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A Framework for Evaluation of Seismic Upgrading through Cost-Benefit Analysis

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Abstract

Existing buildings constructed in earthquake prone locations designed prior to 1970s, in general, fail to meet the criteria of modern seismic design codes. This leads in most cases to unsatisfactory performance in case an earthquake strikes and exposes their residents to unacceptably high risk. Seismic upgrading is one of the most effective strategies to mitigate the seismic risk. However, upgrading the seismic performance of existing buildings to the safety level of new building is often criticized as very expensive, lacking cost efficiency. The present study proposes a probabilistic framework that integrates the principal elements of the performance-based seismic design with the standard actuarial "frequency-severity" method to perform cost-benefit analysis of seismic upgrading accounting for the building property service time horizon. The proposed framework enables engineers to identify cost-efficient seismic upgrading strategies and compare them such that the financially optimal upgrade strategy is identified. The proposed framework is applied to existing residential buildings located in Zurich (Switzerland). A sensitivity analysis of the principal input parameters is conducted to examine the robustness of the proposed framework.

Keywords: Existing; Buildings; Upgrade;



1. Introduction

Earthquakes are among the most feared natural hazards due to their potential to cause loss of lives and direct damage to the infrastructure. Thus, developing holistic strategies for mitigation and management of seismic risk is of critical importance. One of the main earthquake risk management strategies is seismic upgrading. Modern seismic rehabilitation guidelines were first introduced in the 1990s. These guidelines (e.g. [1], [2]) evolved since then to enable engineers to design the seismic upgrade for structures in order to achieve specific seismic performance objectives. However, as discussed in [5] the design procedures described in the aforementioned guidelines remain essentially deterministic and conservative in specifying deformation capacities of both structural components and structural systems. It is indicative that the conservative component-based seismic risk assessment procedures ignore the ability of the structural system to redistribute loads as damage accumulates and deem the performance of the structure not acceptable even if a single component has exceeded the target performance level. Instead, seismic upgrade design procedures that would balance the acceptable level of risk and cost would be better for communities: a further discussion on this topic can be found in [9, 12, 18, 21, 22].

The present study proposes a framework that integrates the principal elements of performance-based seismic design and the standard actuarial "frequency-severity" method to perform cost-benefit analysis [11] for seismic upgrade accounting for different time horizons. The latter is considered in this study as an upfront investment aiming to reduce the risk exposure during the building service time horizon while the benefits correspond to avoided seismic losses during the considered time horizon. The framework is applied to evaluate the benefit of seismic upgrading for existing residential buildings located in Zurich (Switzerland).

2. Earthquake Vulnerability

The present study defines the seismic building vulnerability using a simplified model suggested in [16]. This model addresses the built inventory of typical cities in Western Europe. The model derives the earthquake vulnerability based on three structural parameters that can be estimated using an elasto-plastic idealization of a pushover analysis force-displacement response, namely: the yielding acceleration capacity, the structural ductility capacity μ , and the fundamental vibration period T. Applying a capacity-spectrum based method similar to that adopted by HAZUS MH [13], the probability that a Damage Grade DG_k is exceeded is estimated in [16] using Eq. (1a) and (1b):

$$P(DG_k | S_{ae}) = \Phi\left[\frac{1}{\beta} \left(\frac{S_{d*}}{S_{d,k}}\right)\right]$$
(1a)

$$S_{d*} = \begin{cases} \left[1 + \left(\frac{s_{ae}(T_1)}{a_y} - 1 \right) \frac{T_c}{T} \right] d_y, & T < Tc \text{ and } \frac{s_{ae}(T_1)}{a_y} > 1 \\ \frac{s_{ae}(T_1)}{a_y} d_y & T_c \le T < T_D \text{ or } \frac{s_{ae}(T_1)}{a_y} \le 1 \end{cases}$$
(1b)

where $S_{ae}(T)$ is the seismic demand in terms of (pseudo) spectral acceleration at the fundamental vibration period T of the building, while T_c and T_d define the constant velocity range (the parameter values of T_c and T_d are defined for dense or medium dense sand, gravel or stiff clay, soil type C, according to [6]). For each damage grade k={1,2,3,s45}, the limit states $S_{d,k}$ are directly identified by the pushover curve as a function of the yielding (d_y) and ultimate displacements (d_u) as shown in Table 1. It should be noted that ductility capacity mentioned above is defined as the ratio between the yielding and ultimate displacement parameters, $\mu=d_u/d_y$.



