STRAIN DEPENDENT IMPEDANCE IN SHALLOW FOUNDATIONS

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SUMMARY

The dynamic response of rigid shallow foundations can be analysed by means of the impedance approach, in which dynamic stiffness and damping provided by the ground, along any direction of displacement, are evaluated according the elasto-dynamic theory. However, known impedance solutions do not consider soil inelasticity.

In this paper the degradation of soil stiffness with strain level has been introduced in the finite element code SOFIA, to evaluate the strain-dependent stiffnesses of the ground for vertical, rocking and horizontal motions. The numerical analyses, performed for various foundation dimensions and soil conditions, have indicated the considerable effect of the degradation of the soil stiffness with the strain-level. Thus, the complete dynamic response of the soil-foundation system could be clearly non-linear: the decrease in resonant frequency and the increase in displacement amplitude may be consistent with a decrease of the soil stiffness.

INTRODUCTION

Modelling dynamic soil-structure interaction has become widely used in seismic response analysis and foundation isolation problems.

In the seismic case, not only the displacement of the foundation and the soil must be considered, but also the effect of the foundation stiffness in filtering out high frequency components. In the foundation isolation problems, only the displacement of both the foundation and the soil, induced by the inertial forces, are considered. This second type of analysis is called "*inertial interaction*" and is widely employed for those problems in which dynamic loading acts directly on the foundation or on the superstructure [Gazetas & Mylonakis, 1998].

If the foundation is infinitely rigid, the response to dynamic loading arises solely from the displacement of the supporting ground. In the past decades this particular problem has been investigated in terms of foundation impedances, which account for dynamic stiffness and damping supplied by the ground along any direction of displacement.

The method of impedance comes from the elastic half-space approach [Reissner, 1936; Bycroft, 1956] and has been generalised by Lysmer [1965]; the latter introduces the equivalent lumped mass-spring-dashpot system, which is able to reproduce the results of half-space model for each mode of vibration.

More recently, the foundation impedance approach has received great impetus [Kausel & Roesset, 1975; Dobry and Gazetas, 1985; Novak, 1987; Gazetas, 1991; Wolf, 1994], so that solutions are available for various foundation geometries, boundary conditions and mechanical soil properties. However, these solutions come from elastic analysis and do not take into account soil inelasticity. This paper deals with the effects of non-linearity in the formulation of foundation impedance. The degradation of the soil stiffness has been introduced in the finite element code SOFIA [Massimino, 1999], by considering soil elements provided with hyperbolic stress-strain

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relationship during loading and linear stress-strain relationship during unloading-reloading [Duncan & Chang, 1970]. With this approach residual plastic strains are introduced in the analyses without considering any hardening criterion. The strain dependent stiffnesses of the ground have been determined for vertical, horizontal and rocking motions.

MAIN ASPECTS OF THE IMPEDANCE APPROACH

An isolated rigid foundation resting on the half-space has six degrees of freedom, three traslational and three rotational. Vertical traslation and torsional rotation are uncoupled in symmetric foundations, while swaying and rocking are coupled because the center of gravity of the structure is above the center of pressure of the soil reaction. These coupled impedances are negligible in shallow foundations, but their effects become appreciable in presence of foundation embedment. The eight impedances, which characterise the dynamic soil reaction against the foundation block, can be put in the following form [Gazetas, 1991]:

$$\tilde{K}(\omega) = K(\omega) + i \cdot \omega \cdot C(\omega)$$

(1)

The impedance $\tilde{K}$ is a complex number, where the real component $K(\omega)$ reflects the stiffness and the inertia of the ground, while the imaginary component is the product of the circular frequency ω and the damping coefficient $C(\omega)$; the latter reflects two types of damping: radiation and material damping. Radiation damping depends on the strain level, because of its dependence on the mobilised shear modulus $G(\gamma)$ in the ground; material damping depends again on the mobilised strain-level in the ground, as widely shown by the results of resonant column and cyclic triaxial tests, carried out in both granular and cohesive soils [Seed & Idriss, 1970].

