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SEISMIC RESILIENCE OF MASONRY WALLS ROCKING ON ELASTIC FOUNDATION

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Abstract

The use of rigid body kinematics within a limit analysis framework has become popular for evaluating the seismic capacity of local collapse mechanisms. This approach, combined with the use of linear elastic spectra for evaluating the seismic demand, is now incorporated in current seismic provisions. However, rocking and elastic oscillators are very different and the current assessment procedures have the drawback of not considering properly the dynamic resilience of collapse mechanism and the scale effects occurring in the dynamic response. There is the definite need for an assessment method that takes into account the dynamic resources of the structure and the ability to dissipate energy through the rocking motion. This study aims at proposing a tool for the seismic assessment of historical masonry based on the selection of the local mechanism and the integration of the corresponding equation of motion under seismic inputs. Moreover, according to experimental evidences, the equation of motion includes a term related to an elastic Winkler-type foundation. Eventually, the dynamic analyses of a sample case study under seismic inputs is presented and compared with code-based approaches.

Keywords: rocking; historical masonry; time-history analysis;

1. Introduction

The seismic assessment of old masonry structures lacks in reliable numerical strategies due to the remarkable complexity of the issues involved. However, a careful survey of damages experienced by traditional buildings during historical earthquakes shows that these constructions suffer mostly by out-of-plane rocking mechanisms during seismic events. In particular, a part of the building detaches from the rest and starts moving as a rigid body until its complete separation in the case that overturning occurs. This behavior is possible if it is guaranteed a certain monolithic nature of the boundary wall, such as to prevent isolated breakdowns due to disintegration of the masonry. The Italian provisions [1] adopt a methodology based on the limit equilibrium analyses, according to the kinematic approach which consists on the choice of the collapse mechanism and the evaluation of the horizontal load multiplier that activates this mechanism. The main drawback of code procedures lies in the fact that it does not consider properly the dynamic resilience of collapse mechanism and the scale effects occurring in the dynamic response. Housner [2] was the first to point up that a gravity structure can exhibit significant capacity towards overturning by tilting/rocking. Recently, Mauro et al. [3] has demonstrated that the same model may be extended also to study the response of a wider set of mechanisms developed by masonry constructions. The restoring mechanism of the simple rocking block originates from gravity and the relationship between the restoring moment and the angle of tilting of the block is linear. The block has infinite stiffness until the applied moment is lower than WRsinα, and once the block is rocking, its stiffness assumes a negative value and decreases monotonically, reaching zero when the angle of rotation equals the block slenderness [4]. Different authors have defined various force-displacement relationships for describing the motion of the block from the triggering of the mechanism until the out-of-plane collapse. In 2002 Doherty et al. [5] presented a trilinear curve used to reproduce the experimental results obtained for unreinforced brick masonry walls, while Shawa et al [6] included in the three-branch relationship the effects deriving from the restraint provided by transverse walls as well as the out-of-plumb of the block. These models proved able to reproduce the rocking of masonry blocks

tested in laboratory but, sometimes, the parameters needed to define the curve, are too many and may lack on a physical meaning.
In this paper, a single degree of freedom block resting on a Winkler-type foundation [7] is presented for the time-history analysis of masonry walls. The behavior is governed by the stiffness *k* of the bed of springs in which the block is laying as well as the coefficient of restitution, a fundamental parameter for discontinuous problems.

2. The smooth-rocking model

The simple rocking block formulated by Housner [2] has been recently used for the out-of-plane analyses of masonry walls. However, experimental evidences show that for masonry walls subjected to rocking motion, the relationship between the restoring moment and the angle of tilting is not rigid but can present a smoother behavior. In this paper, aiming to give a better simulation of the actual behavior of masonry walls, the equations of motions describing the rocking block on a Winkler-type foundation are used, as presented in [8]. These equations are implemented in a Matlab code [9] and solved numerically using the algorithms presented in ode23s. The two models are presented in Figure 1 where it can be seen that the main difference between them lay in the contact surface, which is rigid for the classical Housner model (Fig. 1a) and "Winkler-type" for the smooth model (Fig. 1b). As in the rigid model, the vertical component of the displacement is neglected and the rotations instead are considered sufficiently small. In order to model the inability of the ancient mortar to carry tensile stresses, it is assumed that the block is just resting on the springs, without any bond between them. For the description of this smooth-rocking model, the only additional parameter is the stiffness coefficient k of the elastic springs (Eq.1), while M_R is the restoring moment, θ_{cr} is the relative rotation corresponding to the uplift of the block and the others have the same meaning as in Fig.1. In particular, the moment-rotation curve has two fictitious branches corresponding to the case where the coupling of the springs is perfectly reagent ($\theta \leq \theta_{cr}$), and the case where the section is partialized with the consequent loss of contacts ($\theta > \theta_{cr}$). The maximum rotation the block can bear without overturning is not reached for $\theta = \alpha$ as in the simple Housner oscillator but also depends on the stiffness of the springs, tending to α with the increase of the latter. These equations (Eq.1) are valid for both positive and negative angles of tilting and the term u''_g is the horizontal ground acceleration.

