



## A 3D macroelement to model the seismic behaviour of shallow foundations

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### ***Abstract***

In this article, a 3D macroelement to model the seismic behaviour of shallow foundations is formulated, taking into account the non-linearities in the soil and the soil-foundation interface. The main idea of a macroelement is to describe the foundation behaviour using generalized variables (forces and displacements), defined in particular points of the foundation. The forces and displacements at these points are linked by elastoplastic constitutive laws.

The non linearities are studied according to the classical plasticity theory, and the multisurface plasticity theory is used to combine the different mechanisms (punching, overturning, and sliding). This macroelement is implemented in the framework of the finite element method (CESAR-LCPC).

This macroelement permits to simulate the 3D behaviour of a rectangular foundation subjected to complex seismic excitations, and to assess the effects of the limits imposed by the current standards. The time variation of the safety factor during the seismic event is highlighted, as well as the effect of the non linearities on the design demand.

*Keywords: Soil structure interaction, macroelement, shallow foundation, earthquake, plasticity, numerical analysis, design.*



## 1. Introduction

The design of shallow foundation under seismic load requires to account for the nonlinear soil structure interaction, especially the non linearities that may be induced in the soil. This interaction results from the inability of the foundation to match the displacement field imposed by the soil motion underneath (kinematic interaction), and from the inertial effects from the superstructure (inertial interaction).

Regarding the generation of plasticity in the soil and at the soil-foundation interface (overturning and/or sliding) it is caused mainly by the particular properties of the seismic excitation. The occurrence of these non linearities is highlighted based on post-earthquake observations and experimental results.

On the other side, using simplified linear or equivalent linear approaches in seismic design leads to an overestimation of the seismic design efforts induced in the superstructure and on the foundation, and does not predict displacements properly. Accounting for nonlinear soil structure interaction proves to be important in order to avoid conservative design and studies.

## 2. Macroelement approach

### 2.1 Definition

There are many methods that allow the study of soil structure interaction, and its non linearities. Recently, methods based on the macroelement approach have gained the attention of the researchers due to its many advantages. The first application was done by Nova and Montrasio (Nova & Montrasio, 1991) and other contributions were carried out later (Paolucci & Faccioli, 1996) (Pedretti, 1998) (Crémer, Pecker, & Davenne, 2001) (Crémer C. , 2001) (Crémer, Pecker, & Davenne, 2002) (Chatzigogos C. T., 2011) (Grange, 2008) (Grange, Kotronis, & Mazars, 2008) (Grange, Kotronis, & Mazars, 2009). This approach allows to simulate the response of a structure foundation system under static or seismic load, taking into account the non linear soil structure interaction. The macroelement approach is based on the formulation of the response of the foundation using generalized variables (efforts and displacements). These variables are defined in particular points of the foundation, and are linked by elastoplastic constitutive laws.

A macroelement is a rheological element, incorporating these elastoplastic laws. The constitutive laws of this element are formulated using classical plasticity rules and its parameters are calibrated from static response of a foundation (experimental data, FEM simulation). The effects of embedment and soil stratification are taken into account in the definition of the macroelement parameters. The output is in the form of displacements and forces, thus making the macroelement a practical approach to be used in the standard engineering applications, especially for the design of new structures or the analysis of the existing ones.

Furthermore, the calculation cost is significantly reduced in comparison to more complex finite element models. This allows to study the foundation behaviour under many excitations, and perform detailed parametric studies. Hence, the macroelement is both a research and engineering practical tool.

### 2.2 Numerical implementation

The macroelement is installed at specific nodes of the foundation. In the process of resolution by FEM, the nodal forces in these nodes are not considered; the macroelement will bring out this reaction in terms of internal forces, taking into account non linear behaviour.

At a given time step, the macroelement intervenes at each iteration: the corresponding iterative displacement is retrieved and injected into the macroelement model. The macroelement returns a force vector, which represents the nonlinear response of the foundation. These forces are injected in the calculation process as nodal forces, and then we move to next iteration. This process is repeated until the global equilibrium is reached. After that, we move to next time step.



The developed macroelement was implemented in the framework of the Finite Element Method (CESAR-LCPC, software developed by IFSTTAR). The plastic mechanisms are coupled using the multisurface plasticity approach (Simo & Hughes, 1998).

