



ANCHORAGE STRENGTH ESTIMATION ON RAKING-OUT FAILURE OF 90-DEGREE HOOKED BEAM BARS IN R/C RECTANGULAR COLUMN

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Abstract

In this paper, a current formula to estimate "raking-out failure anchorage strength" is verified with experimental data. Besides, extension of the formula is considered for the joint which has stepped column with large depth simplifying a pile cap based on experimental data.

The formula of raking-out failure strength in previous papers is a linear combination force of concrete and hoops: the force of concrete in the assuming fracture at an angle of 45° and the force of hoops across the assuming fracture. This formula gave roughly good calculation results only for the specimens without stepped column, while the calculation values of those with stepped column were about 8 percent lower than the experimental values on average. Then we changed the formula and as a consequence, modified formula provided estimated strength as precisely as the current formula for the straight-column specimens, but as to the stepped-column specimens, the estimated strengths by the modified formula were a little lower than the experimental results. However, the proposed formula can take account of the different substitution of shear reinforcement depending on their distance from beam bars.

Keywords: raking-out failure; beam-column joint; reinforced concrete



1. Introduction

1.1 Background

It is said that there are three failure modes in regards to 90-degree bend used at the end of beam bars in exterior R/C beam-column joint: side split failure, local compression failure, and raking-out failure [1]. The raking-out failure is the mode that concrete of column seems to be raked out, with embedment length being short, and it is also called as local shear failure. Therefore, in AIJ Standard for Structural Calculation of Reinforced Concrete Structures [2], it is recommended that the embedment length should be long enough to avoid such failure, and the capacity formula of raking-out failure is presented in the case that there is no option but to let the anchorage reinforcements short.

1.2 Current capacity formula of raking-out failure

The current capacity formula of raking-out failure is based on an assumption that the tensile force of main beam bars is composed of resistances that shear reinforcements and concrete bear. The capacity formula shown in Eq. (1) derives from former experimental data on which some values of coefficients are based [3].

$${}_{cal}T_{u3} = T_c + T_w \quad (1)$$

where

$$T_c = 2L_{dh} \cdot b_e \cdot \sigma_t (1 + 6.32 \sigma_0 / \sigma_B) / \sin \theta, \quad T_w = k_w \cdot a_w \cdot \sigma_{wy}$$

$$L_{dh} = l_{dh} - d_b / 2 \quad (l_{dh} \text{ is development length})$$

$$b_e = \text{effective width of joint} \quad (= b_b - n \cdot d_b)$$

$$b_b = \text{beam width}$$

$$n = \text{number of beam main bars in a layer}$$

$$d_b = \text{diameter of beam main bars}$$

$$\sigma_t = \text{tensile strength of concrete} \quad (= 0.313 \sqrt{\sigma_B})$$

$$\sigma_0 = \text{axial stress of column (where } \sigma_0 \leq \sigma_B / 6)$$

$$\sigma_B = \text{compressive strength of concrete}$$

$$\theta = \text{strut angle (See Fig. 1)}$$

$$k_w = \text{effective factor of shear reinforcement resistance} \quad (= 0.7)$$

$$a_w = \text{total section area of lateral reinforcements above/below beam bars in the distance of } L_{dh}$$

$$\sigma_{wy} = \text{yield stress of shear reinforcements}$$

The basic idea is as follows. Crack pattern of concrete as shown in Fig. 1 is assumed. The beginning point of the crack lines is the intersection of two axes of beam longitudinal bars and the tail of hook. Then they extend in the angle of elevation and depression that are both 45 degrees. The lateral tensile force is regarded as the sum of concrete resistance, T_c , and shear reinforcement resistance across the crack lines, T_w . The factor $\sin \theta$ in the equation of T_c stands for the effect that the resistance that is directly transmitted from bend point to compressive zone of the beam through the compressive strut is high when the strut angle, θ is low. This is based on the data that the capacity of specimens are generally proportional to $1/\sin \theta$. The effective factor of shear reinforcement resistance, k_w in the equation stands for the effect that the contribution ratio of the shear reinforcements distant from the tensile reinforcement in beams is lower than the average strength of shear reinforcements in the assumed crack area.

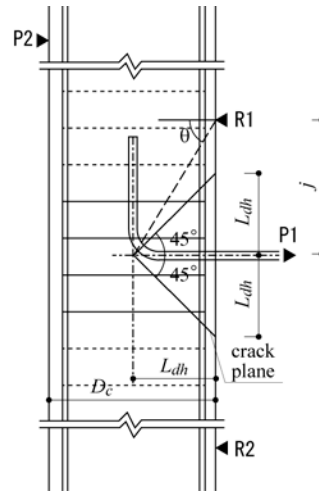


Fig. 1 – Model for current capacity formula of raking-out failure [3]

The coefficients of this formula are not derived from a mechanical model but from experimental data with parameters such as development length, compressive strength of concrete, lateral reinforcement ratio, effective width of joint, axial force ratio in column, and strut angle. Then they should be verified with other experimental data.

