



AN EXTERNAL BRACING SYSTEM FOR THE RETROFIT OF R.C. FRAME BUILDINGS: MODAL PROPERTIES AND SEISMIC RESPONSE

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Abstract

The paper deals with the seismic protection of existing buildings using external viscous damper systems to increase energy dissipation capacity. Usually the addition of dissipative diagonals in existing frames has some drawbacks as increment of internal actions in the columns, need of intervention at foundation level, feasibility limits and indirect costs related to the interruption of the building utilization. These problems can be efficiently avoided by placing the dissipative bracings and the relevant foundations outside the existing building. Dampers and bracings can be arranged in very different configurations and the possible solutions can be grouped into different categories, depending on the specific kinematic behavior, but all permitting the control of both the total amount of the dissipated energy and the frame deformation at the various storeys. In this work, the formulation of the problem involving the coupling of the existing frame with an external damping system is presented in general terms and is employed to investigate both the influence of the external bracing properties on the overall dynamic properties of the coupled system (such as the mode displacement profile, the relevant internal action distribution and the modal damping ratios) and the global effect of the retrofitting on the seismic response. Presented results concern the so called "dissipative tower", a recent solution which exploits the rocking motion of a stiff steel truss hinged at the foundation level for the dampers activation. The influence of the external dissipative bracings on the most important modal properties of the system are shown and it is observed that the bracing system notably influences the stiffness and damping properties while it modifies only marginally the mass properties of the existing frame. Finally the global effect on the seismic response, in terms of both displacements and base shear, is presented by solving the dynamic problem with the modal decomposition method by also investigating the contribution of the higher modes on the dynamic response.

Keywords: passive seismic protection; linear viscous dampers; external seismic retrofit; Dissipative Towers.

1 Introduction

Passive control systems have proven to be very efficient solutions for new constructions and for seismic retrofitting of existing structures. Traditionally, viscous dampers are installed within a building frame in either diagonal or chevron brace configurations connecting adjacent storeys and there are many studies concerning both the dynamic properties of the damped system and the methods for the design [1-4]. However, this type of damping system may present some disadvantages, particularly when employed for retrofitting existing buildings. Usually, the addition of dissipative diagonal in existing frame provides an increment of tension/compression internal actions in the columns and this may lead to premature local failures [5]. Furthermore, there may be some feasibility limits on the strengthening of the existing foundations at the base of the bracing. Also the indirect costs related to the interruption of the building utilization during execution of the retrofit can be very demanding, in particular for special buildings, such as hospitals or schools.

These problems can be efficiently avoided by placing the dissipative bracings and the relevant foundations outside the building. External bracings could also host elevators or emergency stairs, thereby providing accessory benefits; moreover, the arrangement of the dampers in a new structure greatly simplifies the inspection process, maintenance and replacement. The external system is easily removable, and permits to restore the building to its original state [6].

External dampers and bracing components can be arranged in very different configurations and the possible solutions are characterized by substantially different kinematic behaviors. A first solution can be obtained by placing the dampers at the storey levels, between the frame and an external stiff structure [7]. The links are activated by the floor absolute displacements. A similar configuration can be obtained by placing the dampers between adjacent buildings. The solution is efficient if the two buildings have strongly different dynamic properties [8-10]. Another solution can be obtained by coupling the frame with a shear deformable bracing structure. The new and existing structures are connected at the storey levels and the dissipative devices are activated by the relative displacements between adjacent floors, as in the traditional bracings placed within the existing structure [11]. Recently, some applications have been developed by exploiting the rocking motion of a stiff brace hinged at the foundation level [12, 13]. In this configuration (Fig. 1), known as "dissipative tower" [14], the dampers are activated by the base rotation of the tower. All these configurations permit the control of both the total amount of the dissipated energy and the frame deformation at storeys.

This work presents a general formulation for the analysis of the problem involving the coupling of an existing frame with an external damping system. This formulation is then employed to investigate some issues concerning the influence of the bracing properties on the dynamic response of the coupled system. Reported results concern a r.c. frame with limited ductility retrofitted by an external bracing arranged as in Fig. 1 (dissipative tower).

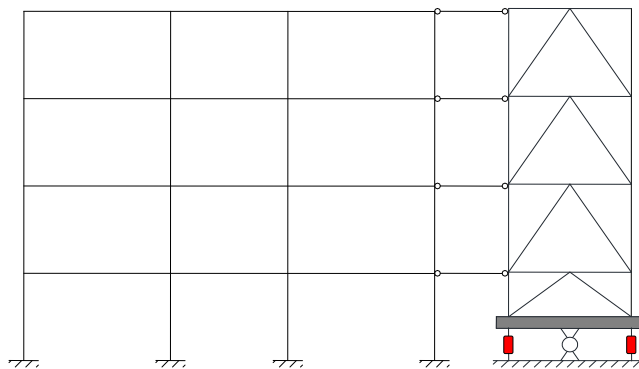


Fig. 1 - Dissipative Tower system



2 Problem Formulation

In the first part of this section, the equation of motion and the state variables of the considered problem are presented by assuming that both the building and the external damping system exhibit a linear elastic response. The two following parts describe the modal properties and the seismic response of the coupled system.