$$
\begin{aligned}
& I_0\ddot{\theta} + WR\,sin(\alpha\,sign(\theta) - \theta) - Wb\,sign(\theta) + \frac{2}{3}\,k\,b^3\theta = -\frac{W}{g}\,\ddot{u}_g\,R\,cos(\alpha\,sign(\theta) - \theta) \\
& for\ \theta \leq \theta_{cr} \\
& I_0\ddot{\theta} + WR\,sin(\alpha\,sign(\theta) - \theta) - \frac{1}{6}\,k\,\theta\sqrt{\left[\frac{2\,W}{k\,\theta\,sign(\theta)}\right]^3} = -\frac{W}{g}\,\ddot{u}_g\,R\,cos(\alpha\,sign(\theta) - \theta) \\
& for\ \theta > \theta_{cr} \\
& \theta_{cr} = \frac{W}{2b^2k}
\end{aligned}
\tag{1}
$$

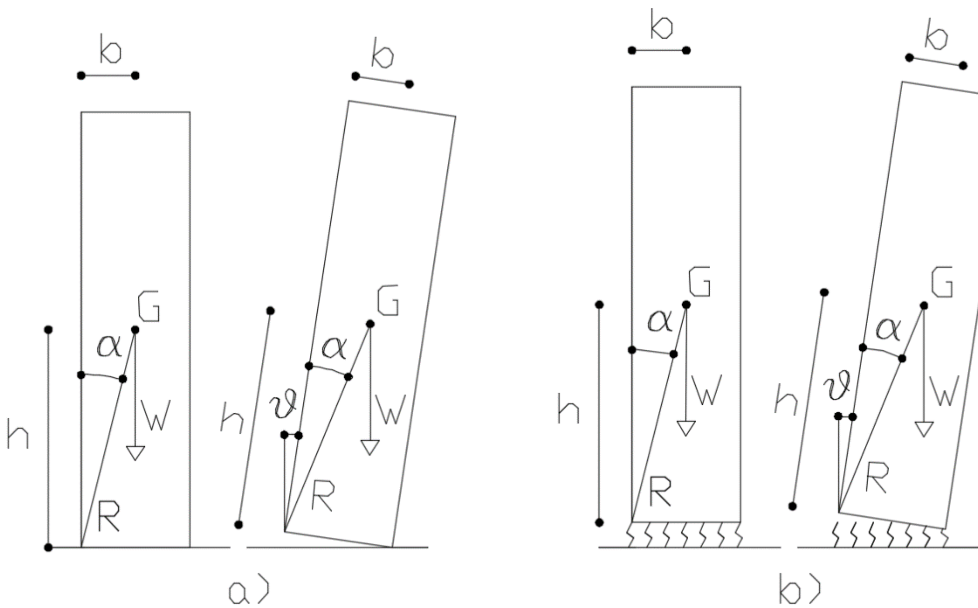

Fig. 1 – Comparison between the rigid model and the smooth model

In Figure 2a) the moment-rotation relationship for a block having 2b = 0.25 m and 2h = 3 m and a specific weight of 12.06 kN/m^3 is presented, comparing the results for different values of the stiffness k. For finite values of k, a linear behavior is exhibited until $\theta \leq \theta_{cr}$, then the response suddenly curves to the maximum allowable restoring moment and then goes to an almost linear branch with negative stiffness until the ultimate rotation is reached. Noteworthy, the maximum rotation the block can bear without overturning is not reached for $\theta = \alpha$ as in the Housner [2] oscillator but it also depends on the stiffness of the springs and is derived here numerically starting from the restoring moment present in equation 1b). When the foundation springs are very stiff, the block is expected to behave as if it was rocking on a rigid foundation, therefore, for $k \rightarrow \infty$, the second branch of the restoring moment-rotation curve ($\theta > \theta_{cr}$) tends to $WR\sin(\alpha - \theta)$ and the first branch tends to infinity in the vertical direction. The free oscillatory motion of a block occurs with a period which, as known, is a function of the maximum angle of rotation that is reached during the motion. In Figure 2b), there are shown the values of the period of a block, obtained numerically according to the maximum rotation θ_u referring to different values of stiffness k. For initial rotation $\theta_0 \leq \theta_{cr}$ the period T is constant and corresponds to oscillations without partialization of the base joints. When approaching the overturning rotation periods tends to infinity.