In addition, the current formula is on a premise that the dimensions of the column section are the same in the vertical direction as in the normal beam-column joint. Thus the definition of strut angle is not clear when the column depth above and beneath the upper tensile reinforcements of beams in the joint are different. Furthermore, in the case there is a pile cap in the joint area, the volume of concrete is so massive that the damage would not easily appear on the surface, and it is foreseeable that the capacity of the joint will be risen. It is also possible the damage of column affect the performance of the joint, which was not enough to be focused on.

Hence, in the earlier paper [4], we made a study on the definition of strut angle for the joint of exterior column, pile and foundation beam, and proposed a modified formula. However, as the coefficient of the shear reinforcement resistance in the current formula and the modified formula (hereinafter referred to as the previously-modified formula) was determined to match the data, it is still doubtful that those formulae are applicable to the case that the cross-sectional shape of column is constant around the joint.

In this paper, we proposed a new mechanical model with as few arbitrary coefficients as possible assuming crack planes of concrete at the maximum strength by comparing to the experimental data.

2. Experiment results of element specimens

2.1 Specimens

We used experimental data of 21 specimens from 2011 to 2013 to compare with calculated values. Table 1 shows the variables of the specimens, Figs. 2 - 4 show the shapes and arrangements of them. The specimens from 2011 to 2012 have a straight column member, then in 2013, to expand the scope of application of the formula, we conducted the experiments that had different column depth in the same specimen. We called the column "stepped column", which imitated a pile cap to consider the condition of having a large cross-sectional area. Table 2 and Table 3 show the material properties of concrete and steel bars obtained from the material tests.



Table 1– Information of specimens

Name of specimen	Number of layers	Development length ^{*2} (mm)			Number of bars in a layer	Dimension b×D (mm)		Type of column arrangement ^{*1}	Shape of the ends of column bars	Side covering depth of beam bars in joint (mm)	Distance from the center of beam bars(mm) ^{*2}																																			
		L_{dh1}	L_{dh2}	L_{dh3}		Column	Joint				Support point of column	Support point of joint	Hoops			Lateral reinforcements of joint																														
													s_{c1}	s_{c2}	s_{c3}	s_{j1}	s_{j2}	s_{j3}																												
L6F3D6	2	304	256	208	2	250×400	A	Welded nut	90	719	478	69	169	269	69	169	269																													
L6F3T6	3									743	454	93	193	293	93	193	293																													
L2F3D6	2	228	180	132						1	150×400	B	Welded nut	75	719	478	69	219	369	69	219	369																								
L2F3T6	3														743	454	93	193	293	93	193	293																								
L6F3D4	2	304	256	208											2	250×300	250×500	C	Welded nut	85	719	478	72	172	272	120	220	320																		
L6F3D8																					719	658	69	169	269	69	169	269																		
L6F6D6	3	304	256	208																	2	250×300	250×500	D	180° Hook	85	719	478	72	172	272	120	220	320												
L6F6T6																											743	454	93	193	293	93	193	293												
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L6F3DB	2	304	256	208																												2	250×300	250×500	C	Welded nut	85	719	478	72	172	272	120	220	320	
L2F3DB		228	180																																			132	719	478	72	172	272	120	220	320
A1F2	2	228	180	180	2	250×300	250×500	C	Welded nut																													85	719	478	72	172	272	120	220	320
A2B2																																							180	120	220	320				
A3F2L										132	170	270	370																																	
A4F2S										180	120	220	320																																	
A6F2C										180	270	420																																		
A7F2C2										180	220	320																																		
A8F2C3										180	270	420																																		
A9F2S2										180	220	320																																		
A10F2W										180	220	320																																		

*1 Type of column arrangement (unit: mm)