### 2.3 Shape of the studied foundation

The developed macroelement models the behaviour of a shallow rectangular foundation (Fig. 1). The considered generalized efforts are: 2 horizontal forces ( $H_x$ ,  $H_y$ ), 1 vertical force ( $V$ ), 2 overturning moments ( $M_x$ ,  $M_y$ ); in addition to their corresponding displacements: 2 horizontal displacements ( $u_{hx}$ ,  $u_{hy}$ ), vertical displacement ( $u_v$ ), 2 rotations ( $\theta_x$ ,  $\theta_y$ ).

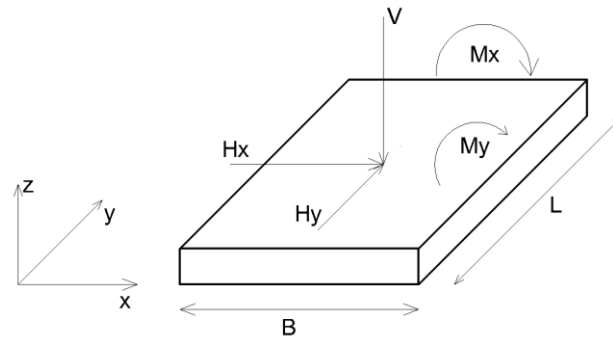


Fig. 1 – Layout of the studied foundation

## 3 Macroelement formulation

### 3.1 Elastic behaviour

The elastic behaviour of the foundation is modelled by its elastic stiffness matrix. The values of the elements of this matrix correspond to the real value of the complex impedance of the foundation. Analytical formulations for these impedances are found in the literature (Gazetas, 1991).

### 3.2 Yield surfaces

In our case, the plastic behaviour of the foundation is modelled by 3 yield surfaces (Table 1). The yield surfaces are expressed in function of generalized efforts; they define the limits of the elastic behaviour. Each yield surface corresponds to a different plastic mechanism:

- a sliding mechanism, based on Coulomb's friction model,
- a bearing capacity mechanism, based on Eurocode criteria (CEN, 2005),
- an overturning mechanism, based on the verification of the compressed area of the foundation.

Those formulas are adapted to account for a 3D loading case by properly adding terms to the corresponding expressions.

### 3.3 Hardening laws

In order to model the evolution of yield surfaces in the space of efforts, a set of hardening laws is added to the elastoplastic model. It consists of two kinematic hardening laws related to the sliding and overturning surface respectively, and an isotropic hardening law related to the vertical bearing capacity surface. The expressions of these laws are presented in table 1.

### 3.4 Plastic flow rule



The plastic flow rule specifies the direction in which plastic deformations are generated. An associated flow rule is used in the developed macroelement.

Table 1 – Elastoplastic formulation of the macroelement

Sliding	Yield surface	$f_1(H, V) = \sqrt{H_x^2 + H_y^2} - V \tan(\delta) - C(B - 2e_x)(L - 2e_y) \leq 0$
	Kinematic hardening law	$h_1(q_h) = H_h du_{h_{pl}} \exp(-\alpha u_{pl}^c)$
	Yield surface after including the hardening variables	$f_1(H, V) = \sqrt{(H_x - q_{hx})^2 + (H_y - q_{hy})^2} - V \tan(\delta) - C(B - 2e_x)(L - 2e_y) \leq 0$
Bearing capacity	Yield surface for monotonic loading	$f_2(H, V, M) = V - V_{lim} \left(1 - \frac{4}{\pi} \arctan\left(\frac{H}{V}\right)\right)^2 \left(1 - 2 \frac{M_x}{BV}\right) \left(1 - 2 \frac{M_y}{LV}\right) \leq 0$
	Isotropic hardening rule	$R(u_{vpl}) = (V_{ult} - V_{lim}) \frac{ u_{vpl} }{ u_{vpl}  + u_0}$
	Yield surface after including the hardening variables	$f_2(H, V, M_x, M_y, q_v) = V - (V_{lim} + (V_{ult} - V_{lim}) \frac{ u_{vpl} }{ u_{vpl}  + v_0}) \left(1 - \frac{4}{\pi} \arctan\left(\frac{H}{V}\right)\right) \left(1 - 2 \frac{M_x}{BV}\right) \left(1 - 2 \frac{M_y}{LV}\right) \leq 0$
overturning	Yield surface for monotonic loading	$f_3(V, M) = \frac{1}{2} - \left(1 - 2 \frac{M_x}{BV}\right) \left(1 - 2 \frac{M_y}{LV}\right) \leq 0$
	Kinematic hardening law	$h_3(q_m) = M_0 \frac{\theta_0}{(\theta_0 + \theta_{pl})^2} du_{m_{pl}}$
	Yield surface after including the hardening variables	$f_3(V, M_x, M_y, q_{Mx}, q_{My}) = \left(\frac{1}{2}\right) - \left(1 - 2 \frac{ M_x - q_{Mx} }{BV}\right) \left(1 - 2 \frac{ M_y - q_{My} }{LV}\right) \leq 0$

