



## Performance-based design of soil-foundation-bridge pier systems by use of sampling techniques

A. Karatzetzou<sup>(1)</sup>, D. Pitilakis<sup>(2)</sup>

<sup>(1)</sup> Civil Engineer, Ph.D., Aristotle University of Thessaloniki, Greece, [akaratz@civil.auth.gr](mailto:akaratz@civil.auth.gr)

<sup>(2)</sup> Assistant Professor, Aristotle University of Thessaloniki, Greece, [dpitilakis@civil.auth.gr](mailto:dpitilakis@civil.auth.gr)

### **Abstract**

A soil-foundation-structure system (SFSS) has in most cases different response from a fixed-base structure subjected to earthquake ground motion. Kinematic and inertial soil-foundation-structure interaction may substantially influence structural performance. There are many parameters in the SFSS that influence the final response, suggesting inherent uncertainty in the solution. The computational effort for performing time history analyses, even for a linear elastic coupled SFSS is important. In order to reduce computational cost without reducing the accuracy of the results, we propose the use of Latin Hypercube Sampling (LHS) technique. Sampling techniques are rarely employed in soil-foundation-structure interaction analyses. However, these methodologies are very helpful, because the analyses determined by sampling can be performed by commercial codes, developed for deterministic analyses without any modifications. The difference is that the number of analyses determined by the sampling size is significantly reduced compared to the one for all combinations of the input parameters.

After determining the important sampling, we evaluate the seismic demand of selected soil-foundation-bridge pier systems using a finite element numerical software. Performance of the system is then readily evaluated by the capacity spectrum method. As stems from the results, LHS reduces computational effort 60%, whereas structural response components (translation, rocking) show remarkable trends for the different systems.

*Keywords: seismic demand; soil-foundation-structure interaction; performance-based design*



## 1. Introduction

The rapid progress in earthquake engineering was very helpful for predicting the performance of complex engineering structures under strong earthquake excitations. There are numerous uncertainties, however, in the analysis of an engineering system, which may affect the result of a deterministic approach using a single set of parameters of interest. For earthquake risk evaluation the uncertainties involved are even larger and concern, among others, the uncertainties in the input motion [1-4] and in the soil properties [5-9].

The goal of the present study is not to treat the general problem of the uncertainties in earthquake engineering, but rather to make an evaluation of the use of the LHS method to a coupled soil-foundation-structure interaction system.

In Monte Carlo analyses, a probabilistically based sampling procedure can be used as a first step to develop mapping from analysis inputs to analysis results [10-13]. In the present study, Latin hypercube sampling as proposed by Olsson and Sandberg [14] is used to generate the sample size.

After the sample has been generated and the corresponding model evaluation using the finite element method has been carried out, the first step of the uncertainty and sensitivity analysis has been carried out. The mapping of the analysis input with the analysis results in a sensitivity analysis can be performed with the representation of the results by scatter-plots, exploration of the results using regression analysis and statistical tests. In the present study the representation of the results by scatter-plots is utilized.

## 2. Description of Latin hypercube sampling

Considering that we examine  $k$  variables and each variable takes  $n$  values, Latin hypercube sampling [14-15], selects  $n$  different values from each of  $k$  variables  $X_1, \dots, X_k$  in the following manner. The range of each variable is divided into  $n$  non-overlapping intervals on the basis of equal probability. One value from each interval is selected at random with respect to the probability density in the interval. The  $n$  values thus obtained for  $X_1$  are paired in a random manner (equally likely combinations) with the  $n$  values of  $X_2$ . These  $n$  pairs are combined in a random manner with the  $n$  values of  $X_3$  to form  $n$  triplets, and so on, until  $n$   $k$ -tuplets are formed. These  $n$   $k$ -tuplets are the same as the  $n$   $k$ -dimensional input vectors described in the previous paragraph. This is the Latin hypercube sample.

The generation, for example, of a sample size equal to 10 from a random vector  $k$  of three variable  $k = [x, y, z]$  is achieved according to the following procedure: the range of each variable is divided into 10 intervals of equal probability content and one value is sampled at random from each interval. The 10 values obtained for  $x$  are paired at random without replacement with the 10 values obtained with  $y$ . These pairs are combined at random with the randomly selected values of  $z$  and the process continuous until a set of 10 3-tuples is formed.