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Damage Description	$\mathbf{S}_{\mathbf{d},\mathbf{k}}$
Slight	0.7d _y
Moderate	1.5d _y
Extensive	$0.5(d_y+d_u)$
Heavy - Complete	d _u
	Damage DescriptionSlightModerateExtensiveHeavy - Complete

Table 1 – List of $S_{d,k}$ parameters per damage state

The seismic vulnerability, i.e. the damage grade exceedance conditional probability curves, are illustrated in Fig. 1 for two typical residential buildings. Building A with T=0.60 sec, $a_E/g=0.05$ and $\mu_E=2.0$ represents an existing building located in Zurich (Switzerland) lacking adequate seismic detailing and having low lateral strength and displacement ductility capacity. Building B, with T=0.45 sec, $a_C/g=0.09$, $\mu_C=6.0$ represents a modern code-compliant building located in Zurich with proper seismic detailing. It should be noted that the yielding acceleration capacity of the code-compliant building is computed as in Eq. (2):

$$a_{\rm C}/g = S_{\rm ae,10\% 50 years}({\rm T})^* \Omega^* \gamma_1 / (g^* q)$$
⁽²⁾

where $S_{ae,10\% 50 \text{ years}}(T)$ is the spectral pseudo-acceleration at the fundamental vibration period T of the building for the seismic hazard with a10% exceedance probability in a 50 year period at the locations of Zurich, Ω =1.5 is the structural system overstrength factor, γ_1 =1.15 is the soil amplification factor (soil type C [6]), g is the acceleration of gravity, q=3.0 is the behavior factor to account for inelastic response. The yielding displacements of the two buildings are assumed to be the same, stemming from the assumption that the buildings have the same geometry and yielding material mechanical characteristics. The fundamental vibration periods of buildings A

and B are computed to satisfy this assumption, by using the elastic period relationship $T = 0.32 \sqrt{\frac{S_a}{S_d}}$, where S_a

is the spectral acceleration (g units) and S_d spectral displacement (inches) of the equivalent Single Degree of Freedom oscillator capacity curve as defined in HAZUS MH [13].



Fig. 1 – Seismic vulnerability curves for building types (a) A; (b) B

$$P(DG_5 | S_{ae}) = 0.09 * \sinh\left(0.6\left(\sum_{k=1}^{3} k * P(DG_k | S_{ae}) + 4 * P(DG_{s45} | S_{ae})\right)\right) * P(DG_{s45} | S_{ae})$$

¹ The following relations to determine the proportions of DG 4 and DG 5 are suggested in [16]: $P(DG_{s45} | S_{ae}) = P(DG_4 | S_{ae}) + P(DG_5 | S_{ae})$,



3. Estimating Seismic Losses

The framework proposed in this study quantifies the (aggregate) seismic risk in financial terms only (no casualties are considered in the evaluation process) for different building service time horizons employing a combination of:

(a) The PEER Performance Based Earthquake Engineering framework [17] used by earthquake engineers to facilitate the earthquake design process aiming to achieve certain seismic performance objectives; and

(b) The "Frequency-Severity" [14] actuarial method originally employed to determine the expected number and size of claims that an insurer will receive during a given time period.

The starting point of quantifying the seismic losses is an estimate of the distribution of the size of losses inflicted, given that a single seismic event with certain seismic intensity occurs. The size of losses in this study is expressed as a percentage of the Present Building Property Value (PBPV). It is assumed that the PBPV remains constant during the considered building service time horizon. The seismic loss is quantified as:

$$P(LOSS \le loss|im) = \sum_{D_k} G_2(LOSS \le loss|dg_k) |dG_1(DG_k|im)|$$
(3)

where *im* is the seismic intensity measure quantified in terms of the elastic spectral pseudo-acceleration at the fundamental building period T; DG_k corresponds to the damage grade k (Table 1) ranging from slight damage to complete collapse; LOSS is the seismic loss incurred by a single earthquake event expressed as a percentage of PBPV (e.g. loss=100% corresponds to complete damage of the building property); G_1 is the damage Cumulative Distribution Function (CDF) estimated using the vulnerability curves presented in the previous section. The CDF G_2 is assumed to follow a beta distribution, Beta(LOSS| $DG_k;\alpha,\beta$), where random variable LOSS has finite support [0,1] and α,β are given by Eq. 4(a), 4(b):

$$\alpha = \frac{1 - MDR}{CoV^2} - MDR \tag{4a}$$

$$\beta = \frac{\alpha * (1 - MDR)}{MDR} \tag{4b}$$

where MDR is the Mean Damage Ratio and CoV is the Coefficient of Variation, defined in Table 2. MDR and CoV are defined by analogy to [4] as a percentage of PBPV. The CDF G_2 conditioned on DG_k is illustrated in Fig. 2. The Damage Ratio (DR) in the figure is damage normalized with respect to PBPV.