*2 Distance from the center of beam bars to shear rebars and development length

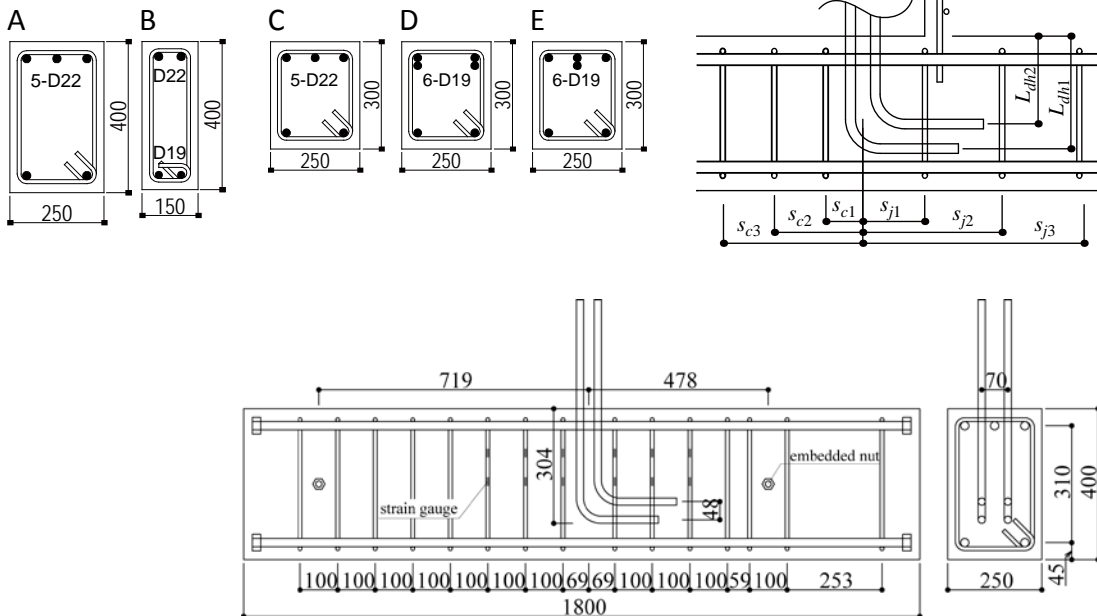


Fig. 2 –L6F3D6 and L6F6D6 (in 2011)

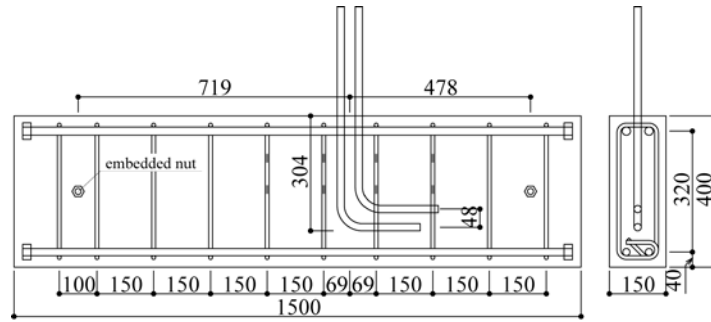


Fig. 3 –L6F3DB (in 2012)

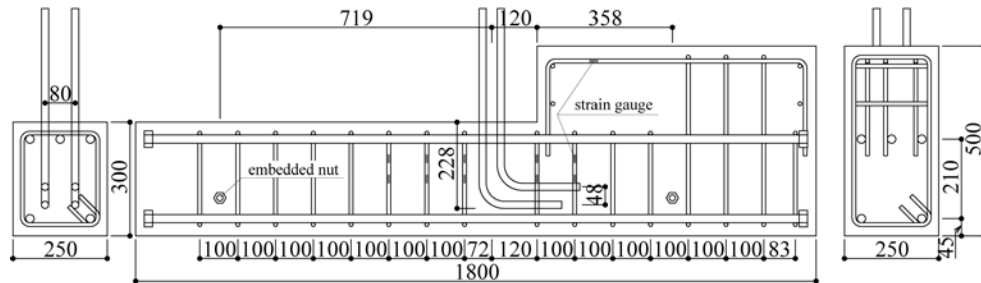


Fig. 4 – A1F2 (in 2013)

Table 2– Properties of concrete

Year	Name of specimen	Compressive strength (MPa)	Splitting tensile strength (MPa)	Modulus of longitudinal elasticity (GPa)
2011	L6F3D6	30.8	2.61	27.0
	L6F3T6	30.7	3.02	26.7
	L2F3D6	29.6	2.97	25.3
	L2F3T6	29.5	2.73	26.0
	L6F3D4	30.0	2.73	25.9
	L6F3D8	30.9	2.69	25.8
	L6F6D6	57.3	3.02	34.1
	L6F6T6	59.1	3.21	34.1
	L2F6D6	59.1	3.79	34.2
	L2F6T6	59.6	4.32	35.3
2012	L6F3DB	34.1	2.40	26.7
	L2F3DB	34.1	2.10	27.1
2013	A1F2	28.9	2.50	23.9
	A2B2	29.5	2.40	22.0
	A3F2L	28.7	2.27	23.0
	A4F2S	31.1	2.45	23.7
	A6F2C	30.1	2.73	23.0
	A7F2C2	34.2	2.30	26.6
	A8F2C3	34.7	2.35	25.4
	A9F2S2	35.2	3.01	25.9
	A10F2W	38.5	3.27	26.4