In the present study for a typical soil-foundation-structure interaction system we will examine the effectiveness of the LHS sampling in reducing the computational effort.

## 3. Configuration of the model

A single-degree-of-freedom (SDOF) structure is used for simplicity, the degree of freedom being the horizontal displacement of the structural mass  $m_s$ . The SDOF structure is characterized by its stiffness  $k_s$ , its dashpot coefficient  $c_s$  and its height  $h$ . The structure is founded on a rigid massless surface foundation of width equal to  $2B$ .

Two-dimensional plane strain analyses are performed in time domain with Opensees software [16] to determine the elastic response of the structure. Elastic bedrock is modelled using Lysmer-Kuhlemeyer [1969] dashpots [17] at the base of the soil. The Lysmer-Kuhlemeyer dashpot is defined in Opensees based on the viscous uniaxial material model and the zero-length element formulation at the same node. Concerning the vertical boundaries, we use the tied lateral boundary approach by declaring equal degree of freedom for every pair of nodes which share the same  $y$ -coordinate as described in Zienkiewicz et al., 1988 [18].



The size of the mesh resulted after a comprehensive sensitivity analysis. The geometry of the mesh is based on the concept of resolving the propagation of the shear waves at or below a pre-defined frequency allowing an adequate number of elements to fit within the wavelength of the chosen shear wave. This ensures that the mesh is refined enough to capture propagating waves. For a maximum frequency of 10Hz, we used quadratic elements of 1m×1m that suggest dense discretization. The soil mesh comprises of 10,000 four-node quadrilateral elements. The width of the finite element soil mesh was sufficiently large, to avoid spurious reflections at the boundaries. The structure and the foundation are modelled by elastic beam elements of 1m length. Full connection is assumed between the foundation and the soil nodes.

The soil domain is homogeneous with thickness of  $H=50\text{m}$ . The bedrock has shear wave velocity equal to  $V_s=1500\text{m/s}$  and density equal to  $\rho=2400\text{kg/m}^3$ . The foundation is a surface, rigid foundation and simulated by linear elastic beam elements and its width is equal to 6m. The structure is simulated by linear elastic beam elements and its height for the current study is equal to 6m. The structure's mass is assumed to be lumped at the top of the pier. The damping for both soil and structure is five per cent for the first mode of both structure and soil profile (Rayleigh damping). The properties and the geometry of the studied models are depicted in Fig. 1. The concrete elasticity modulus is equal to  $E=32\text{GPa}$  for all models. The soil's density in all cases is equal to  $\rho=2000\text{kg/m}^3$  and the Poisson's ratio equal to  $\nu=0.333$ . Three variables are considered as random and are modified, the mass at the top of the structure  $m$  in Mg, the shear wave velocity of the soil  $V_s$  in m/s and the diameter of the pier  $d$  in m. For this specific example we consider the following values:

$$m=[100, 200, 400, 800, 1200] \text{ in Mg} \quad (1)$$

$$V_s=[100, 200, 300, 400, 450] \text{ in m/s} \quad (2)$$

$$d=[0.8, 1.2, 1.6, 2.0, 2.2] \text{ in m} \quad (3)$$

All 125 models are triggered at the level of the elastic half-space by the Northridge 1994 earthquake record (NGA\_1011) with fundamental period  $T_p=0.16$  s and peak ground acceleration equal to  $a_{max}=0.95\text{m/s}^2$ . The dynamic characteristics of soil and structure change at each model resulting at 125 different models to be studied. Each variable can have a different distribution. In this study we consider uniform discrete distribution for all three parameters. In the following paragraphs the efficiency of LHS sampling is going to be evaluated.

#### 4. Sample generation

When considering all the combinations for the input parameters ( $m, d, V_s$ ), the number of the analyses is equal to 125. For this kind of coupled systems, the computational effort is important considering that each analysis lasts about 40 minutes using a common computer. LHS is an efficient method and helps to reduce the computational effort with not important reduction in the accuracy of the result. This method has rarely been used until now to coupled soil foundation structure interaction systems.

In the present study we run the analyses and compare the results in terms of system's periods, accelerations and displacements when considering the results of (i) all 125 combinations of the selected parameters (ii) 50 combinations resulting by the application of the LHS method 10 times and getting each time 5 samples (LHS1) and (iii) 50 samples resulting from the application of the LHS method once (LHS2).