Table 2 – MDR and CoV

for each damage grade

	DG1	DG2	DG3	DG4	DG5
MDR	3.5%	14.5%	30.5%	80.0%	95.0 %
CoV	1.23	0.39	0.36	0.14	0.06





Fig. 2 – Conditional CDF G_2 conditioned on DG_k with DR as a percentage of the PBPV.

To estimate the total (aggregate) seismic losses for a given building service time horizon, the standard actuarial approach "frequency-severity" method is employed. The amount of loss inflicted by a single seismic event (of any earthquake intensity) is defined as a random variable S, where $S \ge 0$ (expressed as a percentage of PBPV) is called the severity. The Complementary Cumulative Distribution Function (CCDF) of S, can be estimated as in Eq. (5), based on the concept of conditional probability:

$$P(S \ge s) = \frac{\int_{\min(im)}^{\max(im)} \lambda_{im} * (1 - P(LOSS \le loss|im)) d(im)}{\int_{\min(im)}^{\max(im)} \lambda_{im} d(im)}$$
(5)

where λ_{im} is the annual rate of occurrence of a seismic event with intensity *im*. The rate of occurrence ranges between min(*im*), which is the lower bound of earthquake intensity that can cause damage or loss to the evaluated building property, and max(*im*), which is the upper bound of earthquake intensity that can occur in the site of interest. Fig. 3 illustrates the CCDF of the severity S for the considered buildings A,B using the uniform seismic hazard curves for the seismic hazard environment of Zurich as provided by European Facility for Earthquake Hazard and Risk (EFEHR) (www.efehr.org).



Fig. 3 - The severity curves for buildings A and B in the seismic hazard environment of the city of Zurich

Aggregate Seismic Losses (ASL) are defined as the sum of losses for a given building service time horizon TH:



$$ASL_{TH} = \sum_{i=1}^{N} S_i \tag{6}$$

where S_i are the earthquake severities with CCDF given by Eq. (5) and index *i* corresponds to the number of occurrences of earthquakes that cause a loss for a given building service time horizon.

Assuming that earthquake occurrence *i* follows a Poisson distribution, ASL can be assumed to follow a compound Poisson distribution, namely ASL ~ CompPoi (λ ,S), where λ is the frequency of earthquake occurrence (total frequency of all seismic intensities *im* considered) for a given building service time horizon (e.g. 50 years) and S is the earthquake severity defined above. Then, the probability distribution of ASL is:

$$ASL_{TH} \sim \sum_{i=1} P(i) \times S^{i*} \tag{7}$$

where P(i) corresponds to the probability of *i* occurrences of earthquakes that cause a loss within the building service time horizon TH and S^{i*} is the severity distribution convolved *i* times with itself. It should be noted, as discussed in [15], that the procedure described above implicitly assumes that every time a loss is incurred the structure is instantaneously upgraded to the original state without any deterioration of the building performance. It is further assumed that the uncertainties are "renewed" after each earthquake event.

Fig. 4 illustrates the CCDF of ASL_{TH} for buildings A and B for building service time horizon of 1 year. The total damage ratio corresponds to the total losses incurred during the building service time horizon expressed as a percentage of the PBPV. It is evident that improved seismic detailing afforded by modern structural design codes makes for a significant improvement in terms of aggregate seismic losses.



Fig. 4 - Aggregate loss exceedance curves for buildings A and B for 1-year service time horizon

4. Evaluating Seismic Upgrading: Example Application to Existing Buildings in Zurich

In this section, the process of selecting a seismic upgrade strategy is described considering different building service time horizons. Furthermore, a cost-benefit analysis that compares the cost of the selected seismic upgrade to the benefit of the avoided seismic losses is used to determine a financially optimal Degree of Seismic Upgrade (DSU). In the present study DSU is defined as in Eq. (8a),(8b):

$$DSU = \frac{\left(\frac{a_U}{g} - \frac{a_E}{g}\right)}{\left(\frac{a_C}{g} - \frac{a_E}{g}\right)}$$
(8a)

$$DSU = \frac{(\mu_U - \mu_E)}{(\mu_C - \mu_E)} \tag{8b}$$

It is assumed that both the yielding acceleration capacity and the ductility capacity are increased proportionally with DSU (the opposite occurs for the fundamental vibration period of the upgraded structure which decreases as DSU increases). For DSU=0% no seismic upgrade is pursued, while for DSU=100% the building is upgraded such that it possesses the same properties with a modern building with proper seismic detailing (e.g. building



type B of the present study), i.e. $a_U = a_C$ and $\mu_U = \mu_C$. The latter case in the present study is named as full seismic upgrade.

