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# Ground motion prediction equation for PGA and SA based on Brune's extended source model

**S. Ólafsson(1), R. Rupakhety(2)**

(1) *Research Professor, Earthquake Engineering Research Centre, University of Iceland, simon@hi.is*
(2) *Associate Professor, Earthquake Engineering Research Centre, University of Iceland, rajesh@hi.is*

## Abstract

In this paper theoretical ground motion prediction model is applied to data from four of the largest earthquakes that occured in South Iceland from 1987-2008. The model is based on the Brune point source model that has been extended by using an exponential function to represent spectral decay. A closed form reprentation for both PGA and response spectra can be obtained by applying Parseval's theorem and carrying out the integration using a frequency domain representation. Peak factors relating rms and peak values are then applied. The model is presented as a GMPE for South-Iceland that can either give PGA or spectral acceleration as a function of distance from fault and seismic moment. Most of the model parameters are estimated by applying optimization in the frequency domain using ground motion records. Applying the stochastic method the theoretical model is used for simulating ground motion records for the four largest earthquakes in South Iceland from1987-2008. Here the near- and far-field equations for the ground motion and response are put foreward. Then the models are applied to the equations and an example given for the attenuation of ground motion for the earthquake on 21st June 2000. A comparison is made with empirical ground motion models based on European data that are commonly used for seismic hazard studies. An example of applying point source models to estimate response spectra for sites close to the earthquake fault and comparing the estimates obtained using near- and far-field models with the average response obtained from strong-motion records obtained during the earthquake.

*Keywords: GMPE, Source parameters, Simulated earthquakes, Brune's model*

## 1. Introduction

In this paper we will put forward a point source model as a GMPE both in the near- and far-field. The model is based on using Parseval's theorem to calculate rms-acceleration, $a_{\mathrm{rms}}$, based on the theoretical spectral representation in the form of Brune's model [1]. We apply a peak factor for linking rms-acceleration and PGA. The resultant model is then defined in terms of source model parameters, duration function and geometric attenuation. After defining the model and determining the parameters parameter, the model can be used to obtain PGA and for simulating ground-motion records using the stochastic method [2]. Point source models for simulation of ground motion are used in current practice and there are programs (for example SMISM, FINSIM, EXSIM) available on the internet (see for example [2], [3]) and these programs also apply a geometric attenuation function. In this paper the application Brune's point source model is presented with minor variations. The solution process is simplified by solving the equations analytically to obtain rms-acceleration and then applying a random vibration type of approach, using a peak factor for relating $a_{\mathrm{rms}}$ to PGA. A hybrid type approach will be applied where we use the near- and far-field models from Brune. It is emphasized here that the point source model can in fact be presented as a theoretical type of GMPE in contrast to the empirical model (see our first article on the subject from 1999 [4]). For a discrete time series approach to the simulation process see [5].

In this article we will apply the point source model to describe strong-motion acceleration records from four largest earthquakes in South Iceland from1987-2008. These are also the four largest earthquakes which triggered the Icelandic Strong Motion Network (IceSMN) and valuable strong-motion records were obtained. The earthquakes are as follows: 25 May 1987 $M_w$ 5.8, 17 June 2000 $M_w$ 6.5, 21 June 2000 $M_w$ 6.4 and 29 May 2008 $M_w$ 6.3. The source parameters for these earthquakes have been estimated (see for example [9]).

## 2. Point source ground motion model

### 2.1 Brune's far-field model

The following two theoretical models are put forward to describe the part of the ground-motion acceleration records that represent S-waves that have travelled near-vertically from below to the measurement station. The main motion is in the horizontal direction. Therefore, the term SH-wave is applied and is what we are modelling with the point source model. A large portion of the energy in the seismic waves at distances less than about 100 km is contained in these wave. Therefor we focus on the signal from the two horizontal components of the ground acceleration measured at each station. The model we apply is formulated in the frequency domain and is named the Brune's model. Brune formulated two models; one for the far field and the other for the near field. The point source model is Brune's far-field model which is written here as an acceleration amplitude spectrum with an exponential term added to account for the high frequency attenuation:

$$\left|A_F(\omega)\right| = \frac{2C_P R_{\theta\phi} M_0}{4\pi\beta^3\rho R}\frac{\omega^2}{(1+(\omega/\omega_c)^2)}\exp(-\tfrac{1}{2}\kappa\omega) \quad (1)$$