Table 3– Properties of steel bars

Year	Use	Standard	Yield stress (MPa)	Maximum tensile strength (MPa)	Young's modulus (GPa)
2011	Beam bars	D19(SD490)	538	678	198
	Column bars	D22(SD390)	468	646	203
	Shear reinforcements	D10(SD295A)	370	524	196
2012	Beam bars	D19(SD490)	532	662	191
	Column bars	D22(SD390)	460	634	163
	Shear reinforcements	D10(SD390)	441	643	203
2013	Beam bars	D19(SD345)	358	546	202
	Column bars	D19(SD390)	459	643	189
		D22(SD390)	503	698	197
	Shear reinforcements	D10(SD295A)	346	492	175

2.2 Loading test

The specimens were set in the apparatus with the column lying down as in Fig. 5, supports were set at the contraflexure point of the column on the side of the tensile face of the beam and at the center of the compressive zone of the beam. The specimens were loaded by the vertical actuator connected with the embedded bars. The test was conducted as a static pullout loading. After loading to the half of calculated maximum value twice, they were loaded to the displacement of 10-20 mm, then unloaded.

The displacement was measured by displacement gauges set on the measurement frame (Fig. 5) supported by reference points at the contraflexure points of column and at the center of compressive zone. Besides, strains of several bars were measured by strain gauges put at 2-3 shear reinforcement bars on the both sides of beam main bars and a pile cap reinforcement as shown in Figs. 2 - 4.

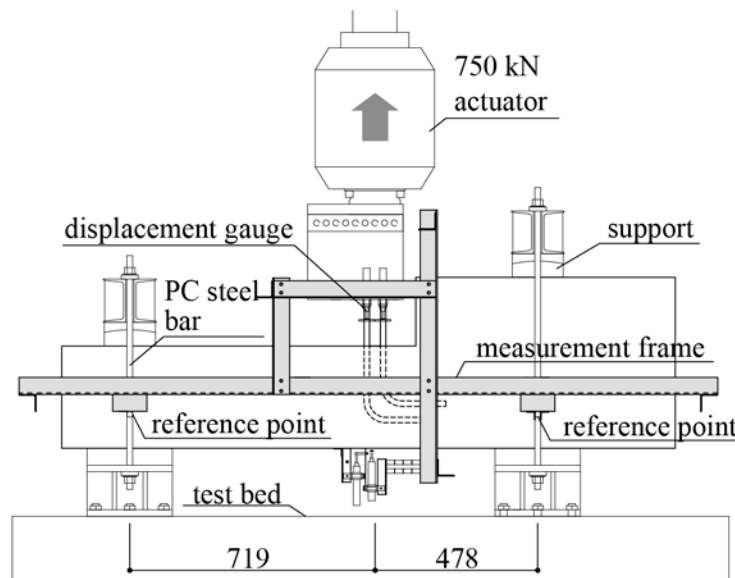


Fig. 5 – Loading apparatus and measurement frame (in 2013)

2.3 Test results

2.3.1 Fracture process

Fig. 6 shows photographs of the specimens in the final fracture states and illustrations of crack patterns at the maximum strength. Every specimen developed cracks along the main reinforcements of the beam first, and the shear cracks in the column and/or the joint area were developed, therefore the stiffness was degraded. After that, the shear reinforcements around the main reinforcements of the beam yielded, and they reached the maximum strength. Although the order of the cracks observed were different depending on the specimens, the fracture mode at the point of the maximum strength seemed similar to that in Joh et al. (1995) [3]. The specimens with short development length of main reinforcements had crack lines mainly in shallower position than those with long development length. Besides, the specimens with triple layer of reinforcements had crack lines mainly in shallower position than those with double layer.