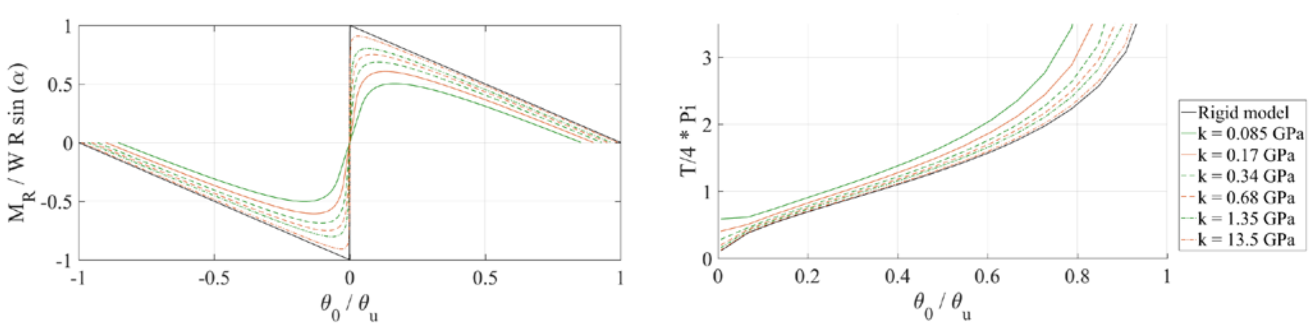

Fig. 2 – Moment rotation relationship a) and period b) for different values of k

3. Parametric analyses for the smooth rocking block

3.1 Italian code

The Italian code [1] recommends the assessment of local-collapse mechanisms of existing masonry buildings through a method based on the limit equilibrium according to the cinematic approach. The method consists in the choice of the mechanism and its representation as a kinematic chain. Once established the portion that acts as a kinematic chain, the horizontal loads multiplier α, that induces loss of equilibrium, is evaluated. This multiplier α is then calculated with increasing displacement d_k of a control point, usually chosen close to the center of gravity of the masses, until its cancellation, as shown in equation 2.

$$\alpha = \alpha_0\left(1 - d_k/d_{k,0}\right) \quad (2)$$

The α-d_k relationship is then transformed in a capacity curve by computing the spectral acceleration a_0^* and displacement d^*, with evaluation of the ultimate displacement of the mechanism. Spectral acceleration a_0^* depends on the equivalent mass M^* and a confidence factor FC as represented in equation 3:

$$a_0^* = \frac{\alpha_0 \sum_{i=1}^{n} P_i}{M^* FC} \quad (3)$$

The equivalent mass M^* is evaluated by considering the virtual displacements of the points of application of the different weights, associated with the mechanism:

$$M^* = \frac{\left(\sum_{i=1}^{n} P_i \delta_{x,i}\right)^2}{g \sum_{i=1}^{n} P_i \delta_{x,i}^2} \quad (4)$$

where n is the number of the weight applied forces P_i, whose masses, for seismic action effect, generate horizontal forces on the elements of the kinematic chain while $\delta_{x,i}$ is the horizontal displacement of the point of application of the i-th weight P_i and g the gravitational acceleration.