Here $C_p = 0.7$ is the factor that accounts for the partition of the energy between two horizontal components, $R_{\theta\phi}$ is the S-wave radiation pattern (average is 0.55), $\beta$ is the shear wave velocity, and $\rho$ is the density of the crust. Anderson and Hough (1984) [8] found it was possible to describe strong-motion acceleration with a frequency independent $Q$ in shallow crust and a spectral decay parameter $\kappa=R/Q\beta$. Here $\omega_c$ is the corner frequency defined as $f_c = \omega_c/2\pi$ [ 1]. The radius of the dislocation, where $r$ is related to the corner frequency as follows according to Brune:

$$r = 2.34\cdot\beta/\omega_c \quad (2)$$

The stress drop is given as follows

$$\Delta\sigma = \frac{(7/16)M_0}{r^3} \quad (3)$$

Eqs. (2) and (3) can be combined to obtain:

$$\omega_c = 2.34(16/7)^{1/3}\beta\left\{\frac{\Delta\sigma}{M_0}\right\}^{1/3} \quad (4)$$

The far-field model described here above has been applied to strong ground motion records obtained in the four largest earthquakes in South Iceland during from 1987-2008 in order to obtain the source parameters ([9]).

### 2.2 Brune's near-field model

Brune's near-field model is described as follows [1]:

$$A_N(\omega) = \frac{7}{8}\frac{C_p M_o}{\rho\ \ \beta\ \ r^3}\frac{\omega}{\sqrt{\omega^2+\tau^{-2}}}\exp(-\tfrac{1}{2}\kappa_o\omega) \tag{5}$$

The term $\tau$ is the rise time (assumed here that $\tau = \omega_c^{-1}$, $\kappa_0$ is the near-field spectral decay parameter. Ólafsson and Sigbjörnsson [10] have used a hybrid ground motion modeling approach applying the near-field model to records obtained close to the fault.

### 2.3 Geometrical spreading function

In order to model the ground motion in the near field it may be necessary to apply a geometric attenuation term in the form of the exponent $n$ = 2 in the near field. The following expression is suggested for the geometrical spreading function [11]:

$$R = \begin{cases} D_2^{1-n}\, D^n & D_1 < D \le D_2 \\ D & D_2 < D \le D_3 \end{cases} \tag{6}$$

where $1 < n \le 2$ and $D$ is a distance defined as:

$$D = \sqrt{d^2 + h^2}$$

Here, $d$ is the epicentral distance and $h$ is a depth parameter. The parameters $D_1$, $D_2$ and $D_3$ are used to set the limits for the different zones of the spreading function. The parameter $n$ is usually in the range of 1 to 2 but this study assumes that $n$ = 2 for $D < D_2$ and that $n$ = 1 for $D$ greater than $D_2$ and less than $D_3$ (taken here as 100 km).

### 2.4 Duration

Duration is a necessary parameter in the ground motion models that are described in this paper. The functional form used to approximate duration is:

$$T_d = c_1\frac{r}{\beta} + c_2 d^{c_3} \tag{7}$$

where $c_1$, $c_2$ and $c_3$ are magnitude dependent parameters.

## 3. Far-field models

### 3.1 Far-field ground motion model

By applying Parseval's theorem a solution can be obtained for the $a_{\text{rms}}$ in terms of the source parameters (see [11]) by applying symbolic integration. The result is the following expression:

$$a_{rms} = \frac{0.85 R_{\theta\phi} C_p \Delta\sigma^{2/3} M_o^{\ 1/3} \Psi^{1/2}}{\beta \rho \sqrt{\kappa T_d}\, R} \tag{8}$$

The new term here, $\Psi$, is a result of the integration, and the parameter, $\lambda$, is defined as $\lambda = \kappa\omega_c$ and is written in terms of sine-integrals as follows (*ci* is cosine integral and *si* sine integral):

$$\Psi = 1 - \frac{1}{2}\lambda ci(\lambda)\left(\lambda\cos(\lambda) + 3\sin(\lambda)\right) - \frac{1}{2}\lambda si(\lambda)\left(\lambda\sin(\lambda) - 3\cos(\lambda)\right) \tag{9}$$

It has been shown in [6] that $\Psi$ can be approximated with a simpler equation in the form of an exponential function.

$$\Psi = \exp(-1.5(\kappa\omega_c)^{0.87}) \tag{10}$$

Peak ground accelearation can be obtained from the root-mean-squared acceleration, $a_{\mathrm{rms}}$, by using a peak-factor as follows [12]:

$$PGA = p \cdot a_{rms} \tag{11}$$

A factor of $p$ = 3 has been found to give good results for Icelandic strong motion records and is assumed in this paper.