The real component $K(\omega)$ can be expressed as the product of the static stiffness K_{st} and the dynamic stiffness coefficient $K_{dy}(\omega)$. The dynamic coefficient $K_{dy}(\omega)$ also depends on the mobilised shear modulus in the ground.

The variation of $K_{dy}(\omega)$ and of the radiation damping coefficient, versus shear modulus, has been summarised by Gazetas [1991] for several foundations and soil conditions; mobilised material damping can only be evaluated through the damping ratio coefficient obtained by laboratory tests. However, the variation of $K_{dy}(\omega)$ and of the radiation damping, versus the mobilised shear modulus, should not be considered so much significant for the impedance prediction, because the soil portion, which undergoes substantial shear modulus degradation, is limited enough and located near the foundation. Instead, the variation of both material damping and K_{st} with the strain-level can be considered very significant in the impedance modification.

The aim of this paper is to investigate only the variation of K_{st} with the strain-level, by using uncoupled numerical analysis.

SOIL PARAMETERS AND FOUNDATION PROPERTIES

Homogeneous cohesive half-space of different properties has been considered in the numerical analyses; three typical rigid shallow foundation, without embedment, have also been considered: a square 3.00x3.00 m foundation, a rectangular 3.00x6.00 m foundation and a strip foundation 3.00 m wide. Geotechnical soil properties are summarised in table 1.

The selected values of undrained cohesion are typical of soils with different stress histories: for example, soil **I** is a normally consolidated clay, soil **II** is a slightly overcolidated clay and finally soil **III** is an highly

Table 1 – Soil parameters considered in the numerical simulations

Soil	***Undrained cohesion*** c_u ***(kPa)***	***Initial shear modulus*** G_0 ***(kPa)***	***OCR***
I	*40*	*12.500*	*1*
II	*80*	*25.000*	*5*
III	*160*	*50.000*	*10*

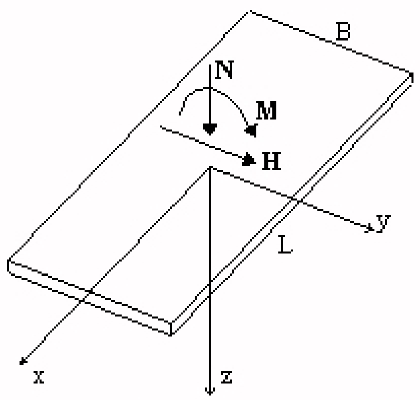

Fig. 1 – Rigid foundation geometry and selected reference system

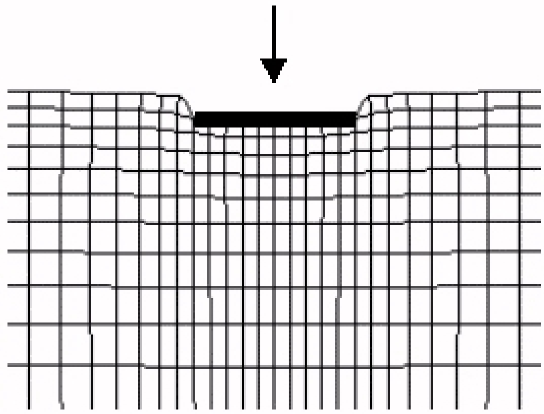

Fig. 2 – Controlled displacement of the foundation to evaluate vertical stiffness K_z

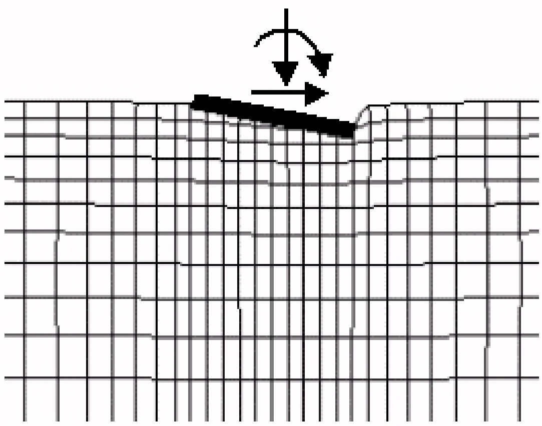

Fig. 3 – Controlled displacement of the foundation to evaluate rocking stiffness $K_{\phi,x}$ in absense of up-lifting, settling and swaying