## 4 Numerical simulations

The 3D macroelement is tested under monotonic static loads in order to verify its accuracy. The response is compared to classic finite element simulations run using CESAR-LCPC. Then, a dynamic excitation is applied, and non linear behaviour aspects will be highlighted.

The parameters of the soil-foundation finite element model, as well as those used in the macroelement are outlined in table 2.

### 4.1 Monotonic static behaviour

Figure 2 shows the comparison of the macroelement response to the response given by a finite element simulation of the foundation under the static loading paths of figure 2(a).

These simulations help to calibrate and validate the parameters of the macroelement.



Table 2 – Parameters of the Finite Element model and the macroelement

Finite Element model			Macroelement		
Foundation properties	Length	4 m	Stiffness matrix	$K_{hx}$	45 MN/m
	Width	3 m		$K_{hy}$	46 MN/m
	Depth	30 cm		$K_v$	60 MN/m
Soil properties	Young's modulus	10 MPa		$K_{mx}$	135 MN/m
	Poisson's ratio	0.35		$K_{my}$	240 MN/m
	Cohesion	50 kPa		Sliding mechanism	$C$
	Internal friction angle	10°	$\varphi$		6,66°
$H_h$			35 MN/m		
Bearing capacity mechanism	$V_{lim}$	4100 kN			
	$V_{ult}$	10 000 kN			

Loading path	Vertical load (kN)	Horizontal load (kN)	Moment (kN.m)
V	8000	0	0
HV	3500	1800	0
VM	3500	0	6000

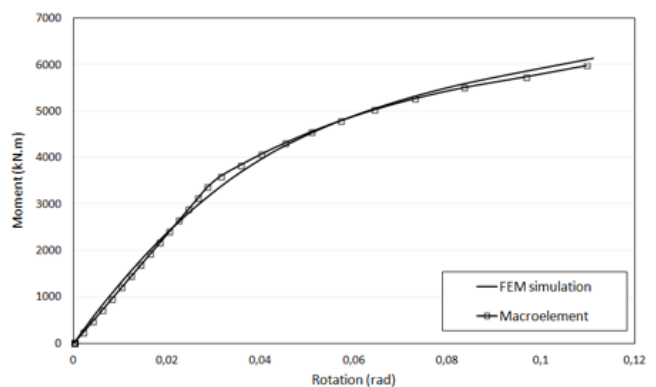
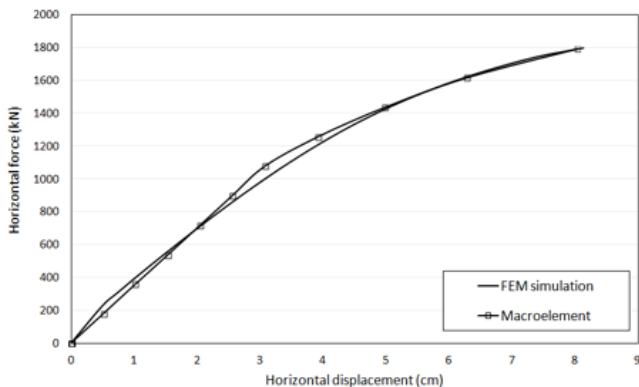
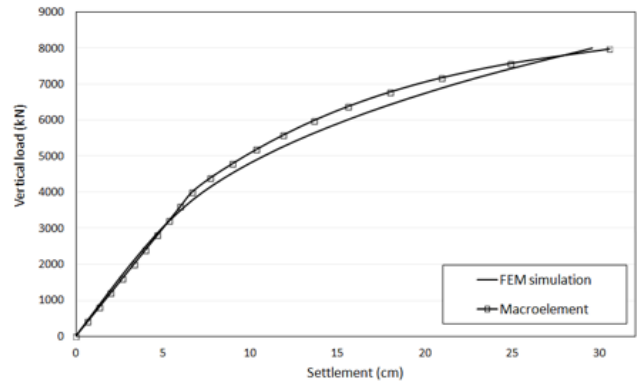