For the above mentioned cases the elastic time history analyses were executed and all needed output parameters were calculated. The output parameters that are examined herein, as mentioned above, are the effective period for the flexible-base system, the spectral acceleration and the spectral displacement for the fixed-base and flexible-base system.

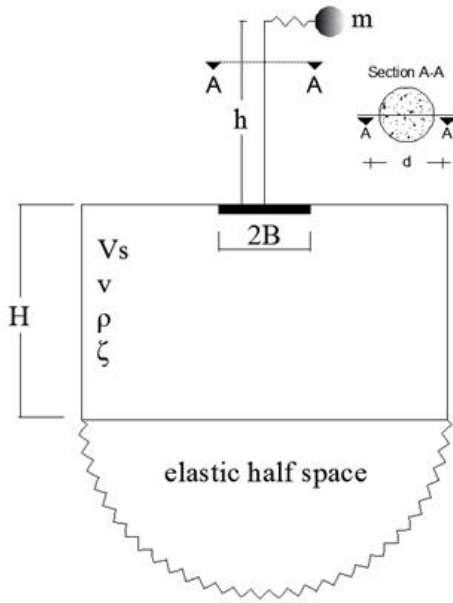


Fig. 1 – Soil foundation structure systems studied

## 5. Evaluation of LHS sampling

The first examined output parameter is the effective period of the flexible-base system. More specifically, two approximations concerning the effective period of the system are examined, the  $T_{SFSI(FF)}$  and the  $T_{SFSI(FND)}$ .  $T_{SFSI(FF)}$  is the effective period and  $T_{SFSI(FND)}$  is the pseudo-effective period.

The  $T_{SFSI(FF)}$  period for each analysis results when dividing the Fourier spectrum at the top of the structure to the one at free-field conditions. This is the effective period as described by Veletsos et al. [19]. The comparison between the numerical  $T_{SFSI(FF)}$  with the ones that result from theoretical expressions [19] are in very good agreement with differences up to 6%. The  $T_{SFSI(FND)}$  period for each analysis results when dividing the Fourier spectrum at the top of the structure to the one at the level of foundation. The difference between  $T_{SFSI(FF)}$  and  $T_{SFSI(FND)}$  is that  $T_{SFSI(FND)}$  includes only the relative to the foundation level displacement at the top of the structure, while  $T_{SFSI(FF)}$  includes also the horizontal displacement of the footing.

Fig. 2a and Fig. 2b present the effective and pseudo-effective to fixed-base period ratio in terms of the relative soil to structure stiffness ratio  $1/\sigma = h/(T_{FIX}V_s)$ , where  $T_{FIX}$  is the fixed-base period of the structure, and in terms of normalized structure mass,  $m_{norm} = m/(\rho h^3)$  for all 125 combinations of the input parameters. It is worth mentioning that both  $T_{SFSI(FF)} / T_{FIX}$  and  $T_{SFSI(FND)} / T_{FIX}$  ratios are increasing with either decreasing soil stiffness or increasing slenderness or mass of the structure. For actual soil profiles, the effective period of the system including SFSI is up to 8 times the fixed-base period, while the pseudo-effective period is up to 6 times the fixed-base period.

The LHS method gives very similar results for both  $T_{SFSI(FF)} / T_{FIX}$  and  $T_{SFSI(FND)} / T_{FIX}$  ratios. As for example, when the relative soil to structure stiffness ratio,  $1/\sigma$ , is equal to 0.2, a common value for actual soil profiles, the resulting ratios for effective and pseudo-effective to fixed-base period, for the five structure's masses ( $m_1$  to  $m_5$ ) are shown in Table 1 and Table 2, respectively. If assuming that the 'correct' value is the one that results from all combinations, it is obvious from Table 1 and Table 2 that both LHS1 and LHS2 give very good approximation of the period's ratio with differences from 0 up to 20%, being in most cases smaller than 5%.