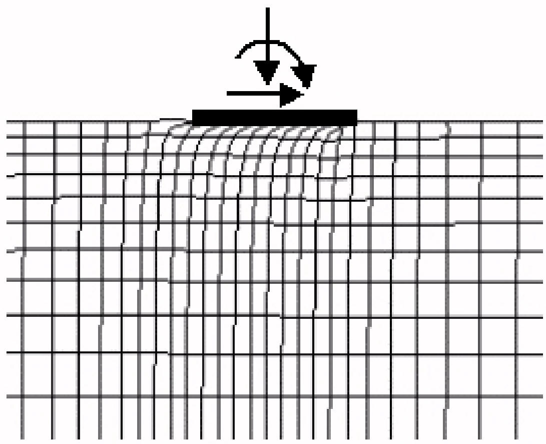

Fig. 4 – Controlled displacement of the foundation to evaluate swaying stiffness K_y in absence of rotation, settlement and up-lift

overconsolidated clay. The initial shear modulus G_0 has been selected by assuming the constant value of the ratio $G_0/c_u \cong 300$. As summarised by Weiler [1988], this ratio is typical of slightly-overconsolidated plastic clays.

Poisson’s ratio has always been fixed equal to $\nu = 0.47$, and the unit weight equal to $\gamma = 20$ kN/m^3. The overconsidation ratio (*OCR*) allowed the initial stress condition to be established in relation to the soil consistency.

The whole soil-foundation system has been analysed by means of the SOFIA code, which employs monodimensional elements for the rigid foundation and isoparametric quadratic plane elements to model the ground [Massimino, 1999]. The interacting soil volume is 21.00 m large and 12.00 m deep and presents a gradual decrease in size of the elements when approaching the foundation. The vertical boundaries consist of vertical rollers, the lowest horizontal boundary is assumed to be a fixed base, finally the highest horizontal boundary represents the free-surface.

Even if the SOFIA code was at the beginning made to work in plane-strain condition, tridimensionality can be taken into account with a simplified procedure based on the comparison with the Boussinesq [1885] elastic solution. The hyperbolic relationship for soil element considers that the limiting pressure approaches twice the undrained cohesion and the initial slope is equal to the Young’s modulus E_0; the latter parameter is linked to the initial shear modulus G_0 by means of the theory of elasticity.

The displacements, which have been taken under consideration for a single foundation, are the vertical, the rocking and the horizontal. The selected reference system is shown in Fig. 1 together to a view of the contact area between the soil and the foundation.

In the vertical displacement mode (Fig. 2) the foundation is constrained to settle uniformly, by increasing the vertical load in the mass center of the basement. The ratio between the applied vertical load and the related displacement w, gives the vertical static stiffness K_z of the foundation.

In the rocking displacement mode (Fig. 3) the applied system of forces to the mass center has been varied appropriately to constrain the block to rotate around a side parallel to the x axis without up-lifting, settling or swaying. Thus neither any relative separation between the soil and the foundation, nor any coupling between vertical, horizontal and rocking displacements has been permitted. The ratio between the acting bending moment M_x and the rotation ϕ gives the rocking stiffness $K_{\phi,x}$.

In the swaying displacement mode (Fig. 4) the applied system of forces to the mass center has been varied appropriately, to constrain the block to translate along the y direction without rotating, settling or up-lifting. The ratio between the acting horizontal force H and the related displacement u gives the horizontal stiffness K_y of the foundation.

The above mentioned procedure has allowed the foundation stiffnesses to be evaluated starting from the linearly elastic initial value up to the full non-linear range. Nevertheless, the coupling between various displacements has not been considered; thus this type of analysis can be employed in problems where plastic coupling is not of primary interest. Moreover, only monotonic loading has been considered in the simulations, so the effects of various cycles of unloading-reloading on foundation stiffness has not been investigated.