Fig. 2 – static loading paths (a), FEM vs. macroelement simulations for V (b), HV (c) and VM (d)



## 4.2 Dynamic behaviour

In this part, a seismic excitation is applied on the foundation. The accelerogram registered during the Friuli earthquake (Italy, 1976, Magnitude=6.1) at Gemona station was chosen. This accelerogram was scaled to a PGA of 0.07 g, it is represented in figure 3(a). The excitation lasts for 10 seconds, and is preceded by a 3500 kN vertical load. (Figure 3a)

First, the FEM and macroelement simulations are compared for the seismic excitation (Figure 3(b)). The figure shows a good correlation between the two responses. Then, the macroelement response of the foundation under this excitation is analyzed in both linear and non linear cases.

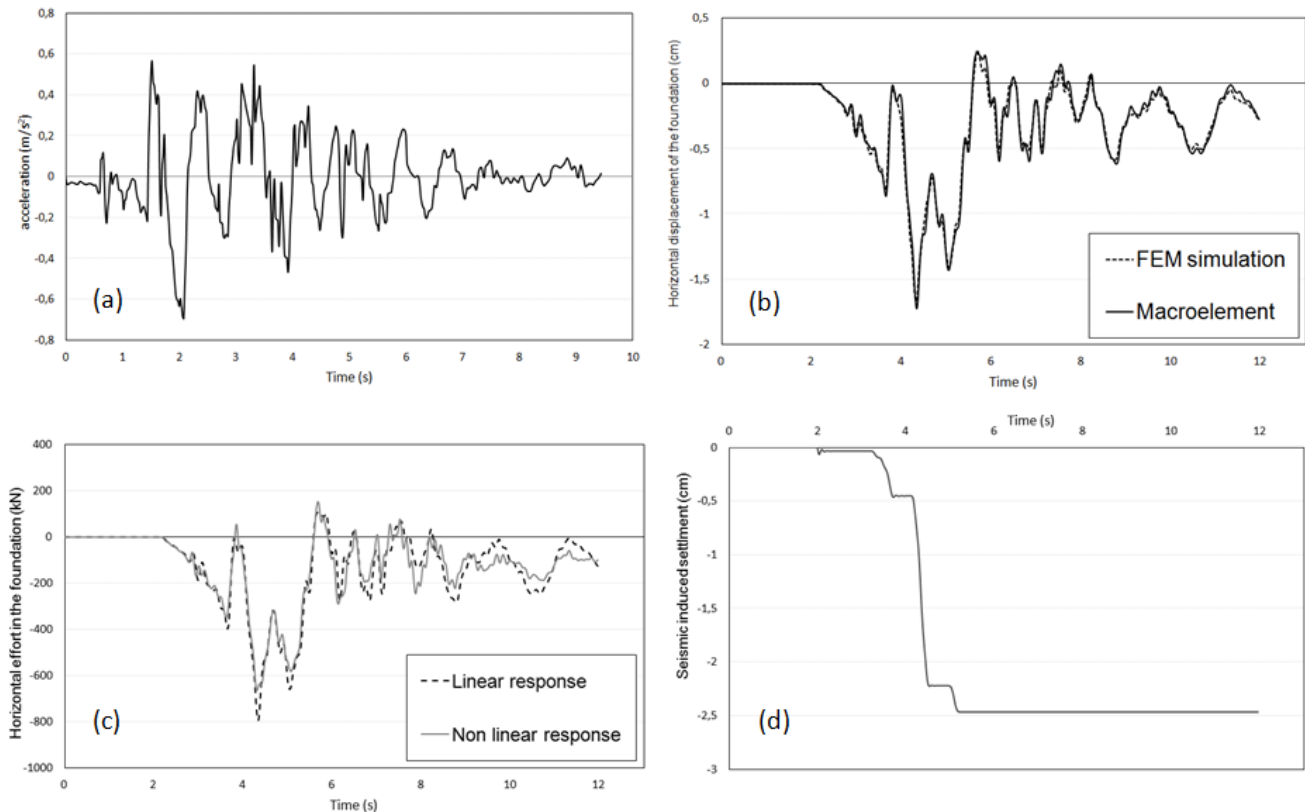


Fig. 3 – (a) Seismic excitation, (b) FEM vs Macroelement simulations for seismic excitation (c) horizontal effort (linear vs. non linear response), (d) earthquake induced settlement

Figure 3c shows the comparison in terms of horizontal efforts induced in the foundation for linear and non linear response. We highlight the difference between the amplitudes of the efforts. The comparison shows that taking the non linearities into account reduces the induced efforts on the foundation, thus reducing the design demand, and avoiding possible over conservative judgment in any required analysis.

The reduction of the induced efforts comes at a cost, with the generation of permanent displacements. The value of these displacements is a direct result of the macroelement simulation process, its evolution with time is plotted in figure 3d. The values of permanent settlement could be analyzed quantitatively later; it's up to design engineers to decide whether this displacement could be accepted or not, based on the sensitivity of the analyzed structure, and the return period of the design seismic event.

On the other side, the variation of the induced horizontal force on the foundation can be compared to the limits imposed by the applied standards. In this case, this force is compared to the elasticity limit in the bearing capacity criterion for both ultimate limit and service limit state (figure 3c). The temporal variation of the safety



factor can be taken into account, by computing the cumulated time during which the value of the effort exceeds one or other of the limits.