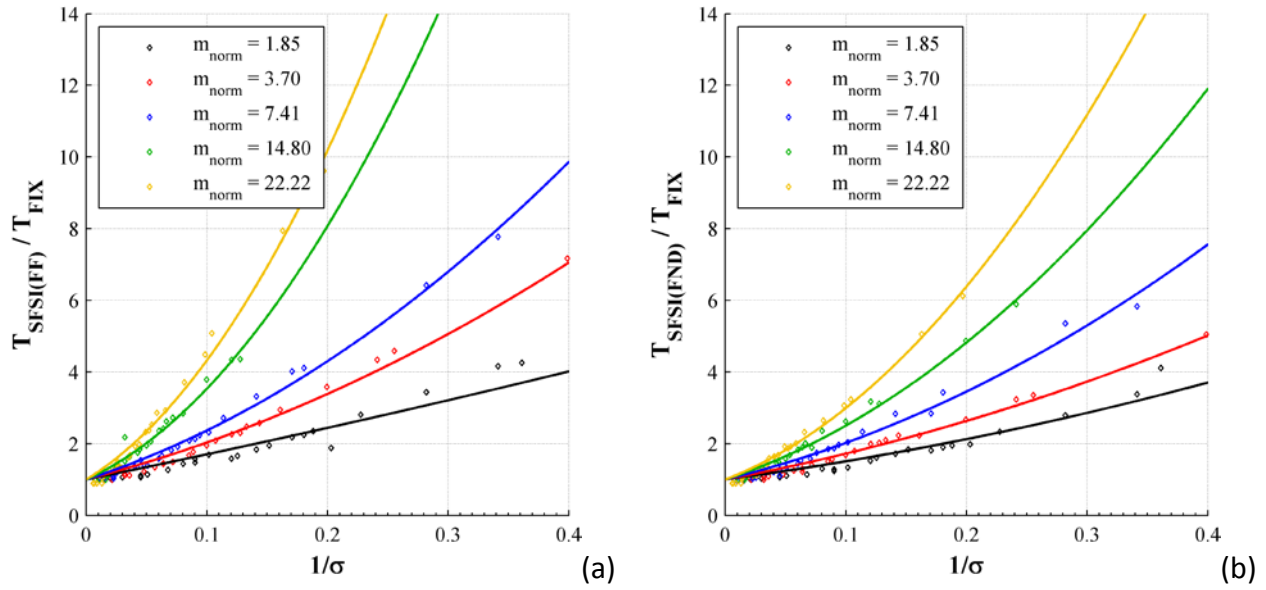


Fig. 2 - (a) Effective to fixed-base period and (b) pseudo-effective to fixed-base period in terms of relative soil to structure stiffness ratio  $1/\sigma$  and normalized mass  $m_{norm}$  for all 125 combinations

Table 1 - Effective to fixed-base period ratios for the five mass ( $m_1$  to  $m_5$ ), assuming the  $1/\sigma$  ratio equal to 0.2, when utilizing the three studied approaches: all combinations, LHS1 and LHS2

	all	LHS1	LHS2
	$T_{SFSI(FF)}/T_{FIX}$	$T_{SFSI(FF)}/T_{FIX}$	$T_{SFSI(FF)}/T_{FIX}$
$m_1$	2.5	2.5	2.4
$m_2$	3.5	3.5	3.5
$m_3$	4.2	4.5	5.0
$m_4$	8.0	7.7	7.0
$m_5$	10.0	10.8	10.0

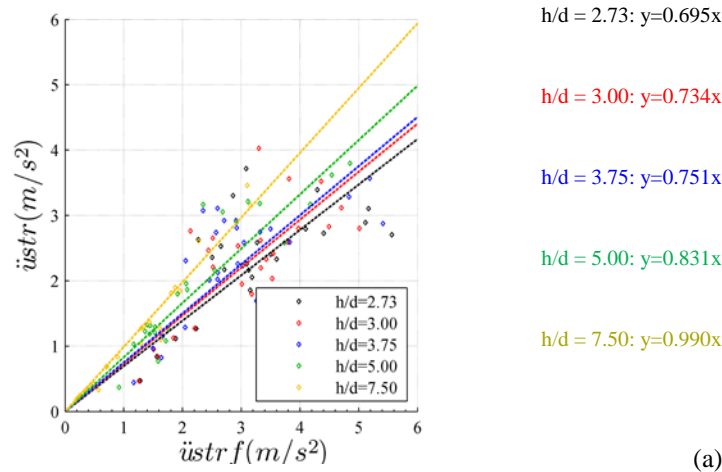
Table 2 - Pseudo-effective to fixed-base period ratios for the five mass ( $m_1$  to  $m_5$ ), assuming the  $1/\sigma$  ratio equal to 0.2, when utilizing the three studied approaches: all combinations, LHS1 and LHS2