RESULTS OF THE ANALYSES

The variation of the vertical load N with respect to vertical displacements w are summarised in Fig. 5a for all the examined foundations and soil conditions. The initial elastic behaviour of the whole soil-foundation system can be apprised only for small displacements, less than 10 mm, and for the soil conditions **II** and **III**, in which case an initial linear relationship between N and w is evident.

The threshold displacement at which non-linearity takes place can be easily evaluated by means of the normalised representation of foundation stiffness, shown in Fig. 5b. Local slope of the non-linear N-w relationship gives the tangent vertical stiffness K_z; if this local stiffness is normalised with respect to the initial value, the ratio K_{tang}/K_{init} versus loading level gives the degradation law of the initial foundation stiffness. The elastic threshold can be fixed in correspondence of a foundation stiffness degradation not exceeding 10 %, to which corresponds a ratio $K_{tang}/K_{init} \geq 90$ %. The same normalised representation of the foundation stiffness allows us to appreciate the limiting vertical load supported by the foundation; for a degradation of foundation stiffness of about 90 %, to which correspond a ratio $K_{tang}/K_{init} \cong 10$ %, liming vertical load is always achieved.

The variation of the bending moment M, with respect to the rocking displacement ϕ around the x axis, is summarised in Fig. 6a. The initial elastic behaviour of the whole soil-foundation system is less evident respect to the vertical loading case. Referring to the normalised rocking stiffness, shown in Fig. 6b, it can be observed how an elastic threshold is present only for the soil condition **III**. Also in this case the limiting values of the bending moment are achieved for degradation of rocking stiffness near to 90 %.

The variation of the horizontal load H with respect to the horizontal displacement u along the y direction is shown in Fig. 7a. For this condition of loading, the elastic threshold is practically absent for every soil condition (Fig. 7b) and the limiting horizontal load is approached with almost linear degradation of the horizontal foundation stiffness.

The initial soil stiffnesses K_z, $K_{\phi,x}$ and K_y, obtained in this study, have been compared with those summarised in Gazetas [1991]. This comparison has indicated a general good agreement between the two apporaches, with a maximum difference of about 5 %, depending on the departure of the actual foundation from the plane-strain conditions. As an example, table 2 reports the rocking stiffnesses evaluated by the numerical analysis and those computed according Gazetas [1991].

Table 2 – Initial rocking stiffness $K_{\phi,x}$

SOIL TYPE	Square foundation $B = L = 3$ m		Rectangular foundation $B = 3$ m; $L = 6$ m		Strip foundation $B = 3$ m	
	Numerical analysis [MNm]	**Gazetas (1991) [MNm]**	**Numerical analysis [MNm]**	**Gazetas (1991) [MNm]**	**Numerical analysis [MN]**	**Gazetas (1991) [MN]**
I	280	294	510	523	78	79
II	560	589	1010	1046	157	158
III	1115	1178	2020	2092	315	316