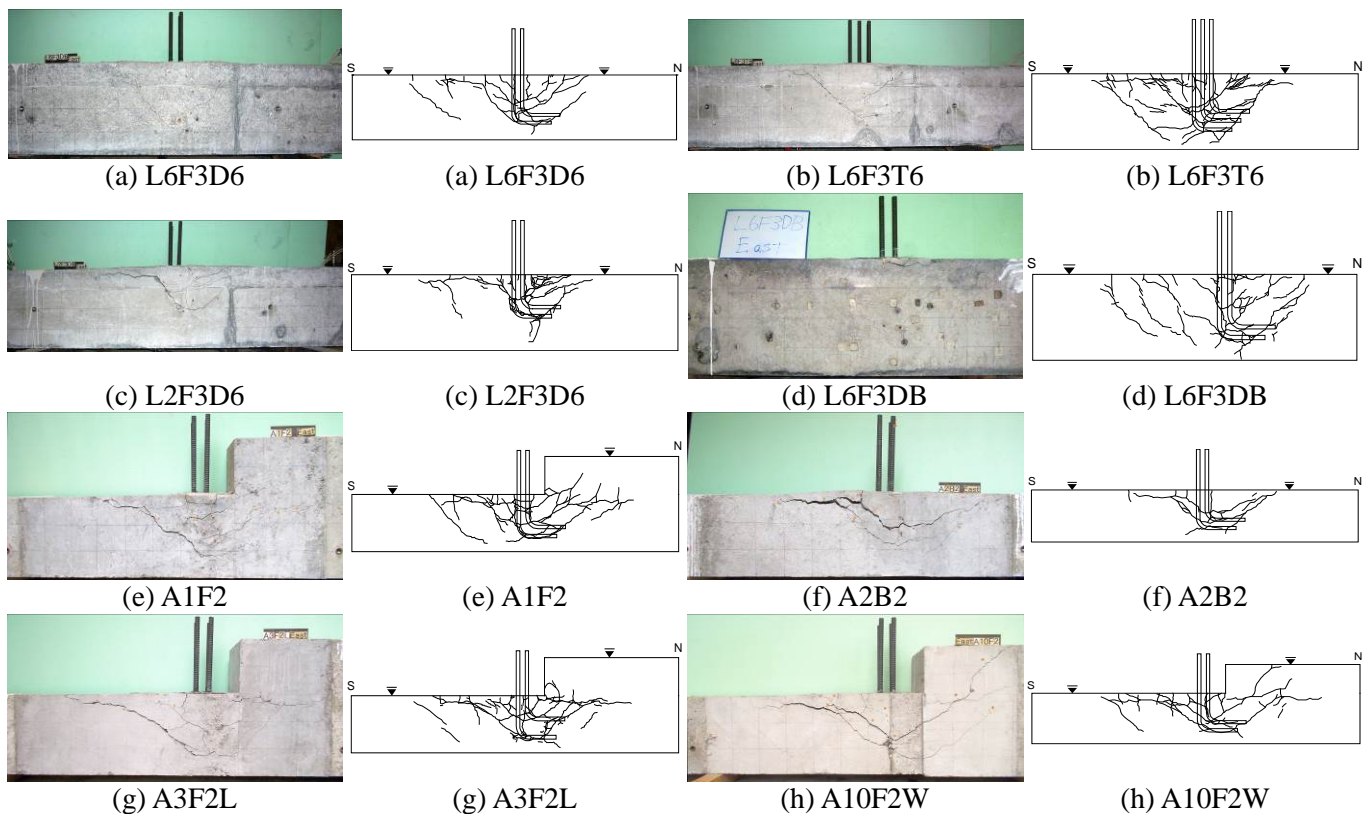


Fig. 6 – Photos in the final fracture states and crack patterns at the maximum strength

As to the specimens with stepped column, cracks around the main reinforcements of the column extended beyond the center of the support on the joint side when the deformation was large. Besides, the damage on the column side was more severe than that on the joint side at the end of the test. Furthermore, because of the fact that the shear cracks that developed at the joint extended toward the support in the compression zone of the beam, and that the value of the strain gauge at the pile cap reinforcement (see Fig. 4) showed compression through the test, it can be concluded that the compressive force transmitted from the end of the main reinforcement of the beam to the face of the pile cap.

2.3.2 Relationship between tensile force and displacement at loading point

Fig. 7 shows the relationship between tensile force and displacement at loading point of all the specimens shown above. In each graph, X mark indicates the maximum strength, and dotted dashed line indicates the calculated

capacity by the previously-modified formula. In the calculation, the strengths of the materials in Table 2 and Table 3 were used, and the development length of multi-layer reinforcements was the average of all the tensile reinforcements. As to the specimens with stepped column, the angle of the line toward the face of the pile cap at the center of compressive zone was defined as the strut angle, θ . In all the graphs except for the specimen in 2012, which had main reinforcement in a single row, the degradation after the raking-out failure was more gradual than shear failure. The number of shear reinforcement bars counted in calculated shear reinforcement resistance was the number of bars across the crack lines at an angle of 45° above/below the horizon developed from the point that the central axis of main reinforcements and the tail of hook in the first layer were crossing.