As an example, a residential building located in the seismic hazard exposure environment of Zurich, with structural properties specified in Section 2, is considered.

Only a few studies relating the structural parameters (such as those discussed in Section 2) to the costs of implementing a seismic upgrade are available in literature. One such approach, used to investigate optimal seismic design strategies for new buildings in Mexico City, was discussed by several authors including [19]. In this approach, the cost of the structure is assumed to be an exponentially increasing function of the building base shear coefficient. Although this cost function was originally developed for newly designed buildings, in this study the relationship will be assumed to hold, after adjustments, for existing buildings as well. Thus, the cost of a seismic upgrade to a degree DSU is:

$$C_{DSU} = \begin{cases} C_{\rm c} (\gamma_m + \gamma (DSU)^{\delta}) , & \text{if } DSU > 0\\ 0 , & \text{if } DSU = 0 \end{cases}$$
(9)

where C_c is the cost of a full seismic upgrade (i.e. DSU = 1) and γ_m is the mobilization cost factor, assumed to be equal to 10% of the full seismic upgrade cost C_c by analogy to [3]. Coefficient δ controls the curvature of the seismic upgrade cost with respect to DSU and is set to 1.10 as proposed in [19], and coefficient γ is calibrated such that when $DSU=100\% C_{DSU=1}=C_c$. In the present study C_c was varied between 5% and 30% of PBPV. The values chosen in this study are based on the experience of the authors, but can be changed depending on the state of the particular construction market. Furthermore, the uncertainties reflecting the incomplete knowledge of the exact parameter values of the cost function are not considered.

Assuming risk-neutral behavior of the person who makes the seismic upgrade decision, the financially optimal seismic upgrade level can be determined as the one that maximizes the NPV of seismic upgrading compared to seismic losses due to damage, computed as:

$$NPV = -C_{DSU} + \sum_{t=1}^{TH} \frac{E(ASL_{TH=1,E}) - E(ASL_{TH=1,U})}{(1+r)^t}$$
(10)

where C_{DSU} is the cost of a seismic upgrade to a degree DSU estimated in Eq. (9), TH is the building service time horizon, $ASL_{TH=1,E}$ corresponds to the annual aggregate losses due to damage for the existing structure, $ASL_{TH=1,U}$ corresponds to the annual aggregate losses due to damage for the structure that is seismically upgraded to a degree DSU. The probability distribution of $ASL_{TH=1}$ is calculated according to Eq. (7) discussed in the previous section for TH=1. The discount rate compounded on an annual basis r is set equal to 0.03.

Optimal degrees of seismic upgrade DSU, that maximizes the NPV (Eq. 10), for different service time horizons are shown in Fig. 5. Each curve in the graph corresponds to a different value of full seismic upgrade cost C_c , ranging from 5% (black line) to 30% (red line) of PBPV. First, seismic upgrading to as little as 10% (DSU=0.1) is beneficial even for building service time horizons as short as 5 years. Second, a full seismic upgrade (DSU=1) is optimal only if such costs are relatively low, and then only for long building service time horizons. Third, high costs of full seismic upgrading, even as high as 10% of PBPV, make partial seismic upgrades the optimal choice. This gives engineers a large design space to design seismic upgrades.





Fig. 5 – Optimal DSU for different building service time horizons, assuming 6 different levels of full seismic upgrading (DSU=1) cost

4. Sensitivity Analysis of Seismic Upgrading Evaluation

The sensitivity of the optimal DSU is examined with respect to three principal input parameters: the pushover structural ductility capacity of the existing structure, the assumed MDR for DGk k=1-5, and the exponent parameter δ of the seismic upgrading cost relationship.

Parameter	Values for Sensitivity Testing		
μ_E	2.0, 3.0		
$\mathbf{MDR} \left[\mathbf{DG}_1, \mathbf{DG}_2, \mathbf{DG}_3, \mathbf{DG}_4, \mathbf{DG}_5 \right]$	[3.5%, 14.5%, 30.5%, 80%, 95%] ² ; [2%, 20%, 55%, 90%, 100%] ³		
δ	1.1 , 2.0		
ТН	1-50 years		

Table 3 – Input parameter values for sensitivity testing

In the literature there are multiple studies that provide significantly different estimates regarding the MDR for the range of damage grades considered. However, the effect of variation of MDR values will be considered in this section only by employing the estimates from the study of Fah et al. [8] for illustration purposes. Since no data is given for CoV of the MDR estimates provided in the latter study, the CoV estimated in the study of Dolce et al. [4] will be assumed to apply for the results provided by Fah et al. [8].