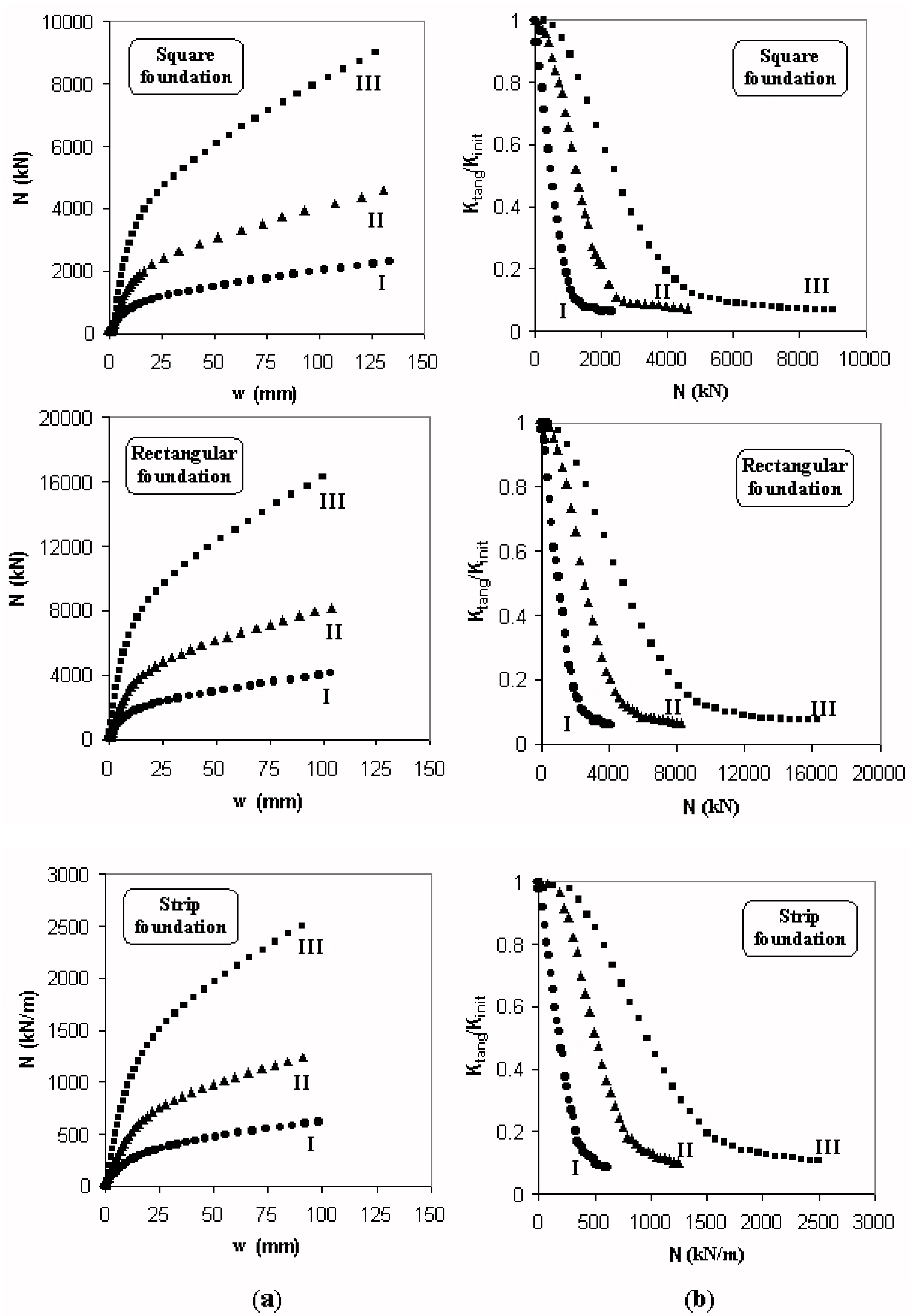

Fig. 5 – Response of the foundations to vertical displacement for different soil conditions
(a) Vertical load versus vertical displacement; (b) Normalised tangent vertical stiffness versus vertical load