2.1 Equation of motion

The equations of motion for the system can be expressed as follows:

$$\mathbf{M}\ddot{\mathbf{u}}(t) + \mathbf{C}\dot{\mathbf{u}}(t) + \mathbf{K}\mathbf{u}(t) = \mathbf{M}\mathbf{p}a_g(t) \quad (1)$$

where $\mathbf{u}(t) \in R^l$, is the vector of nodal displacements, dot denotes time derivative; $\mathbf{p} \in R^l$ is the load distribution vector, l denotes the total number of degrees-of-freedom, and $a_g(t)$ is the external scalar loading function describing the seismic base acceleration. The time invariant matrices \mathbf{M} , \mathbf{K} , \mathbf{C} describe the mass, stiffness and damping operators $R^l \rightarrow R^l$; they result from the sum of the contribution due to the existing frame and the one coming from the external dissipative bracing system. Generally, the bracing system notably influences the stiffness and damping operators while only marginally contributes to the mass operators. The displacement vector $\mathbf{u}(t)$ collects both the displacements required for the description of the frame response and the displacements involved in the bracing deformations.

In order to study the dynamic response of the system it is useful to separate the displacements associated with the masses, and thus involving inertial forces, from those of the internal degrees of freedom, related to stiffness and damping forces only. Accordingly, the total displacement vector $\mathbf{u}(t)$ can be split into the active components collected in the vector $\mathbf{x}(t) \in R^m$ and the other components $\mathbf{y}(t) \in R^n$ ($l = m + n$). The matrices describing the linear operators and the distribution vector can be consequently partitioned as follows

$$\begin{bmatrix} \mathbf{M}_{xx} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{x}} \\ \ddot{\mathbf{y}} \end{bmatrix} + \begin{bmatrix} \mathbf{C}_{xx} & \mathbf{C}_{xy} \\ \mathbf{C}_{yx} & \mathbf{C}_{yy} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{x}} \\ \dot{\mathbf{y}} \end{bmatrix} + \begin{bmatrix} \mathbf{K}_{xx} & \mathbf{K}_{xy} \\ \mathbf{K}_{yx} & \mathbf{K}_{yy} \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \mathbf{y} \end{bmatrix} = \begin{bmatrix} \mathbf{M}_{xx} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{p}_x \\ 0 \end{bmatrix} a_g \quad (2)$$

As usual, only the masses related to the horizontal floor displacements are considered in order to reduce the dimension of the dynamic problem and to simplify the interpretation of the results.

The distribution of the damping in the structure and, in particular, the location of the concentrated dampers of the external bracings, leads to a non-classically damped system and it is convenient to formulate the problem by introducing the vector $\mathbf{v}(t) = \dot{\mathbf{x}}(t)$ and the state vector $\mathbf{z}(t)$ collecting the displacements and the velocities of the active displacements and the displacements of the internal nodes

$$\mathbf{z}(t) = \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{v}(t) \\ \mathbf{y}(t) \end{bmatrix} \quad (3)$$

Eqn. (1) can be reduced to a first-order state space form:

$$\dot{\mathbf{z}}(t) = \mathbf{A}\mathbf{z}(t) + \tilde{\mathbf{p}}a_g(t) \quad (4)$$

where



$$\mathbf{A} = \begin{vmatrix} \mathbf{0} & \mathbf{I} & \mathbf{0} \\ -\mathbf{M}_{xx}^{-1}(\mathbf{K}_{xx} - \mathbf{C}_{xy}\mathbf{C}_{yy}^{-1}\mathbf{K}_{yx}) & -\mathbf{M}_{xx}^{-1}(\mathbf{C}_{xx} - \mathbf{C}_{xy}\mathbf{C}_{yy}^{-1}\mathbf{C}_{yx}) & -\mathbf{M}_{xx}^{-1}(\mathbf{K}_{xy} - \mathbf{C}_{xy}\mathbf{C}_{yy}^{-1}\mathbf{K}_{yy}) \\ -\mathbf{C}_{yy}^{-1}\mathbf{K}_{yx} & -\mathbf{C}_{yy}^{-1}\mathbf{C}_{yx} & -\mathbf{C}_{yy}^{-1}\mathbf{K}_{yy} \end{vmatrix} \quad (5)$$