In our case, the ultimate limit corresponds to a horizontal effort of 800 kN. It can be seen from figure 3c that this limit is reached for linear analysis, while for non linear analysis, a security factor of 1.25 is retained. This increase of the security factor is due to the dissipation of energy by plastification of the soil. On the other side, the final value of the earthquake induced settlement (figure 3d) can be compared to the limits, which can vary from a structure to the other. This verifications is adequate with the new concepts of displacement based verifications of structures under seismic load.

## 5 Conclusion

In this article, a 3D macroelement to simulate the seismic behaviour of shallow foundations is presented. Based on the verifications, this approach is able to represent the non linearities in soil structure interaction to an acceptable accuracy. The calculation cost is significantly reduced. Moreover, time based verifications of foundations could be performed using this approach.

Many improvements could be brought into this macroelement: the effect of seismic inertial forces in the soil could be studied, as well as the influence of the embedment of the foundation. Later, this macroelement will be completely validated by centrifuge tests on foundations.

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## 8 List of symbols

$B, L$  : width and length of the foundation (m)

$C$  : soil cohesion (kPa)

$\phi$  : internal friction angle of the soil (rad)

$H_x, H_y$ : horizontal forces in x and y directions respectively (kN)

$u_h, u_{hpl}$  : total and irreversible horizontal displacement (m)

$V$  : vertical load applied on the foundation (kN)

$u_v, u_{vpl}$  : total and irreversible vertical displacement (m)

$M_x, M_y$ : overturning moments around x and y axis respectively (kN.m)

$\theta, \theta_{pl}$  : totale and irreversible rotation (rad)

$e_x, e_y$ : eccentricities of the load (m)

$q_{hx}, q_{hy}$  : kinematic hardening variables related to sliding mechanism

$q_{Mx}, q_{My}$  : kinematic hardening variables related to overturning mechanism

$V_{lim}$  : vertical load limiting the elastic behavior (kN)

$V_{ult}$  : vertical failure load of the foundation (kN)

$H_h$  : coefficient of the sliding hardening law

$\alpha$  : coefficient of the sliding hardening law

$M_0, \theta_0$ : overturning moment of the foundation (KN.m) and corresponding rotation (rad)