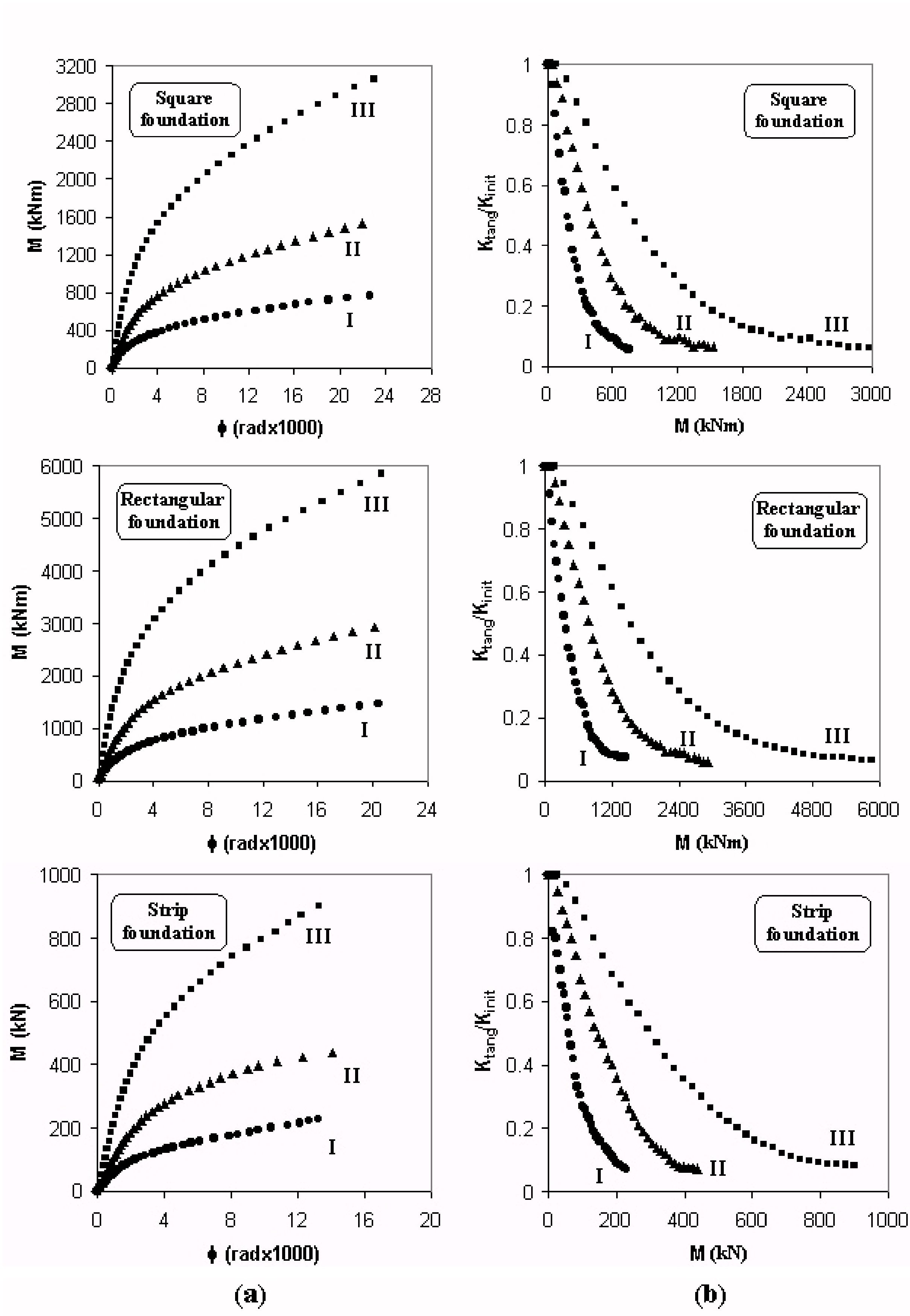

Fig. 6 – Response of the foundations to rocking displacement for different soil conditions
(a) Bending moment versus rotation; (b) Normalised tangent rocking stiffness versus bending moment