Vector $\tilde{\mathbf{p}}$ is defined as:

$$\tilde{\mathbf{p}} = \begin{vmatrix} 0 \\ \mathbf{M}_{xx}^{-1}\mathbf{p} \\ 0 \end{vmatrix} \quad (6)$$

2.2 Free vibrations and modal properties

The free vibration problem can be solved by assuming a solution of the form $\mathbf{z}(t) = \boldsymbol{\varphi}e^{\lambda t}$, where $\lambda, \boldsymbol{\varphi}$ are a eigenvalue-eigenvector pair of \mathbf{A} , such that:

$$\mathbf{A}\boldsymbol{\varphi} = \lambda\boldsymbol{\varphi} \quad (7)$$

Complex eigenvalue has the following form:

$$\lambda_i = -\xi_i\omega_{0i} + i\omega_{0i}\sqrt{1 - \xi_i^2} \quad (8)$$

and contains information regarding both the damping ratio ξ_i and the corresponding undamped circular frequency ω_{0i} of the i -th mode. These information can be extrapolated as follows:

$$\begin{aligned} \omega_{0i} &= |\lambda_i| \\ \xi_i &= -\text{Re}(\lambda_i) / |\lambda_i| \end{aligned} \quad (9)$$

Known the modal properties, the problem solution can be obtained as a linear combination of the single mode contributions. Let $\boldsymbol{\Lambda}$ be the diagonal matrix containing the complex eigenvalues and $\boldsymbol{\Phi} = [\boldsymbol{\varphi}_1, \boldsymbol{\varphi}_2, \dots, \boldsymbol{\varphi}_{2m+n}]$ the complex eigenmatrix containing the eigenvectors, such that the orthogonality property $\boldsymbol{\Lambda} = \boldsymbol{\Phi}^{-1}\mathbf{A}\boldsymbol{\Phi}$ holds.

2.3 Seismic response via modal decomposition method

The motion can be obtained as a linear combination of modes:

$$\mathbf{z}(t) = \boldsymbol{\Phi}\mathbf{q}(t) \quad (10)$$

where $\mathbf{q}(t)$ is a vector collecting the modal coordinates. The orthogonality property leads to the diagonal problem

$$\dot{\mathbf{q}}(t) = \boldsymbol{\Lambda}\mathbf{q}(t) + \boldsymbol{\gamma}a_g(t) \quad (11)$$

where $\gamma_i = [\boldsymbol{\Phi}^{-1}\tilde{\mathbf{p}}]$ is the i -th (complex-valued) modal participation factor.

Introducing the normalized complex modal response vector $\mathbf{s}(t)$ such that: $q_i(t) = \Gamma_i s_i(t)$, the problem can be posed in the normalized form

$$\dot{s}(t) = \Lambda s(t) + \mathbf{I} a_g(t) \tag{12}$$

Assuming that the system is initially at rest, the solution can be obtained by the Duhamel integral

$$s(t) = \int_0^t \mathbf{h}(t - \tau) a_g(\tau) d\tau \tag{13}$$

where the components $h_i(t) = e^{\lambda_i t}$ are the solutions related to an impulsive unitary input.

3 Case study

3.1 Case study description

The application of the proposed approach is illustrated by considering a r.c. frame structure typical of many buildings designed during the 80s in Italy without any particular seismic detailing. Along the longitudinal x direction, the structure consists of two external frames and a central one with 6 or 7 spans (Fig. 2). The building has 5 storeys plus the roof.

