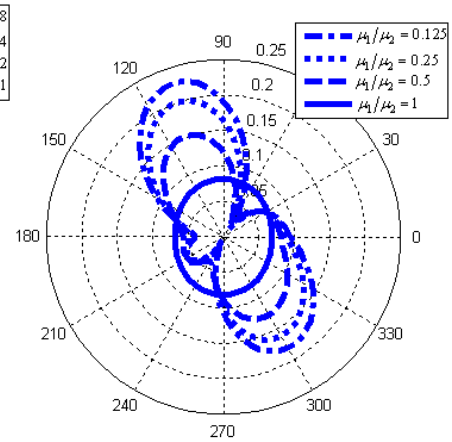


(c) $\alpha = 90^{\circ}$

Figure 3 Influence of Crack to the Displacement Mode when $k = 2$ $\mu_1/\mu_2 = 2$



(a) $\mu_1/\mu_2 = 1-8$



(b) $\mu_1/\mu_2 = 1-0.125$

Figure 4 Influence of μ_1/μ_2 to the Displacement Mode when $k = 1$ $\alpha = 90^{\circ}$

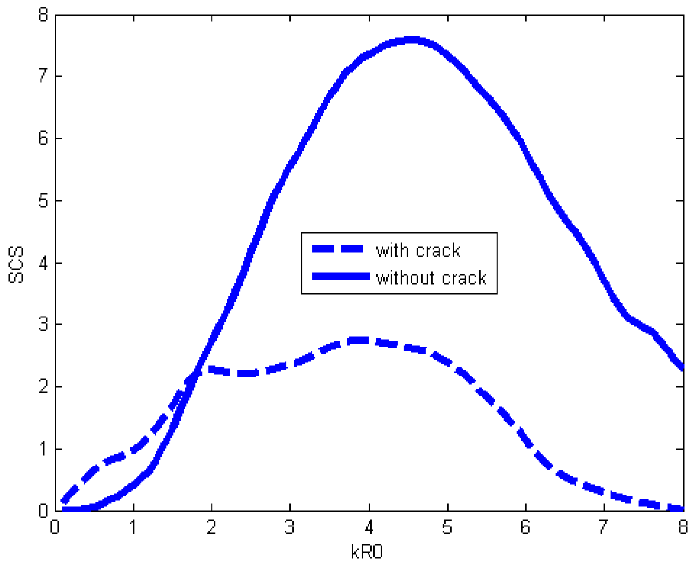


Figure 5 Variation of SCS vs. kR_0 when $\mu_1/\mu_2 = 2$

From the instances above-said, it can be shown that the influence of the crack should not be neglected, and by using the conclusion we should be capable to estimate the position of the crack through analysing test data of the displacement mode.

8.SUMMARY

In this paper, by using the technique of crack-division, far field solution of circular inclusion and linear crack impacted by incident SH-wave is given.. By using the method an example is solved, and some new conclusion is given. The method in the paper could be used to study some other correlative problem.

REFERENCES

Pao Y. H. (1983).Elastic Waves in Solids. *ASME Journal of Applied Mechanics* **50:4,** 1152-1164.

Zheng Zhemin, Zhou Heng, Zhang Hanxin. (1995) Trends of Development in Mechanics in the Early 21st Century. *Advances in Mechanics* **25:4,** 433-449

Wang Duo, Wang Yuesheng. (1993). Recent Progress in Dynamics of Interface. *Shanghai Journal of Mechanics* **14:4,** 1-15

Liu Diankui, Liu Hongwei. (1999).Scattering of SH-wave by Cracks Originating at A Circular Hole Edge and Dynamic Stress Intensity Factor. *Acta Mechanica Sinica* **31:3,** 292 –299

Liu Diankui, Liu Hongwei. (1998).Scattering and Dynamic Stress Concentration of SH-wave by Interface Circular Hole. *Acta Mechanica Sinica* **30:5,** 597-604

Liu hongwei, Liu Diankui. (1999).Far Field Solution of SH-wave Scattered by Interface Circular Hole. *Acta Mechanica Solida Sinica* **20:4,** 349-355

Li HongLiang, Liu DianKui. (2004).Interaction of SH-Waves by Cracks with Circular Inclusion.*Journal of Harbin Engineering University* **25:5,** 618-622.

Li HongLiangi. (2004).The Interaction of Circular Cavity, Inclusion with Beeline Cracks by SH-wave, *Harbin Engineering University*.