	all	LHS1	LHS2
	$T_{SFSI(FND)}/T_{FIX}$	$T_{SFSI(FND)}/T_{FIX}$	$T_{SFSI(FND)}/T_{FIX}$
$m_1$	2.1	2.0	2.0
$m_2$	2.5	2.6	2.5
$m_3$	3.5	3.5	3.7
$m_4$	4.8	5.8	5.5
$m_5$	6.3	6.7	6.2



The next output parameter that is going to be examined for the assessment of the LHS sampling is the absolute acceleration at the top of the coupled SFSI system ( $\ddot{u}_{str}$ ) plotted at the same graph with the absolute acceleration at the top of a fixed-base pier with period equal to the pseudo-effective period  $T_{SFSI(FND)}$  subjected to the free-field motion ( $\ddot{u}_{strf}$ ). The  $\ddot{u}_{str}/\ddot{u}_{strf}$  ratio is an index of how the acceleration motion at the foundation level is affected by interaction effects [20].

Fig. 3a shows the resulted  $\ddot{u}_{str}$  to  $\ddot{u}_{strf}$  plot for all 125 combinations with the results categorized according to the structure's height to pier diameter ( $h/d$ ) ratio. Each line represents the radial resulted after linear regression of the scatter plots for each  $h/d$  ratio. Additionally, in the right side of the figure the equations for all resulted radials are illustrated. Fig. 3b presents the same plots for LHS1 case and Fig. 3c for LHS2 case. The results are given with an equation  $y=ax$ . The maximum per cent differences between the resulted 'a' factors from all the analyses compared to the 'a' factor from LHS1 and LHS2 case are 10% and 5.5%, respectively. Both approaches are in a very good agreement with the exact solution (all combinations). However, LHS2 case gives slightly better results for the same sample size. From Fig. 3a it is obvious that as the  $h/d$  ratio increases, the  $\ddot{u}_{str}/\ddot{u}_{strf}$  ratio increases also. This conclusion shows that the more intense the interaction effects are, the more the acceleration time history at the foundation level is affected. Considering the fact that in the present study the structure's height is the same for all analyses, the  $h/d$  ratio is modified only through the modification of the pier diameter,  $d$ , and thus as the  $h/d$  ratio increases the structure becomes more stiff. From the above-mentioned, it is obvious that LHS method can be used in order to reduce the computational effort. It is worth mentioning that when executing all 125 analyses, the time needed is about 5000 minutes, while for LHS1 and LHS2 case the time needed is 60% reduced.



(a)

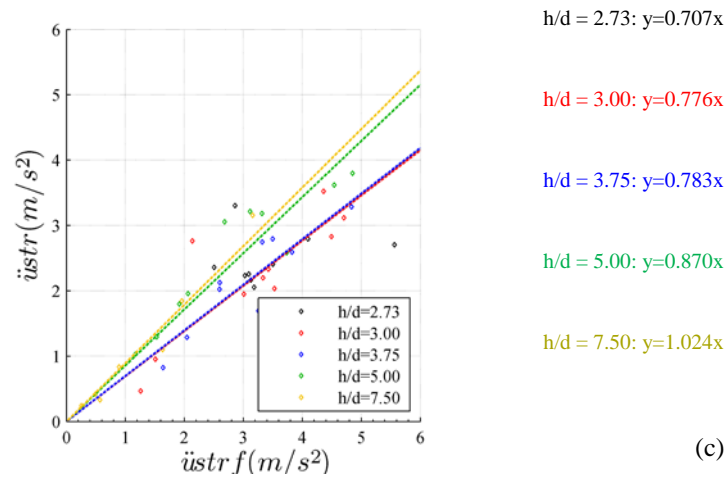
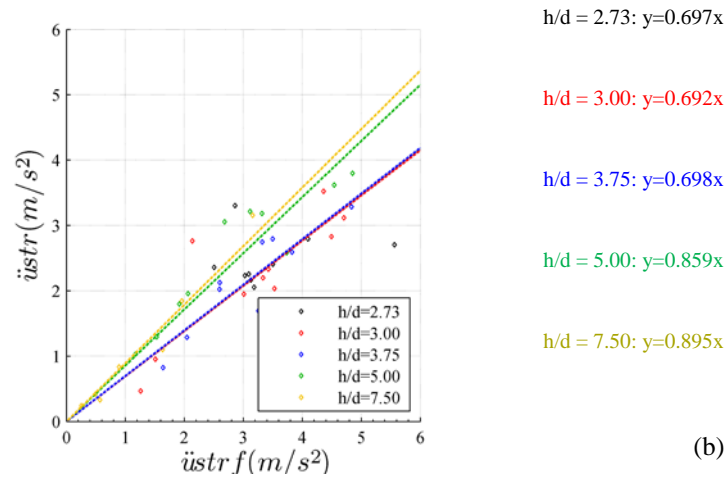