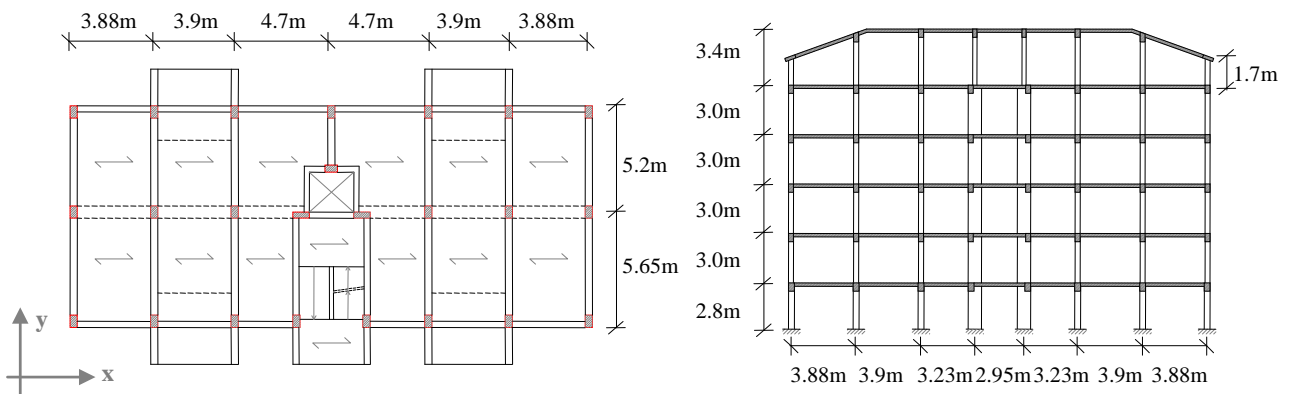


Fig. 2 - Planar view and longitudinal section of the bare building

The presented results concern the r.c. frame coupled with two dissipative towers, as shown in Fig. 3, hinged at the foundation level and equipped with linear viscous dampers located at the base (*retrofit configuration*), whose performances are compared with the ones of the bare existing frame (*frame configuration*). First, the previously described formulation is employed to investigate the influence of the external bracings properties on the overall dynamic properties of the coupled system, such as the mode displacement profile, the relevant internal action distribution and the modal damping ratios. Successively, the global effect of the retrofit on the seismic response is evaluated by solving the seismic problem with the modal decomposition method.

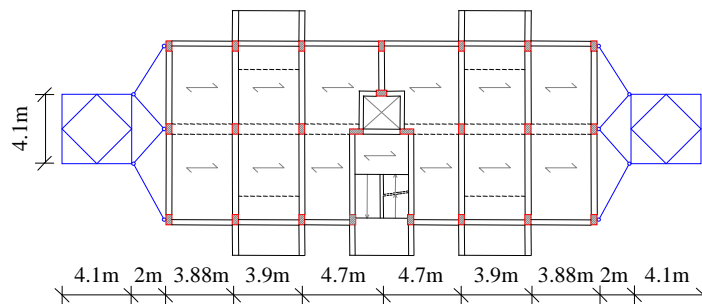


Fig. 3 - Planar view of the *Retrofit* case



The dynamic system is described by considering only the motion along the longitudinal x direction and the masses have been concentrated at the storey levels, so that the vector of active degrees of freedom \mathbf{x} collects the five storey motions only.

3.2 Modal properties of the undamped system

The bracing systems influence both the stiffness and the damping properties of the coupled system and it is useful to separately analyze these effects. For this purpose, the case of added towers without dampers is considered separately from the case of towers with dampers.

The modal analysis of the bare building (*frame*) shows a value of the first and second mode periods equal to 1.021s and 0.300s; the participant mass ratio, defined as

$$M_i^* = (\mathbf{M}\mathbf{p} \cdot \boldsymbol{\psi}_i)^2 / \mathbf{M}\boldsymbol{\psi}_i \cdot \boldsymbol{\psi}_i \quad (14)$$

where $\boldsymbol{\psi}_i$ are the eigenvectors of the undamped frame, is $M_1^*=79\%$ for the first mode and $M_2^*=12\%$ for the second mode.

Table 1 reports, for the first two modes of the bare building and of the *Retrofit* case, the vibration periods (T_i), the modal displacements normalized to the highest value (x^i), and the corresponding interstorey drifts (δ^i). The modal analysis of the coupled system (frames plus bracings) gives a value of the first and second mode periods equal to $T_1=0.964$ s and $T_2=0.147$ s; the participant mass ratio is 81% for the first mode and 14% for the second.

Fig. 4 a) and b) report and compare the values of the interstorey drifts (δ^i) along the building height corresponding to the first two modes of vibration for the *frame* and for the *retrofit* configuration. It can be observed that the addition of the towers generally yields a reduction of the drift demands and a regularization of their distribution along the building height.

Fig. 5 a) and b) report the distributions of the peak values of the total shear forces of the *frame* and of the *retrofit* configuration for the first two modes, whereas Fig. 6 a) and b) show, for the *retrofit* configuration, the relative contribution to the total shear forces by the building and by the tower. It is noted that the peak values of the shear in the frame and in the towers occur at different times and that they exhibit different signs at some levels. For the considered case, the first mode shear forces acting on the building are higher than the total shear force at the first level.

Table 1 – Modal analysis results of the analyzed configurations

Floor	<i>frame</i> configuration					<i>retrofit</i> configuration				
	Mass	$T_1 = 1.021$ s		$T_2 = 0.300$ s		Mass	$T_1 = 0.964$ s		$T_2 = 0.147$ s	
	[kNs ² /m]	x^i	δ^i	x^i	δ^i	[kNs ² /m]	x^i	δ^i	x^i	δ^i
5	523.93	1.000	0.149	-0.719	-0.899	533.41	1.000	0.196	-0.693	-0.842
4	325.81	0.851	0.222	0.180	-0.760	339.15	0.804	0.210	0.150	-0.686
3	325.81	0.630	0.267	0.940	-0.060	339.15	0.594	0.215	0.836	-0.164
2	325.81	0.363	0.251	1.000	0.586	339.15	0.379	0.216	1.000	0.430
1	330.26	0.112	0.112	0.414	0.414	343.42	0.162	0.162	0.570	0.570