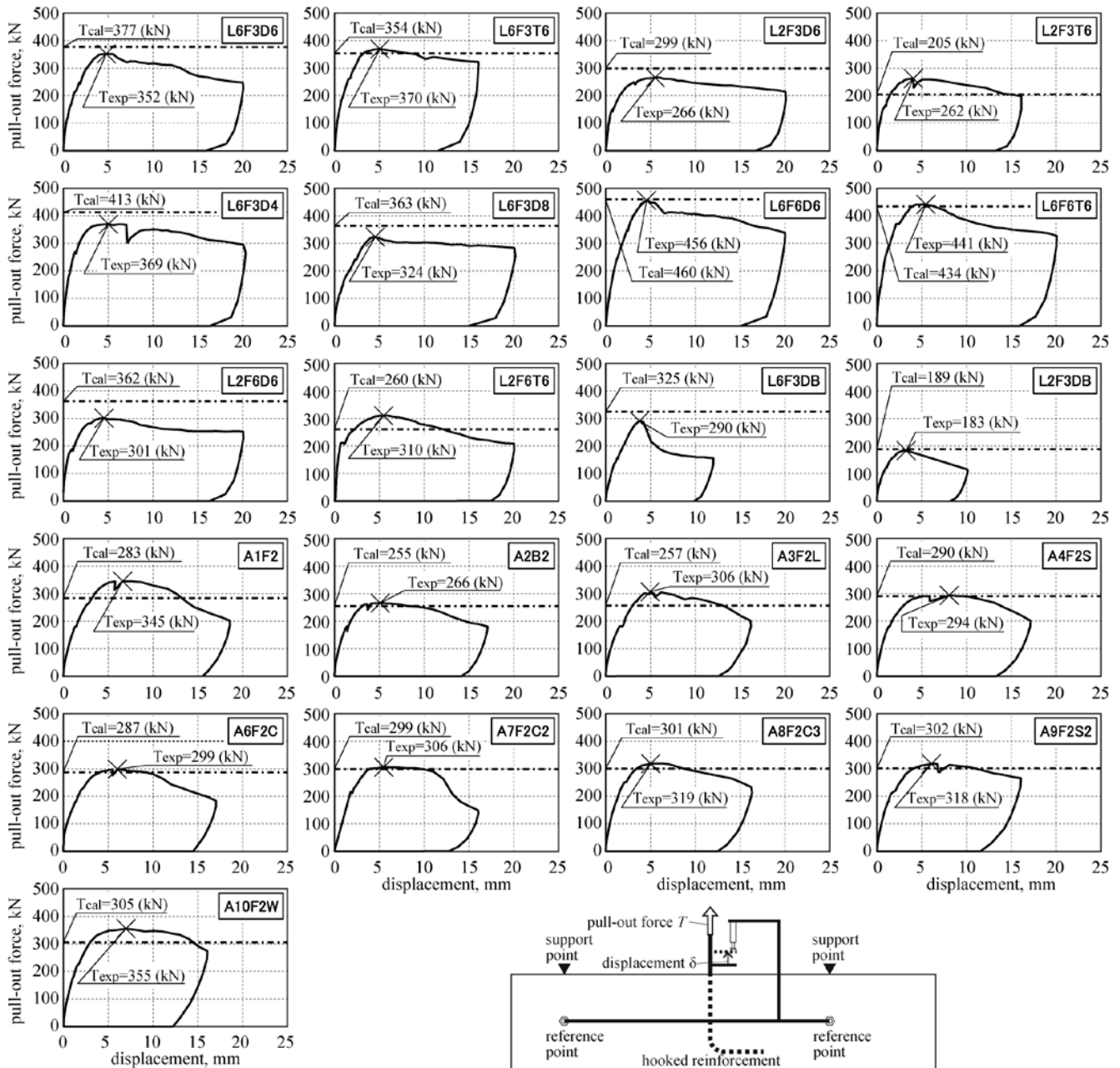


Fig. 7 – Relationship between tensile force and displacement at loading point

3. Modified capacity formula by lever model

The current formula has coefficients of 0.7 and $1/\sin\theta$ to adjust to the experimental data, but there was no mechanical consideration. Moreover it seems that the crack along the tail of hook was not taken into account. Then we used a calculation model for resistance of shear reinforcements as follows.

3.1 Lever model for shear reinforcement resistance

With respect to shear reinforcements, we proposed a model that we called a lever model, shown in Fig. 8 (a). In this model, we assumed crack planes in reference to the crack patterns of the specimens. The surface lines of the crack planes consists of three lines. One is the 45° line in the column area, another is the vertical line along the tail of hook, and the other is the 45° line in the joint area. The line in the column area begins from the point that the central axis of tensile reinforcements of the beam crosses the axis of the tail of hook of the first-layer reinforcement. The line in the joint area begins from the point at a distance of $3d_b$ from the beginning point of the column-side line.