The spectral displacement of the equivalent oscillator d^* can be obtained as the average displacement of the different points in which weights P_i are applied, weighed on the same. Approximately, once known the displacement of the control point d_k is possible to define the equivalent spectral displacement with reference to the virtual displacements measured on the initial configuration:

$$d^* = d_k \frac{\sum_{i=1}^{n} P_i \delta_{x,i}^2}{\delta_{x,k} \sum_{i=i}^{n} P_i \delta_{x,i}} \quad (5)$$

where $\delta_{x,k}$ are the horizontal virtual displacement of the point k, taken as a reference for determining the displacement d_k.
The security check consists of the compatibility control of displacement and/or resistance requested to the portion involved in the mechanism. In particular, for the strength-based approach, it must be verified that the spectral acceleration a_0^* is bigger than the PGA multiplied by a coefficient S which takes into account the subsoil category and the topographical conditions, and divided by the structure factor usually set as 2 for historical masonry buildings.

$$a_0^* \geq \frac{(PGA \cdot S)}{q} \quad (6)$$

For the displacement-based procedure, the check consists in comparing the spectral displacement d^* with the demand $S_{De}(T_s)$ corresponding to the period of vibration of the structure T_s as shown in Eq. 7.

$$d_u^* \geq S_{De}(T_s) \tag{7}$$

Within a framework of demand/capacity ratios, the two approaches present the fractions according to Eq. 8:

$$\frac{(PGA)}{2\,a_0^*} \quad strength-based \qquad\qquad \frac{S_{De}(T_s)}{d_u^*} \quad displacement-based \tag{8}$$

3.2 Parametric analyses

In this section, in order to assess the reliability of the proposed model, a comparison of the results from non-linear time-histories analyses of the smooth rocking block and the Italian code [1] is made. In particular, four walls, having (b, h) = (0.125, 1.5), (0.20, 3.0), (0.25, 3.0), (0.375, 3.0) m and four accelerograms, as shown in table 1, are considered. The amplitudes of the four accelerograms are scaled with ten coefficients varying from 0.2 to 2.0, while, in order to account for the strong numerical integration sensitivity, time-axis and the acceleration-axis are multiplied with random coefficients belonging to a normal distribution having 3% coefficient of variation and unity mean.

Table 1 - Time-histories selected to perform numerical analyses

Earthquake	Year	Mw	Station	Soil type	Record	PGA (g)	PGV (m/s)
Irpinia	1980	6.9	Bagnoli Irpino	A	BagnirWE	0.167	0.377
Umbria-Marche	1997	6.0	Nocera Umbra	B	R1168EW	0.438	0.280
Irpinia	1980	6.9	Calitri	B	CalitWe	0.181	0.317
Irpinia	1980	6.9	Sturno	B	SturWE	0.313	0.700

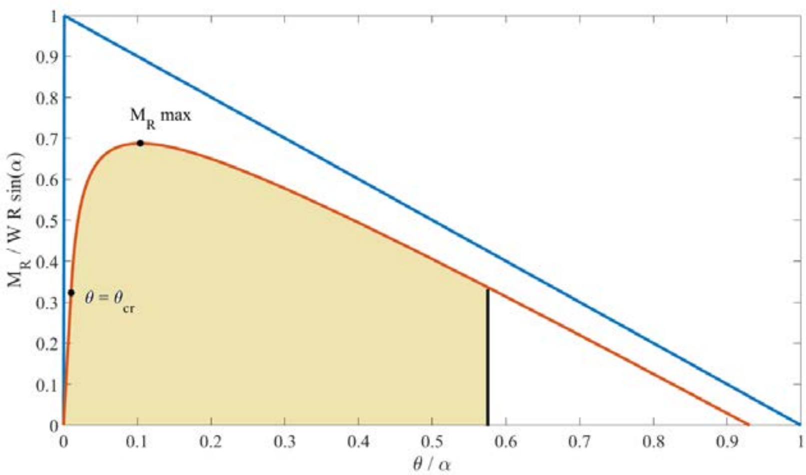

Fig. 3 – Calculation of the demand of the smooth block as the area underneath its moment-rotation curve

Comparisons are made in terms of demand/capacity ratios. The capacity of the smooth block is calculated by computing the area underlying the moment-rotation curve (Fig. 3). The demand instead is given by the area under the above curve, truncated at the value of the maximum rotation reached during the analysis, in the case where no overturning occurs. When overturning occurs, the demand is calculated by adding to the capacity the kinetic energy possessed by the block the instant before the collapse.