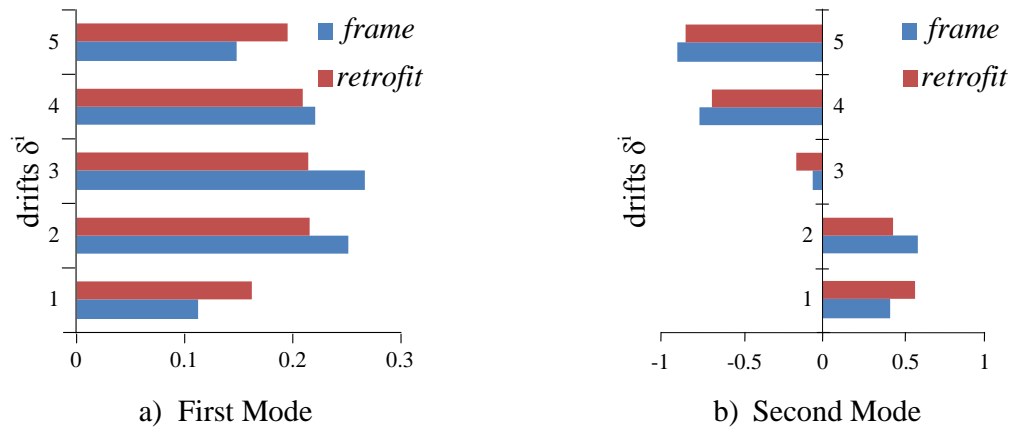


Fig. 4 - Interstorey drifts along the building height for mode 1 a) and mode 2 b)

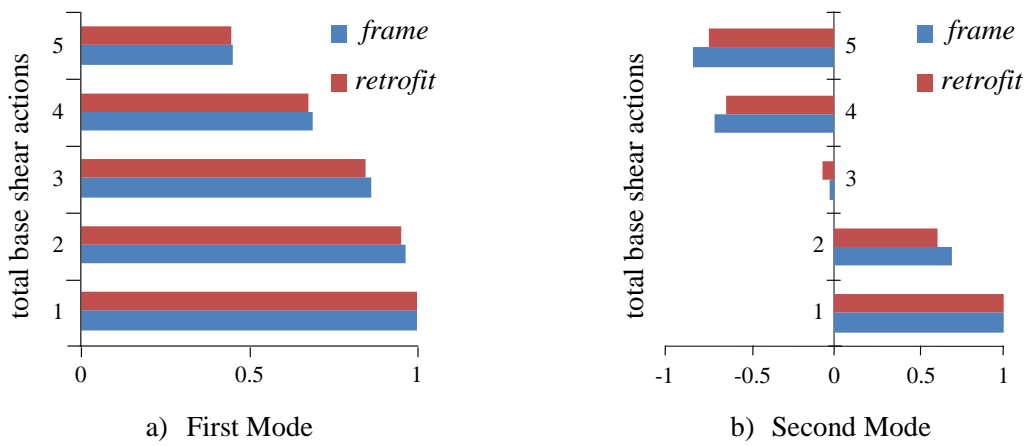


Fig. 5 - Total shear force comparison for mode 1 a) and mode 2 b)

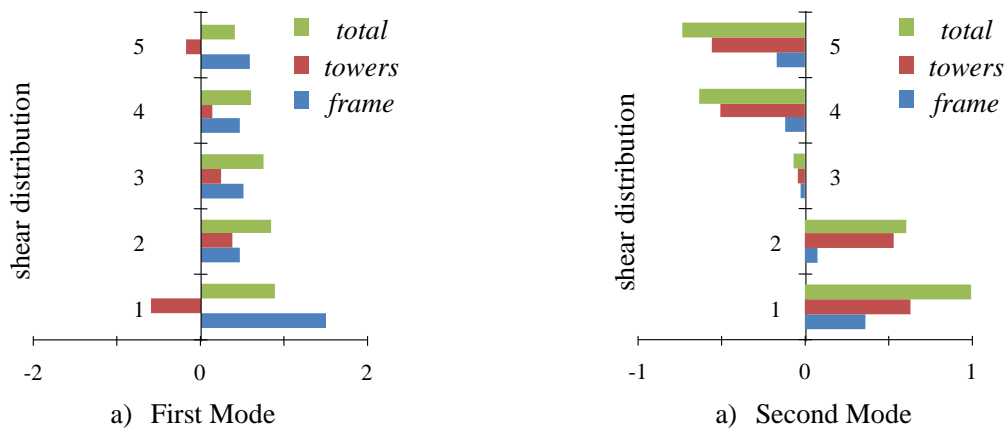


Fig. 6 - Shear force distribution in the retrofit configuration for mode 1 a) and mode 2 b)



3.3 Modal properties of the damped system

It is assumed that the damping is the sum $\mathbf{C} = \mathbf{C}_R + \mathbf{C}_D$, the former contribution is described by a Rayleigh damping matrix $\mathbf{C}_R = \alpha\mathbf{M} + \beta\mathbf{K}$, providing an inherent damping $\zeta=0.05$ at the first two vibration modes, whereas the latter contribution is due to the dampers and it is directly related to their displacements.