### 3.2 Far-field response model

It has been shown in [6] and [11] that the response of a second order system with an undamped natural frequency of $\omega_0$ can also be written in closed form as a function of the source parameters. Application of the above described models lead to the following expression after the integration has been carried out:

$$x_{rms}(t) \approx \frac{1}{\omega_o^2}\sqrt{I_F + \frac{1}{\pi T_d}\left|A_F(\omega_o)\right|^2\left(\frac{\pi\omega_o}{4\zeta} - 1\right)} \tag{12}$$

where $\omega_o$ denotes the undamped natural frequency, $\zeta$ is the critical damping ratio, $T_d$ is the duration $\left|A_F\right|$ is given by Eq. (1) and the term $I_F$ is given as follows:

$$I_F = \frac{1}{\pi}\left(\frac{7}{16}\right)^{2/3}\left(\frac{C_P \left\langle R_{\theta\varphi}\right\rangle \Delta\sigma^{2/3}}{\beta\rho R}\right)^2 \frac{\Psi}{T_d \kappa} M_0^{2/3} \tag{13}$$

where $\Psi$ is the far-field function described by Eq. (9) or (10). The peak response is similarly obtained from the rms-response using a peak factor of $p$ = 3.

## 4. Near-field models

The model described in the previous section is not valid in the near-field and can, therefore, not be expected to accurately describe the response close to the fault. To obtain an approximation that is valid for shear waves in the near-fault area the Brune near-field model can be used. Hence, the near-field acceleration spectrum is approximated as given in Eq. (5), after modifying the high frequency part with an exponential term and accounting for the free surface and partitioning of the energy into two horizontal components.

### 4.1 Near-field ground motion model

Based on the Brune near-field source model the $a_{\mathrm{rms}}$ ground motion model can be represented by the following equation:

$$a_{rms} = \frac{2}{\sqrt{\pi}} \frac{C_p \Delta\sigma}{\rho\beta\sqrt{\kappa_0}} \sqrt{\frac{\Psi_0}{T_0}} \tag{14}$$

Here $T_0$ is the source duration defined as follows (see [13]):

$$T_0 = \frac{0.6}{f_c} \tag{15}$$

The near-field spectral decay parameter is represented by $\kappa_0$, and here is 0.042 s for South Iceland [9].

The parameter $\Psi_0$ can be represented by the following function fit to the resulting sine-integral function for the near-field (see [7]):

$$\Psi_0 = \exp(-1.1(\kappa_0\omega_c)^{0.92}) \tag{16}$$

### 4.2 Near-field response model

The following expression is obtained after the integration has been carried out (see reference [10]):

$$x_{rms}(t) \approx \frac{1}{\omega_0^2} \sqrt{I_N + \frac{1}{\pi T_0} \left| A_N(\omega_0) \right|^2 \left( \frac{\pi\omega_0}{4\zeta} - 1 \right)} \tag{17}$$

where $\omega_o$ denotes the undamped natural period, $\zeta$ is the critical damping ratio, $T_0$ is the source duration, and $\left| A_N \right|$ is given by Eq. (5) and

$$I_N = \frac{1}{\pi} \left( \frac{7}{8} \frac{C_p}{\rho\beta r^3} \right)^2 \frac{\Psi_o}{T_o \kappa_o} M_o^2 \tag{18}$$

Here, the source duration is denoted by $T_0$ and $\Psi_0$ is a dispersion function given in Eq. (16)

## 5. Results

### 5.1 Ground motion prediction models

The near- and far-fault point source ground motion models introduced in sections 3 and 4 have been applied to modelling of PGA from strong-motion records obtained from the IceSMN in South-Iceland from the four largest earthquakes recorded in South Iceland from 1987-2008. The models use the source parameters $\omega_c$, $M_0$, $\kappa$ and $\kappa_0$ estimated from the strong-motion acceleration records obtained in the four earthquakes. In addition duration represented by the parameter, $T_d$, (Eq. (7)) is necessary.