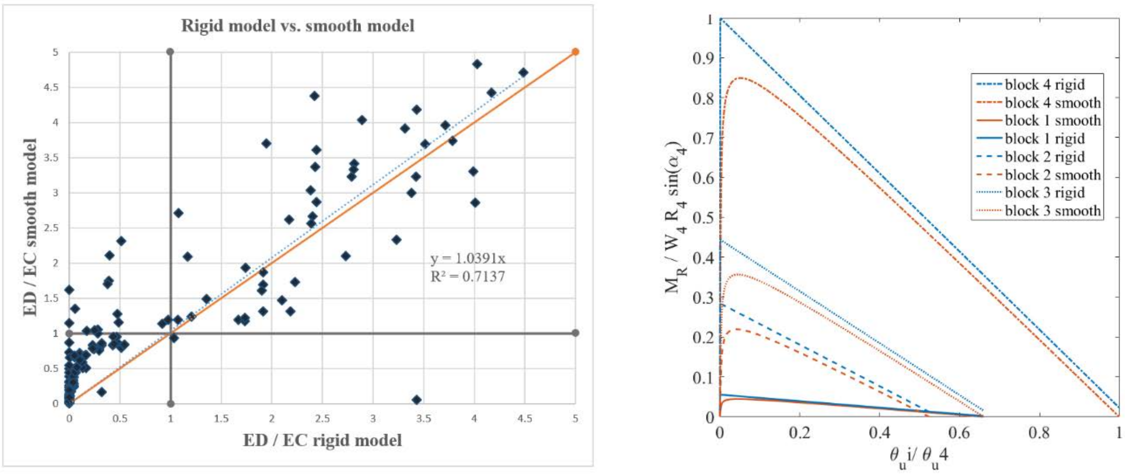

Fig. 4 - Comparison between the smooth and rigid model in terms of demand/capacity ratios (a) capacity curves for the four blocks considered (b)

Figure 4 shows the relationship between energy demand and capacity for the four analysed blocks comparing between rigid and smooth results. The capacity for the different blocks is depicted in figure 4b) where it can be seen that the smooth model presents less capacity than the rigid one. As shown in figure 4a), the smooth block moves even for small accelerations while the rigid model is motionless until a certain threshold is reached. This behaviour seems somewhat realistic given that even for small external perturbations, so before the start of the rocking motion, some local disruptions of the material along the contact are usually possible.

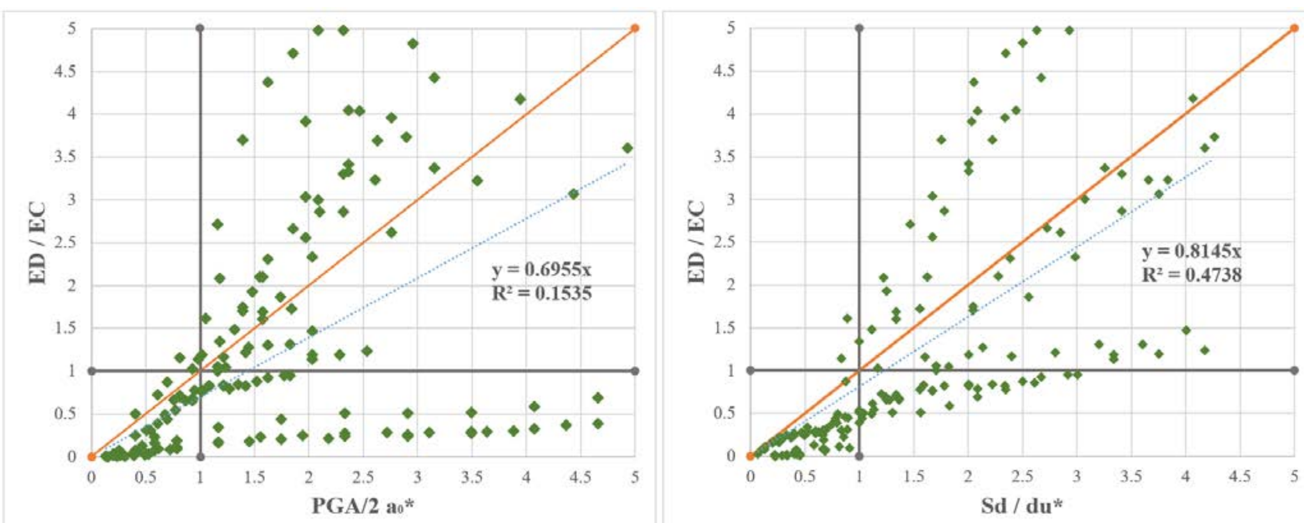

Fig. 5 - Comparison between the smooth model and code demand/capacity ratios (a) force-based and (b) displacement-based

As it can been seen from fig.5, where code demand/capacity ratios are compared with dynamic analysis, both models provide similar outcomes. Neglecting the results in which both numerical analyses and code estimates give demand/capacity ratio greater than one, the strength based seems less conservative than the displacement-based one. Anyway, the two code procedures may yield very different results, depending on ground motion characteristics that are almost neglected by the code procedures.