A reference value of the damper dimensions is obtained by fixing a target total amount of the effective damping ratio β_{eff} as 0.25 (0.05 due to the building and 0.20 due to the dampers), and by using the expression reported in ASCE/SEI 41-13 (2013)

$$\beta_{eff} = \beta + \frac{\sum W_j}{4\pi W_k} \quad (15)$$

where β is the damping in the structural frame (0.05); W_j the work done by j -th device in one complete vibration cycle and W_k is the maximum strain energy in the frame.

The retrofit configuration consists of two dampers placed as in Fig. 1, whose viscous constant values are designed by employing eqn. (15) assuming that the system deforms according to its first undamped vibration mode. The total amount of added damping constant is $C_0=135020$ kNs/m, which is the sum of the viscous damping constant of the devices.

Different damping levels can be analyzed by introducing a damper multiplier c for the total added damping C_0 and by evaluating the variation with c of the modal properties.

Fig. 7 a) shows the variation with c of the vibration periods of the first two modes. For $c=0$ the first two natural periods (T_{iu} dashed lines) are, respectively, 0.964 s for the first and 0.147 s the second. The vibration periods of the coupled system decreases for increasing damping; for $c=3.0$ the first period becomes 0.595 s and the second 0.141 s. Thus, the amount of damping introduced influences significantly only the first vibration period of the system. Fig. 7 b) shows the variation, with the total added damping c , of the damping ratio of the first and second vibration modes, ζ_1 and ζ_2 ; it also reports the variation with c of the estimate of the damping ratios β_{eff} obtained by employing the approximate formula of Eqn. (15).

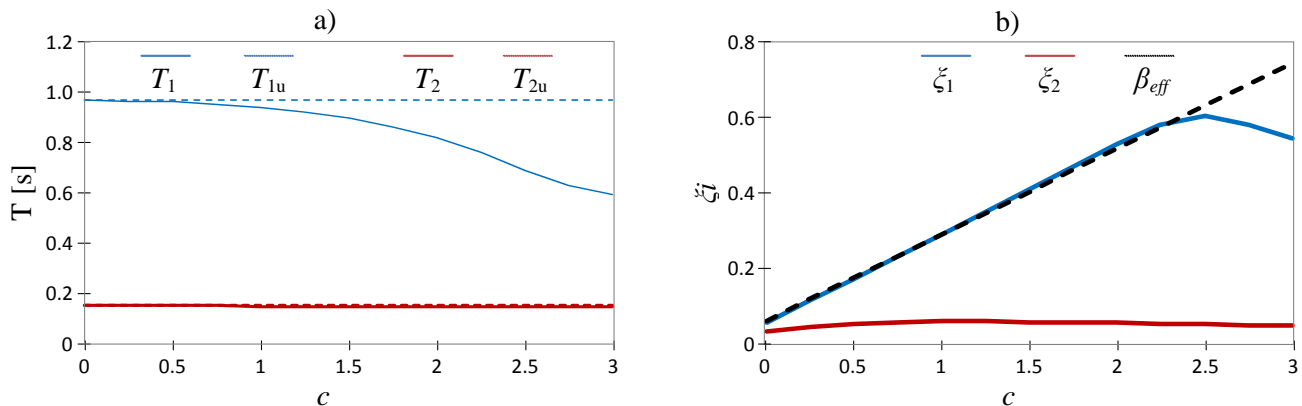


Fig. 7 - First two period trends (a) and first three modes damping trends (b) for increasing damping levels

The trend of first modal damping ratio ζ_1 is approximately in line with the design value β_{eff} up to the value 0.50 ($c=2.25$); for larger value the period continues to decrease and the amount of the effective damping starts to decrease. As already discussed for the periods, the influence of the damper dimensions on the second mode is small and the amount of damping introduced varies in the range 0.050-0.067 with a maximum 0.074 when $c=1.25$.