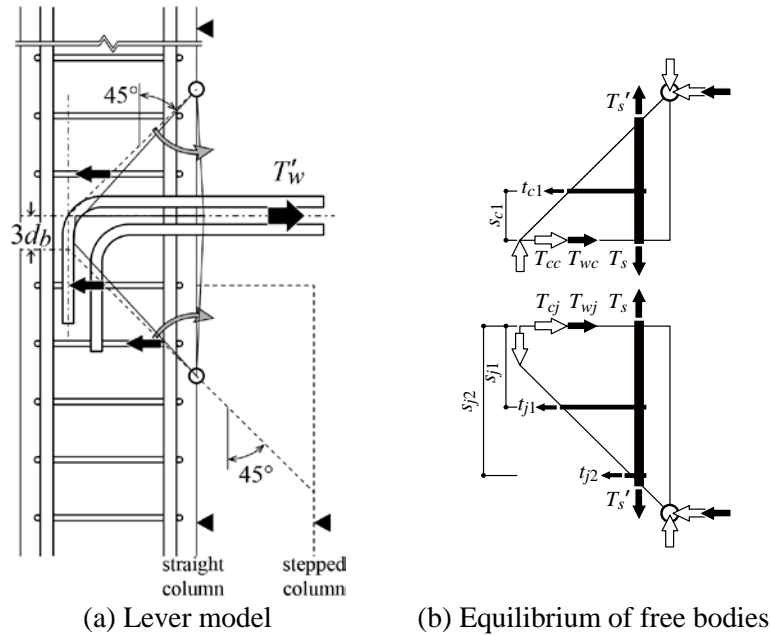


Fig. 8 – Proposed model

The concrete around the main reinforcement bars of the beam was modelled as two blocks of concrete. They can rotate around the point that the assuming crack plane crosses the face of the column on the beam side. The shear reinforcement resistance was calculated from the balance of moment of each free body in the column and joint area with respect to the tensile force of the main and shear reinforcements through the free bodies as shown in Fig. 8 (b). In the calculation, the tension at the both ends of the main reinforcements in the free bodies were assumed to be equal ($T_s = T_s'$ in Fig. 8 (b)), and bars outside the assuming crack lines were not included.

$$T'_w = T_{wc} + T_{wj} \quad (2)$$

$$T_{wc} = \frac{\sum (L_{dh1} - s_{ci}) t_{ci}}{L_{dh1}} \quad (3)$$

$$T_{wj} = \frac{\sum (3d_b + L_{dh1} - s_{ji}) t_{ji}}{3d_b + L_{dh1}} \quad (4)$$



where

T_{wc} = shear reinforcement resistance on the column side

T_{wj} = shear reinforcement resistance on the joint side

L_{dh1} = development length to the center of the tail of hook of the first-layer bars of the beam

s_{ci} = distance from central axis of beam bars to the i -th hoop in the column

s_{ji} = distance from central axis of beam bars to the i -th lateral bar in the joint

t_{ci} = yield tensile strength of the i -th hoop in the column

t_{ji} = yield tensile strength of the i -th lateral bar in the joint

Fig. 9 shows the ratio of the value, T_w of Eq. (2) to the value, T_w/k_w of Eq. (1). The ratio is correspond to k_w . In the previously-modified formula, k_w is 0.7 in the case of 90° bend to the joint. According to the graph, the values of k_w of the normal specimens were approximately 0.5-0.8, while that of all the stepped-column specimens were about 0.5.

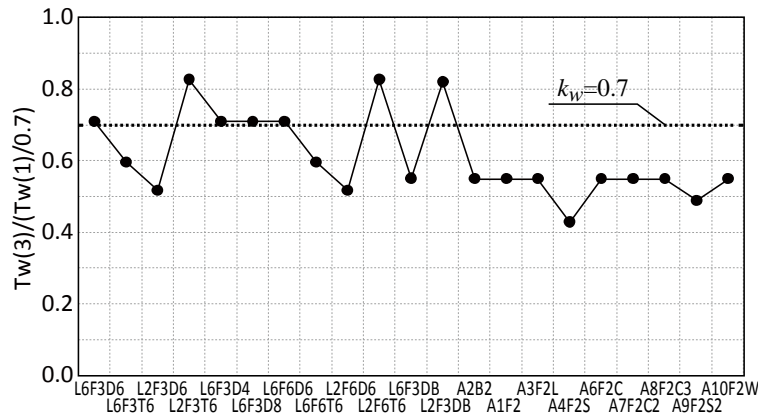


Fig. 9 – Ratio of the value, T_w of Eq. (2) to the value, T_w/k_w of Eq. (1)

The number of shear reinforcement bars across the assuming crack planes would differ depending on whether the beginning point of an assuming crack line on the column side is on the central axis of main beam bars or the first-layer axis of main beam bars. By the method of Eq. (1), the difference of the values of shear reinforcement resistances between those cases can be very large. However, by the method of Eq. (2), shear reinforcements distant from the main reinforcement of the beam make less influence on the calculation, so that the difference between those cases is little. Therefore, we have chosen the latter method, namely, the beginning point of an assuming crack line is on the first-layer axis of main beam bars. The assumption that the beginning point is shifted $3d_b$ to the joint side is supposedly based on the experimental fact that there were cracks along the tail of hook and 45-degree cracks were observed a little offset to the joint side. However, it is necessary to consider the effect of the length of the tail of hook and the diameter of bar.