If a hybrid modelling approach is used the near-field duration $T_0$ and near field kappa, $\kappa_0$ are also necessary parameters. For the larger and shallower earthquakes a geometric spreading function $R$ (Eq. (6)) is applied, if necessary, where R ~ $D^{-2}$ closer to the fault. We applied both type of models i.e. with a geometric spreading function applied with values of $D_2$ = 20 km for $M_w$ = 6, and $D_2$ = 30 km for $M_w$ = 6.5. The lower bound is taken as $d = r$ (radius of dislocation). The alternative is proportional to $R = D$ out to distance $D_3$ = 100 km. Most of the records in the four earthquakes are obtained within100 km from the epicenter.

The models were applied using the source parameters that have been estimated for the four earthquakes. For an account of the estimation of the source parameters, see [9]. The estimated stress drop for the larger events was approximately 80 bar. The spectral decay parameter were estimated as $\kappa$ = 0.045 s and $\kappa_0$ = 0.042 s. The near source model, Eq. (14), was used to determine the PGA close to the epicenter, called $PGA_0$ in Table 1. The estimated values were consistent with PGA values obtained from records in the near-fault regions.

As an example the results for the modelling of the earthquake at Hestfjall on 21st June 2000 ($M_0 = 41 \text{x} 10^{24}$ dyne cm or $M_w$ 6.4 with Hanks-Kanamori relation [14]) are shown in Fig. 1 and in Table 1 which shows the duration model parameters $c_1$, $c_2$, $c_3$, the near-fault $PGA_0$, and source duration, $T_0$. Here the far-field point source model of Eq. (8) is applied with R ~ $D^n$ with $n$ = 1 for all distances less than 100 km. The model (here called O&R 2016 model) is shown in Fig. 1 for the estimated stress drop $\Delta\sigma$ = 83 bar (black line and triangles) and $\Delta\sigma$ = 50 bar (dot-line). The GMPE's of Ambraseys et al. 2005 [15]. and Akkar et al. 2014 [16] are also shown. It can be seen from Fig.1 that the Akkar et al. 2014 model, which is here approximated by the point source model with $\Delta\sigma$ = 83 bar, gives values of PGA close to the fault that are better agreement with what the PGA values obtained by from the recorded strong-motion. The Ambraseys model which is here similar to the point source model with a stress drop of $\Delta\sigma$ = 50 bar gives values that are too low. This trend has also been observed from some of the GMPE's in the literature based on data from other regions. This can be expected if the models are developed based on earthquakes with stress drop that are on average lower than 80-100 bar.

In general the Akkar et al. 2014 model gives a good approximation to the PGA close to fault of the four earthquakes from South Iceland. There is also a correspondence with the PGA values predicted by the near-fault point source model of Eq. (14). It can, however, be observed from Figs. 1 and 2 that the Akkar et al. 2014 model does not follow the slope of the attenuation function particularly well and for example predicts to high value further removed from the fault. The theoretical point source model (black line in Fig. 2) is an attempt to follow more closely the slope of the attenuation and also capture the steeper rate of attenuation close to the fault using a geometrical spreading function that proportional to $D^2$ in the region where we have near-fault effects that we approximate as $D_2$ = 30 km for a $M_w$ 6.5 earthquake and $D_2$ = 20 km for $M_w$ = 6. The magnitude dependent duration function can also be adjusted to account for the regionally dependent rate of attenuation.

Table 1 – Estimated parameters for earthquake 21.06.2000 $M_w$ 6.4 applying different models. See curves plotted in Figs. 1 and 2.

| **Model** | $c_1$ | $c_2$ | $c_3$ | **$PGA_0$(g)** | **$T_0$ (s)** |
|---|---|---|---|---|---|
| O&R 2016 ($R \sim D^{-1}$ if $D < 100$ km) | 0.23 | 0.023 | 1.16 | 0.56 | 0.41 |
| O&R 2016 ($R \sim D^{-2}$ $D_2 < 30$ km) | 1.1 | 0.008 | 1.78 | 0.57 | 1.88 |
| Near-Source model ($\Delta\sigma$ = 83 bar) | | | | 0.58 | 2.76 |
| Near-Source model ($\Delta\sigma$ = 50 bar) | | | | 0.34 | |
| A&B 2014 | | | | 0.57 | |
| Ambraseys et al.2005 | | | | 0.34 | |