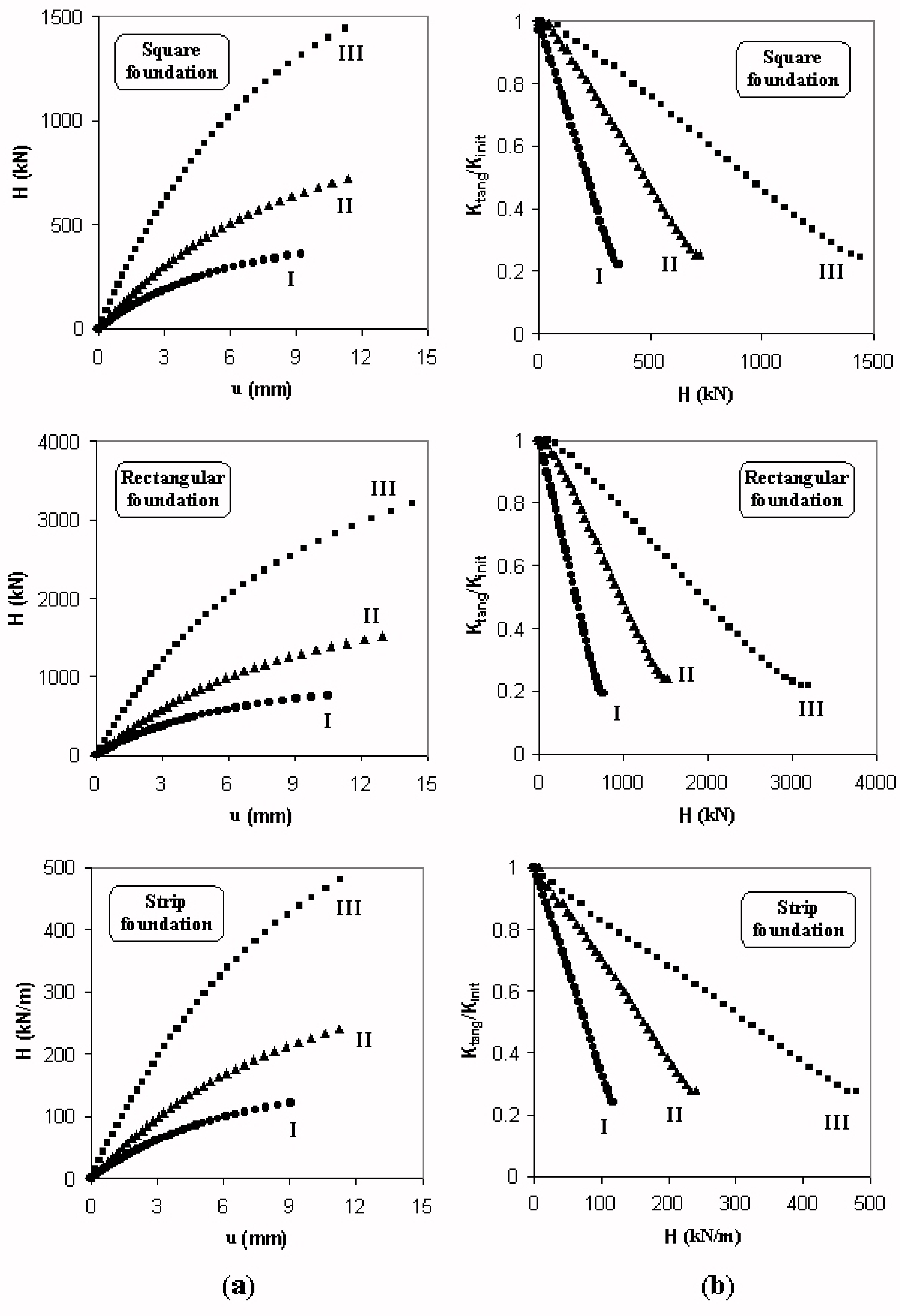

Fig. 7 – Response of the foundations to swaying displacement for different soil conditions
(a) Horizontal load versus horizontal displacement;
(b) Normalised tangent swaying stiffness versus horizontal load

CONCLUSIONS

In the last decades, the dynamic response of rigid shallow foundation have been extensively studied in terms of foundation impedances. With this approach the effect of elastic half-space is reproduced by means of appropriate lumped soil spring and dashpot.

The known impedance solutions do not take into account soil inelasticity. Non-linearity could rise from increasing of material damping and decreasing of foundation stiffness with strain level [Novac, 1985]. The effect of static stiffness degradation has been considered in this paper by performing non-linear finite element analysis of some shallow foundation resting on cohesive soil. The analyses have given initial stiffnesses in a good agreement with those proposed by Gazetas [1991] for vertical, rocking and swaying displacements. The proposed laws of degradation of normalised stiffnesses take into account the stiffness reduction with the stress level, starting from the initial elastic value up to almost failure. The non-linear stiffness can be employed in dynamic uncoupled analysis to emphasise any decreasing in resonant frequency and increasing in displacement amplitude in non-linear foundation respect to the linear case.

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