Fig. 3 - Absolute acceleration at the top of the structure for the coupled SFSI system ( $\ddot{u}_{str}$ ) plotted at the same graph with the absolute acceleration at the top of a fixed-base pier with period equal to the pseudo-effective period  $T_{SFSI(FND)}$  and subjected to the free-field motion ( $\ddot{u}_{str f}$ ) according to the  $h/d$  ratio for (a) all 125 combinations (b) LHS1 case and (c) LHS2 case.

## 6. Sample size

In literature almost no recommendations exist for the sample size of LH sampling. Considering that we do not know the output when executing all combinations of the input parameters, we have to know the exact sample size in order to be sure that the result is not affected.

We consider the soil-foundation-structure system of Fig. 1. The mass at the top of the structure is equal to 400 Mg, while the pier diameter and shear wave velocity of the soil profile change according to Eq.2 and Eq.3. The total analyses of the coupled SFSI system are equal to 25. Our goal is to examine which is the sample size that gives a good approximation of the result in terms of the acceleration ratio.



We generate afterwards three sample sizes of 5, 10 and 15 samples using the LHS sample generation methodology which is described in the second session of the present study. Fig. 4 shows these three sample generation. More specifically, Generation- I (Fig. 4a) concerns 5 samples, Generation- II (Fig. 4b) concerns 10 samples and Generation- III (Fig. 4c) concerns 15 samples. Both x-x and y-y axes of Fig. 4 depict the probabilities.

After generating the three samples, we run the time history analyses for these samples. In this paragraph we will examine the  $\ddot{u}_{str} / \ddot{u}_{str f}$  ratio for the three samples and for all combinations. Fig. 5 depicts the  $\ddot{u}_{str} / \ddot{u}_{str f}$  ratio, which result after the linear regression of the scatter-plots, for all studied cases. It is obvious that when the sample size is equal to 15, the result is very close to the solution for all combinations with this difference being equal to 2.5%. On the other hand when we assume that the sample size is equal to 5, the difference with the exact solution is equal to 18%. The black continuous radial line concerns the 1:1 line. When the result is on this line,  $\ddot{u}_{str} = \ddot{u}_{str f}$  which means that the response at the foundation level which is actually the input to the superstructure is identical to both cases.