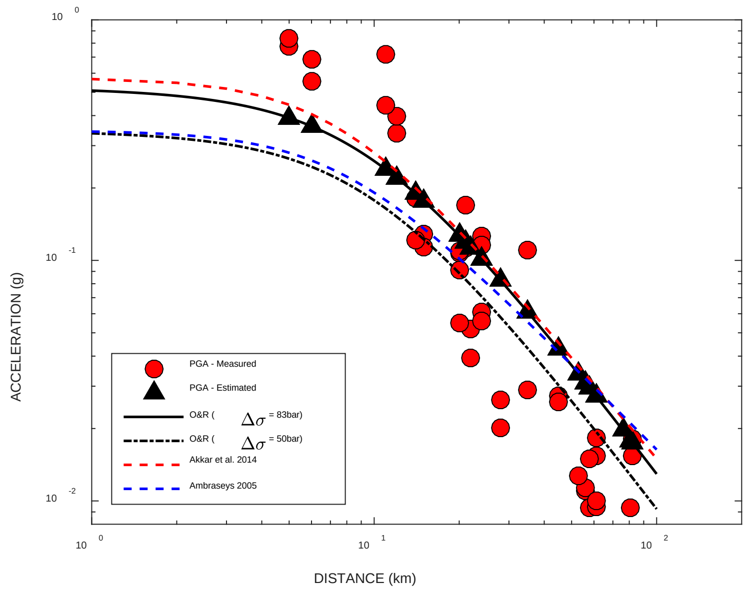


Fig. 1 – Point source model of Eq. (8), ($R$ = $D$ for all $D$ < 100km) applied to earthquake PGA values of horizontal acceleration from earthquake of 21st June 2000 ($M_w$ 6.4) shown here for stress drop $\Delta\sigma$ = 83 bar (black line and triangles) and $\Delta\sigma$ = 50 bar (dot-dash). Also shown are the models of Akkar et al. 2014 and Ambraseys et al. 2005 for $M_w$ 6.4

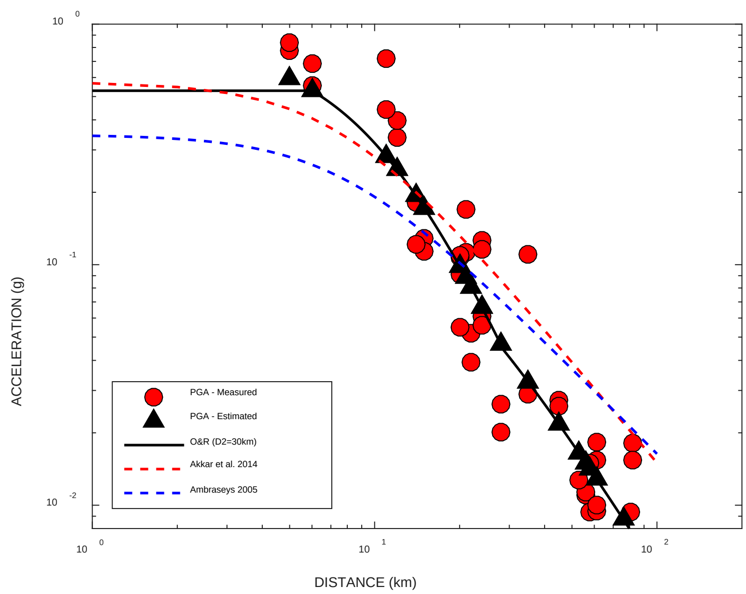


Fig. 2 – Point source model of Eq. (8) (with $R \sim D^2$ for all $D < D_2$ = 30 km) applied to earthquake PGA values of horizontal acceleration from earthquake of 21st June 2000 ($M_w$ 6.4). Also shown are the models of Akkar et al. 2014 and Ambraseys et al. 2005 for $M_w$ 6.4

5.2 Response spectra base on a point-source model

In Fig. 3 response spectra based on the near- and far field Eqs. (17) and (12) are shown as estimated for a site at 5 km distance from the fault of the earthquake in Hestfjall 21st June 2000 $M_w$ 6.4 earthquake (black line and blue dash-dot line respectively). The red dashed line represents the average response spectrum based on the two horizontal components measured at the station Thjorsa Bridge west pillar. The near-fault model is shown to fit the data far better at higher frequencies.