First, the effect of an increase in the exponent parameter of Eq. (9) that controls the curvature of the upgrading cost with respect to DSU is shown in Fig. 6. Evidently, increasing the exponent parameter δ makes the upgrading costs increase more slowly with the increase of DSU.

² According to Dolce et al. [4] ³ According to Fah et al. [8]





Fig. 6 – Illustration of the relationship between the upgrading cost and DSU for two different values of δ

Then, using the value of full seismic upgrading cost, $C_c=15\%$ PBPV as basis a benchmark, Fig. 7 demonstrates how two principal input parameters affect the optimal DSU and how these parameters interact.



Fig. 7 – Optimal DSU for μ_E =2.0, for different MDR assumed values and (a) δ =1.10, (b) δ =2.0

In Fig. 7 the blue curve correspond to the values of MDR as suggested by Dolce et al. [4] while the red curve corresponds to the MDR values proposed by Fah et al. [8]. In Fig. 7(a) the seismic upgrading cost exponent parameter is assumed δ =1.1 as suggested by the initial study of Rosenblueth [19] while in Fig. 7(b) δ =2.0, yielding low marginal increase in upgrading cost for low DSU and increased marginal upgrading cost for higher values of DSU. It can be observed that, for both δ =1.1 and δ =2,0, for long time horizon equal to 50 years the optimal DSU is approximately the same. However, for shorter time horizons the difference is more significant since for δ =2.0 low DSU is inexpensive and, thus, preferred.

So far in this study only one pushover structural ductility value of μ_E was considered, namely μ_E =3.0. In Fig. 8 we examine the effect of the parameters MDR and δ , but for a higher μ_E =3.0, assuming that the full upgrading cost remains the same as in the case of μ_E =2.0. The effect of the MDR follows the same trend as in Fig. 7 demonstrating relatively low effect in the optimal DSU. Comparing Fig. 7 and 8 we observe that the most significant difference between them is coming from the effect of μ_E and δ values for short time horizons (lower than 15 years) while for longer time horizons (higher than 15 years) perturbations of μ_E and MDR seem to have approximately the same influence in the optimal DSU.



Fig. 8 – Optimal DSU for μ_E =3.0, for different MDR values and (a) δ =1.10, (b) δ =2.0

5. Conclusions

Selecting the optimal seismic upgrading level for existing buildings lacking earthquake design is a cumbersome procedure requiring the engineer to connect sophisticated nonlinear structural analysis and earthquake engineering with actuarial loss estimation. The present study makes an attempt to do this. We demonstrated the relation between seismic losses, quantified in terms of the Present Building Property Value and three structural parameters, namely the fundamental vibration period (T), the yielding acceleration capacity (a_y) and the pushover structural ductility (μ) .

Combining the principles of performance based earthquake engineering and actuarial science we suggested a method that engineers in practice could utilize to probabilistically estimate seismic losses. Consequently, we applied a cost-benefit analysis to identify the financially optimal degree of seismic upgrading such that it maximizes the Net Present Value, where the seismic upgrading is viewed as an upfront investment which reduces the seismic losses during the building service time horizon considered. For the example application of an existing residential building located in Zurich (Switzerland) we found that: 1) seismic upgrading to as little as 10% of a full seismic upgrade is beneficial even for building service time horizons as short as 5 years; 2) a full seismic upgrade is (financially) optimal only if such costs are relatively low, and then only for long building service time horizons; and 3) high costs of full seismic upgrading, even as high as 10% of PBPV, make partial seismic upgrades the (financially) optimal choice. This gives engineers a large design space to design seismic upgrades.

Moreover, the study demonstrated that the curvature of the seismic upgrading cost function δ are most influential in determining the optimal degree of seismic upgrading for short time horizons lower than 15 years. The Mean Damage Ratio (MDR) values on the other hand seemed to be more influential in determining the optimal degree of seismic upgrading for longer time horizons, while for shorter time horizon their effect was minimal. Overall the pushover structural ductility capacity of the studied existing building seemed to be the most influential parameter since it seemed to be influential for both short and long time horizons.



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