To quantify the extent of non-classical damping in the retrofitted system, the coupling index ρ is evaluated by following the approach of Claret e Venancio-Filho (1991). This index is expressed as:

$$\rho = \max \left| \frac{\Xi_{ij}^2}{\Xi_{ii}\Xi_{jj}} \right| (i, j=1,2,\dots,m) i \neq j \quad (16)$$

where $\Xi_{ij} = \mathbf{C}_{xx} \boldsymbol{\psi}_i \cdot \boldsymbol{\psi}_j$ is the modal damping matrix component, m and $\boldsymbol{\psi}_i$ are the degrees of freedom and the eigenvector of the undamped system, respectively. The index assumes the value 0 for classical damped systems and it spans the range [0,1] for non-classical damped system. Fig. 8 shows the values of ρ obtained for the damping target value $c=1$. The maximum value of ρ is equal to 0.35 and corresponds to the coupling among the first and the second mode. This results implies that the external retrofitting configuration investigated is notably non-classical damped due to the fact that all the dampers are concentrated at the towers base.

5	0.01	0	0	0	1
4	0.02	0.01	0	1	0
3	0.14	0.08	1	0	0
2	0.35	1	0.08	0.01	0
1	1	0.35	0.14	0.02	0.01
	1	2	3	4	5

Fig. 8 - Coupling index for the first five modes of the system

3.4 Seismic response

This section reports some results concerning the response of the system, before and after the retrofit, to a seismic input. The seismic action has been determined by assuming the building located in Camerino (MC, Italy), with soil category C and topographical one T1, according to Italian code [15].

The results reported concern the time-history of the displacement of the fifth floor and of the base shear, for both the *frame* and the *retrofit* configuration. The seismic input is an artificial earthquake generated in accordance with Italian Standards (total duration 25 s and stationary part duration at least equal to 10 s) for a reference period V_R of 50 years.

Fig. 9 shows the time-history of the 5th floor displacement before and after the retrofit for the case corresponding to the total added damping multiplier $c = 1$. The maximum value of the measured displacement is 0.108 m in the *frame*, while in the *retrofit* configuration it becomes 0.056 m. The relative reduction of the maximum displacement with respect to the bare frame after the addition of the tower is nearly 50%.

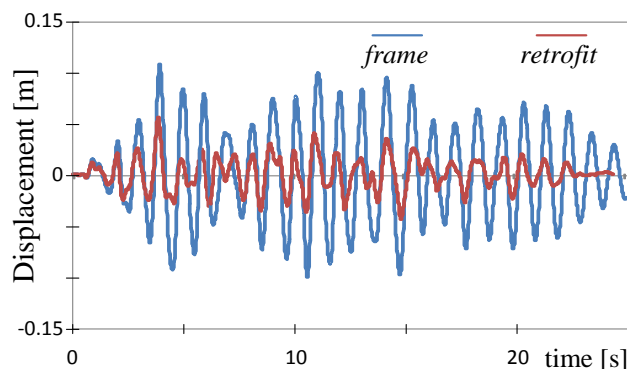


Fig. 9 - Time history of the displacement of the 5th floor before and after the retrofit.



Having employed the complex mode superposition approach to solve the seismic problem, the contribution of the higher modes to the response can be estimated by comparing the full response accounting for all the modes (continuous line) to the response obtained by considering the contribution of the first mode only (dashed line). Fig. 10 and Fig. 11 report this comparison for, respectively, the *frame* and the *retrofit* configuration. In both the cases, the first mode contribution nearly controls the total response, while the effects of higher order modes are negligible. Fig. 12 reports the time history of the base shear of the bare building (*frame*) and the upgraded system (*retrofit* configuration). In the case of the *retrofit* configuration, the base shear reported in Fig. 12 and Fig. 14, is the sum of the base shear of the building and of the towers. As already discussed for displacements, also the total amount of shear decreases after the retrofit from the initial value of 5269 kN (*frame*), to the value of 3864 kN (*retrofit* configuration), with a relative reduction of 27%.

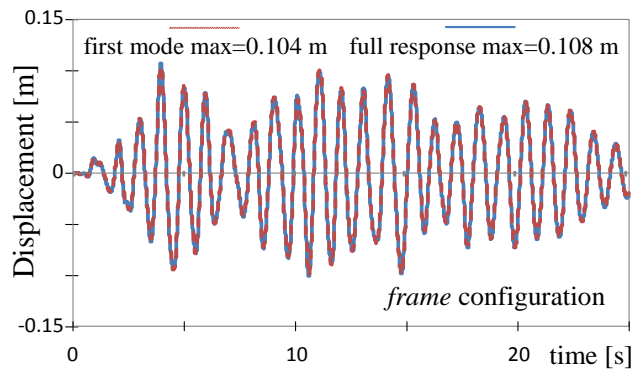


Fig. 10 - Contribution of the first mode to the time history of the displacement - 5th floor *frame* configuration