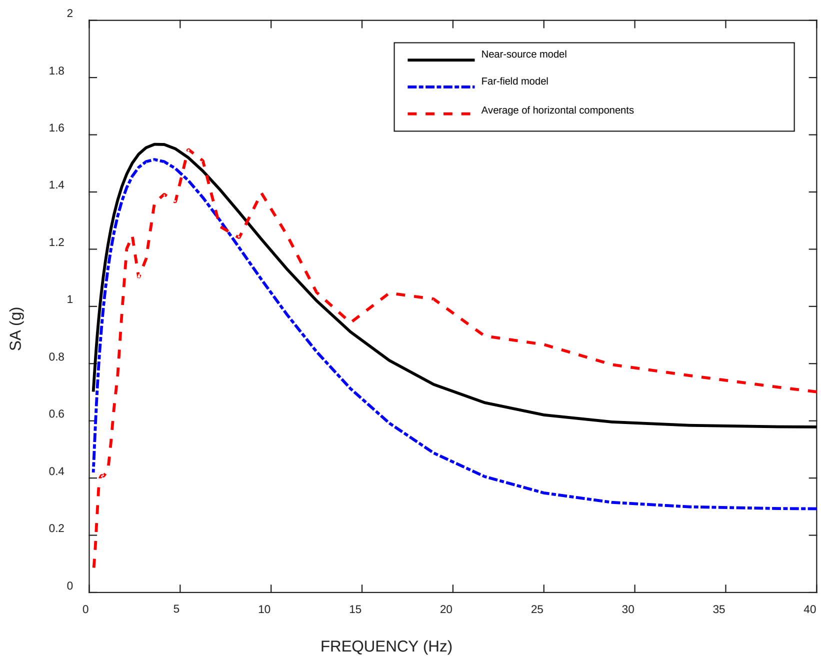


Fig. 3 – Comparison of the mean response spectrum for horizontal of ground motion at Thjorsa Bridge in earthquakes of 21st June 2000, $M_w$ 6.4 (red dashed line) and estimated response spectra with near field model (Eq. (17)) and far field model (Eq. (12)).

## 6. Discussion and conclusions

In this paper the application of a point source model to strong-motion acceleration records from the four largest earthquakes in South Iceland, are demonstrated. The model is presented as an alternative to the more traditional empirical regression type GMPE's. The model is based on the Brune source model with an exponential term to account for spectral decay at higher frequencies. Applying Parseval's theorem the root-mean-squared acceleration, $a_{rms}$ , can be written in closed form in terms of the parameters of the Brune's model and a residual term that can be approximated with an exponential function. A peak-factor is then used to obtain PGA. Ground motion model base on Brune's near- and far-field model are presented. Similarly the equations for the near- and farfield Brune's model are presented and in addition to models estimating PGA then theoretical models for response spectra are presented in closed form.

The source parameters estimated from the strong-motion records are used as input into the point source models. A comparison is made of the attenuation curves obtained using the point source models with PGA from recorded data from the four earthquakes The results are also compared with two GMPE's based on regression analysis using European strong motion records. Furthermore, an example of applying the theoretical near-and far-field response equations are presented. The near-field model is found to provide considerably better estimate for the case considered.

The near-field equation Eq. (14) is found to provide a good estimate of PGA close to the fault for all the four earthquakes (demonstrated for 21st June 2000, $M_w$ 6.4 earthquake). It also turns out that Akkar et al. 2014 gives PGA values in the near-source área that are very close to the values given by the near-source theoretical point source model (Eq. (14)). The point source model with an 83 bar stress drop (estimated for the larger South Iceland earthquakes) gives samilar values as the Akkar, et al. 2014 model. The Ambraseys et al. 2005 model fits well with the same point source model, but with a stress drop of 50 bar.

Similar to how closed form equations are arrived at for a point source based GMPE for PGA, equations in closed form are obtained for response spectra in the near- and far field. A demonstration is presented for the application to near-fault records obtained in the 21st June 2000, $M_w$ 6.4 earthquake and the near-field model is found to give an excellent fit to the average spectrum from the two horizontal components. But this is only one point and to assess the quality of the model a test has to be made of the fit at other distances and a test of the residuals.

The examples presented in this paper demonstrate the advantage of theoretically based GMPE's where the model parameters have physical meaning. This is important where few recorded ground motion records are available The compact closed form equations will also considerbly simplify the modelling procedure.

## 7. Acknowledgements

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