4. Case study

In this section, we study the seismic vulnerability of a church façade. The behaviour assumed for this macro element is the out-of-plane one-sided rocking. In particular, the results obtained with kinematic linear and non-linear analysis as prescribed by the Italian provisions [1] are compared with dynamic non-linear analysis. The accelerograms used [10] as input for the direct integration of the equations of motion are acquired from the software Rexel v.3.5 developed by Iervolino et al. [11].

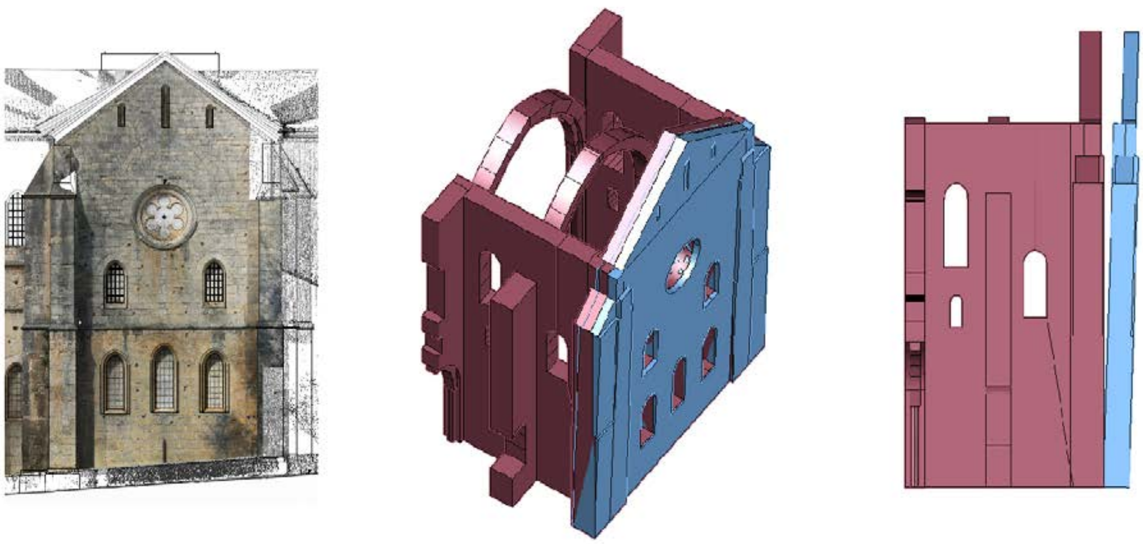

Fig. 6 - Façade of the church and relative mechanism

For the generation of the spectrum, after the exact definition of the location using the site latitude and longitude, a soil class C and a topographic category T1 is hypothesized. The nominal life of the structure is 50 years while the functional type is II category.
The church wall is studied using the linear and non linear kinematic analysis, resulting not verified for both. In particular, the ultimate limit state checks yielded the following results:

Strength-based: $a_0^* = 0.432\ m/s^2 \quad < \quad a_{SLV} = 1.341\ m/s^2$
Displacement-based: $d_u^* = 0.173\ m \quad < \quad S_D\ (Ts) = 0.196\ m$

In addition, non-linear dynamic analysis were conducted which records were chosen to be compatible with the response spectrum for the indicated area. These analyses are carried out for 7 time-histories and for 3 different coefficients of restitution, a theoretical coefficient equal to -0.48 [12], another equal to -0.65 and a third one equal to -0.85. The stiffness value k, to be used for the integration of the equations of motion, is chosen starting from the characteristics of the masonry.
In figure 7a) the time histories spectrums, which records are then integrated numerically in Matlab [9] are shown, while in figure 7b) the responses in terms of normalized rotations for the each time history are reported.