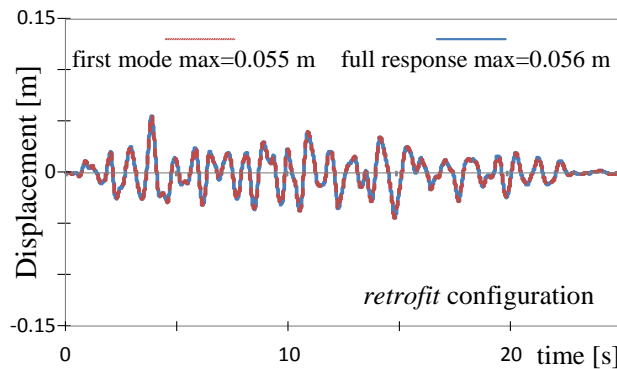


Fig. 11 - Contribution of the first mode to the time history of the displacement - 5th floor *retrofit* frame

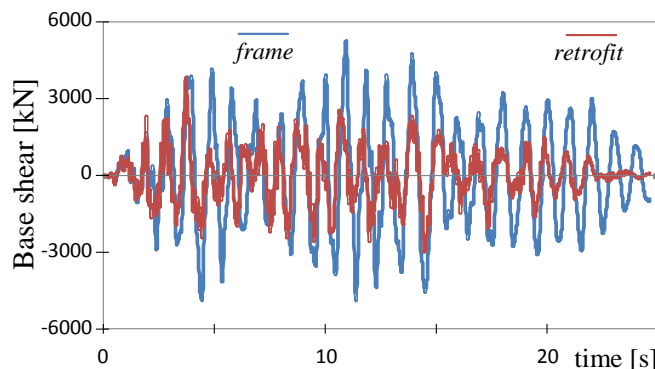


Fig. 12 - Base shear comparison before and after the retrofit

Fig. 13 and Fig. 14 report the time histories of the contribution of the first mode to the total base shear response. Differently from the case of the displacements, the contribution of higher order modes is of great importance since the values of the total base shear response are higher than the corresponding values obtained by considering the first mode only. This observation, together with the fact that the increase of damping ratio after the retrofit is lower for higher modes than for the first mode, explains why the reduction of the displacement demand is higher than that of the base shear.

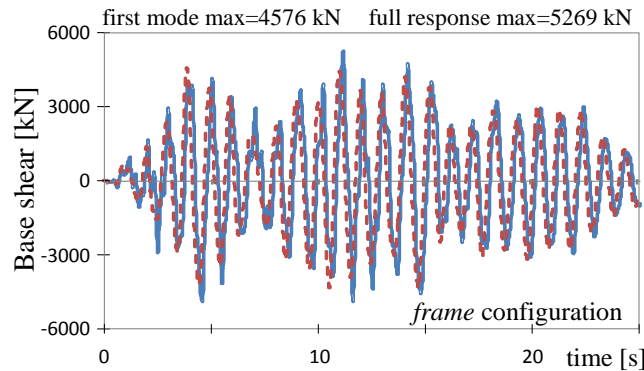


Fig. 13 - First mode contribution on the total response – base shear *frame configuration*

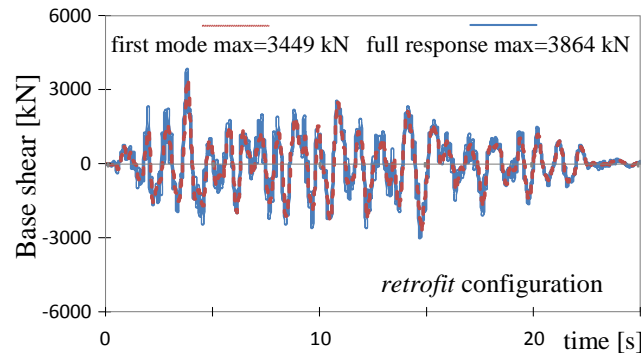


Fig. 14 - First mode contribution on the total response - base shear *retrofit configuration*

4 Conclusions and further studies

In this paper, three alternative categories of external retrofitting systems equipped with viscous dampers are presented, each one characterized by a different kinematic behaviour.

A problem formulation concerning the dynamic behaviour of the coupled system formed by an existing frame and an external damping system is presented in general terms. The results reported concern the so called “dissipative tower” configuration, an innovative solution which exploits the rocking motion of a stiff steel truss hinged at the foundation level for the dampers activation.

From the analysis of the case study results it can be observed that the addition of the towers yields a regularization of the drift demand along the building height. After the retrofit, the shear force distribution in the existing frame changes significantly and the system becomes non-classical damped due to the fact that all the dampers are concentrated at the bases of the towers. Moreover, the obtained results show that the contribution of higher order modes is of great importance for the internal actions, while it is negligible for the displacements control.



A deeper and wider investigation is still necessary for a full comprehension of the problem, and it should address the evaluation of the changes of the response parameters that are significant for the performance of structural and non-structural elements such as the absolute accelerations, interstorey drifts, and shear forces. Moreover, it should be oriented to provide information of design interest and useful for the identification of the optimal stiffness range of the dissipative towers.

5 References

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