3.2 Comparison to the previously-modified formula

When the concrete resistance calculated with the previously-modified formula are added to the shear reinforcement resistance calculated with the proposed formula given in Eq. (2) in the previous section, you can get a tensile force of main beam bars as follows.

$$T_{cal1} = T'_w + T_c \quad (5)$$

Fig. 10 shows the calculated value of Eq. (1) and Eq. (5). The left bar of each specimen is the value of Eq. (1) and the right is Eq. (5). Table 4 shows the means and standard deviations (SD) of the ratio of the calculated value to the experimental value. As to the straight-column specimens, Eq. (1) gave the mean, 1.02, and SD, 0.12, while Eq. (5) gave the mean, 0.99, and SD, 0.10. As to the stepped-column specimens, the mean of Eq. (1) was 0.92, while that of Eq. (5) was 0.84, which was smaller. SD of each specimen group was 0.07 and 0.06, respectively, the variability was small. This may be because the effect of the difference of the parameters was small as the experimental values are similar.

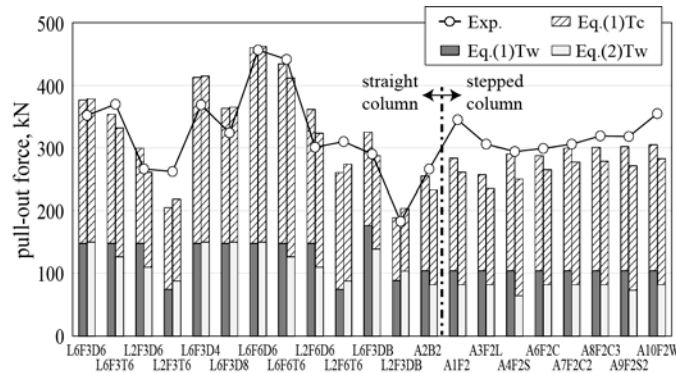


Fig. 10 – Calculated value of Eq. (1) and Eq. (5)

Table 4– Means and standard deviations of the ratio of calculated values to experimental values

		Straight column	Stepped column	Total
Eq. (1)	Mean, μ	1.02	0.92	0.98
	SD, σ	0.12	0.07	0.11
Eq. (5)	Mean, μ	0.99	0.84	0.93
	SD, σ	0.10	0.06	0.12

Compared the specimen A2B2, A3F2L, A10F2W with each other of photographs in Fig. 6, the stepped-column specimens had cracks on the column side with shallower angle, and the damage of the joint were less than the straight-column specimens. Due to the larger deformation of the column than that of the joint, the number of hoops bearing the tensile force of beam bars increased to the outside of the area for the lever model. As a result, it seemed that shear reinforcement resistance was larger than the calculation.

4. Conclusion

In order to reevaluate the current formula and the previously-modified formula with the experimental data from 2011 to 2013, we proposed a new mechanical model for shear reinforcement resistance of tensile force of main beam bars. Then we obtained the following conclusions:

- 1) As to the current formula for the raking-out failure, the mean and standard deviation of the calculated value of the straight-column specimens were 1.02 and 0.12, respectively. It shows that the current formula has high accuracy and precision. In addition, as to the previously-modified formula in the earlier paper, even with the modification, the mean of the ratio of calculated value to the experimental value was 0.92, which was a little underestimated.
- 2) We tried to replace the effective factor of shear reinforcement resistance, k_w , in the current and previously-modified formula. Assuming two centers of rotation at the crossing point of the plane of the column surface and the crack planes in both the column and the joint, the values which correspond to k_w were about 0.5 for the stepped-column specimens, and were approximately 0.5-0.8 for the straight-column specimens. This fact roughly corresponds to the effective factor of shear reinforcement resistance, $k_w = 0.7$, in the current and



previously-modified formula. In addition, because the contribution of the shear reinforcements far from the main bars of the beam is less than the nearer shear reinforcements in the proposed formula, it will make little difference in calculation values of the formula if the number of shear reinforcements across the assuming crack plane differs due to a subtle difference of condition of the arrangement around the crack plane, which is one of the faults of the current and previously-modified formula.

- 3) It is shown that the maximum strength of raking-out failure mode was able to be estimated by Eq. (5) as well as the previously-modified formula. However, as to the stepped-column specimens, the formula underestimated the value by 16 percent on average. It is conceivable that the shear reinforcement resistance can be higher than the calculated value as a consequence of the contribution of shear reinforcements out of the region considered in the lever model.

Acknowledgements

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