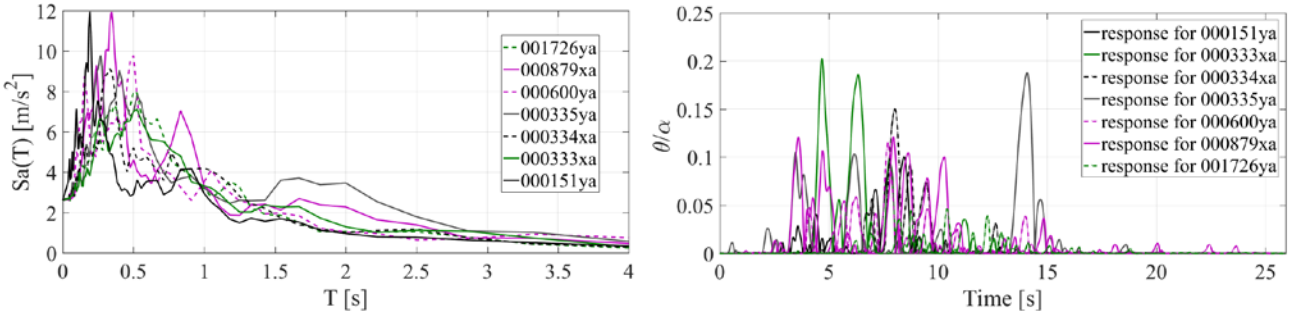

Fig. 7 - Time histories spectrums a) and one-sided rocking responses of the facade b)

In particular, no overturing occured for the coefficient of restitution of -0.48 and -0.65, while for the highest coefficient, 3 out of 7 cases of overturning were observed. Anyway, since the performance if evaluted in terms of demand/capacity ratios, the overall demand is still lower than the capacity. It is worth to say that ξ = -0.85 is really high for one-sided rocking problem but it was interesting to observe the potentiallity of dynamic resources of the façade.

The results in demand over capacity ratios are shown in figure 8. It is seen that the façade is not verified for both linear and non-linear kinematic analyses while, if we account for energy dissipation through rocking motion, the check is positive.

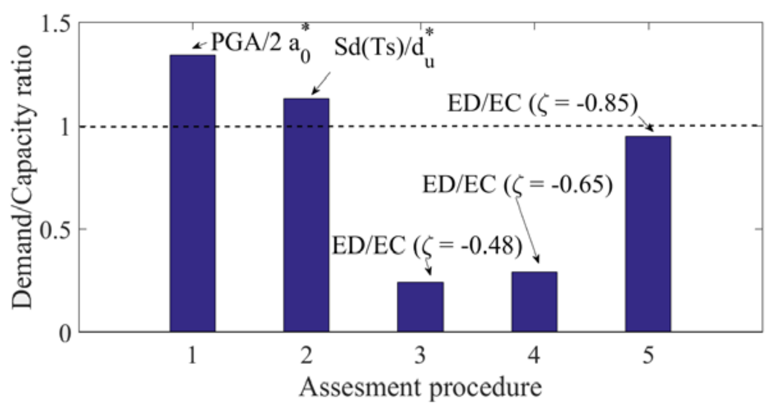

Fig. 8 - Demand/Capacity ratios for different approaches

5. Conclusions

In this paper the dynamic behavior of a rigid block laying on elastic springs is presented as a tool for the analyses of the out-of-plane seismic collapse of masonry structures. Compared to the classical Housner rocking model, the additional parameter represented by the stiffness of the springs allows taking into account the effective experimental behavior of rocking masonry walls, as well as the decay in out-of-plane strength deriving by the non-monolithic characteristics of masonry. The parametric analyses carried out by computing the response of different walls under various accelerograms, show the differences, not only in the range of small displacements, where the model responds in the elastic range, while the Housner model does not begin rocking up to a certain threshold, but also in the range of large displacement, where the overturning of the block appears to be less conservative and less sensitive to the characteristics of the accelerograms. Finally, the one-sided smooth rocking model is applied to a real case study, the façade of a church. The displacement-based code procedure consisting of non-linear kinematic analysis with response spectrum, is compared with the nonlinear dynamic analysis through direct integration of the equations of motion, using seven real records, suitably scaled in order to be compatible with the response spectrum according to current seismic provisions. The results show the underestimate of the actual assessment based on kinematic analysis with the response spectrum, pointing at the importance to consider the rocking phenomenon to correctly assess the out-of-plane capacity of masonry and preserving masonry structures from non-necessary interventions.

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