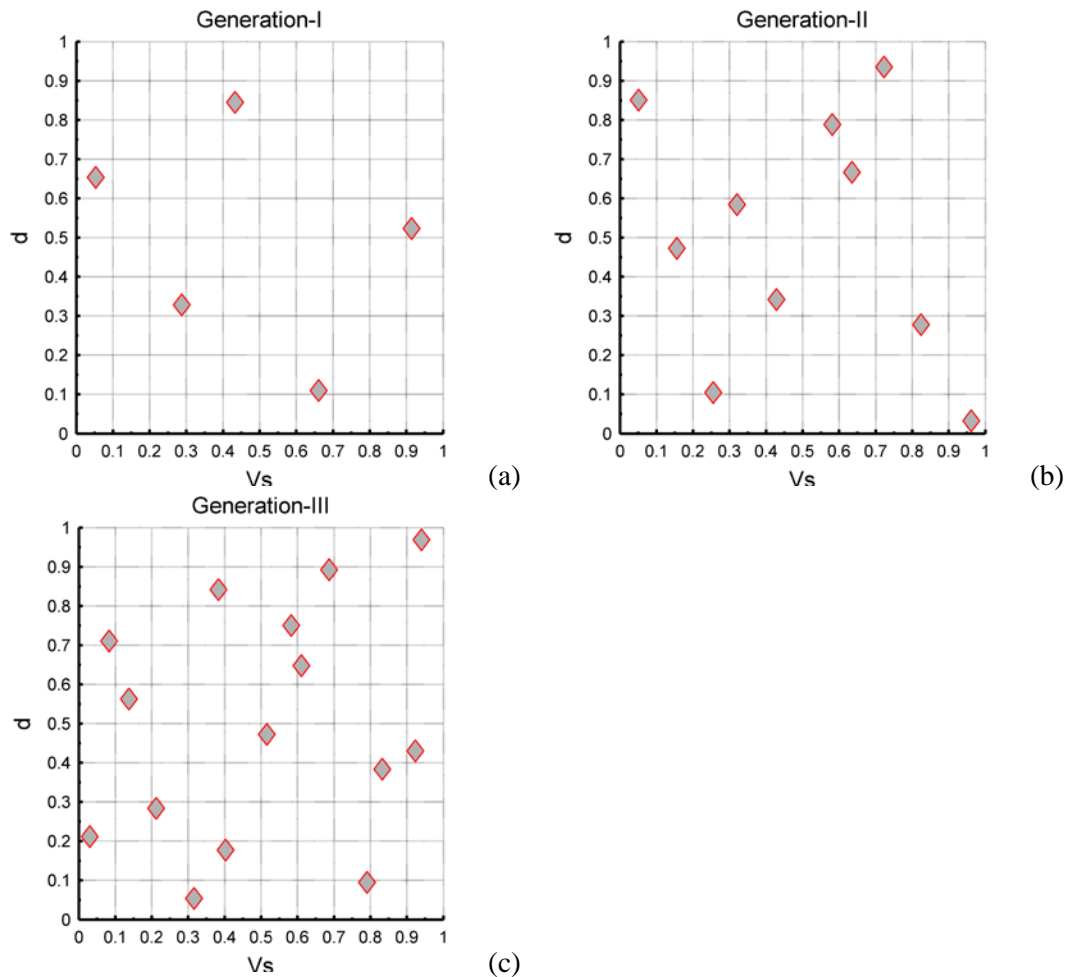


Fig. 4 - Sample generations of (a) 5 (b) 10 and (c) 15 samples



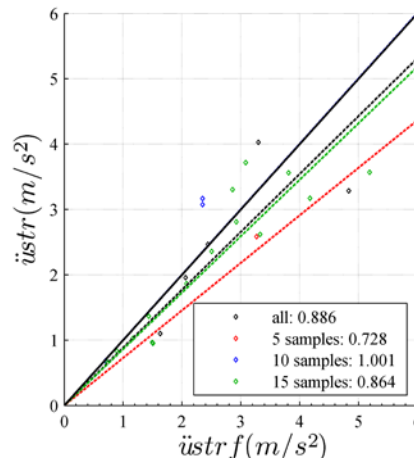


Fig. 5- Absolute acceleration at the top of the structure for the coupled SFSI system ( $\ddot{u}_{str}$ ) plotted at the same graph with the absolute acceleration at the top of a fixed-base pier with period equal to the pseudo-effective period  $T_{SFSI(FND)}$  and subjected to the free-field motion ( $\ddot{u}_{str f}$ ) and the radial lines of the scatter-plots for all 25 combinations and for 5 to 15 samples.

## 7. Conclusions

The main goal of the present study, except examining the soil-structure- interaction phenomenon, is to evaluate the use of LHS method in soil-foundation-structure interaction. As seen from the results, the LHS method is very effective and the computational cost can be reduced from 40% to 60% with minor reduction of the accuracy of the results. Regarding the effects of interaction, for actual soil profiles the effective period of the system including SFSI is up to 8 times the fixed-base period, while the pseudo-effective period is up to 6 times the fixed-base period. In all cases the effective period of the systems is higher compared to the pseudo-effective as it includes all counterparts of the total displacement. As the structure becomes stiffer, the acceleration time history at the foundation level is less affected by interaction, and it resembles the motion at free field conditions.

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