Parametric Simulation of Multi-level Rigid Blocks under Horizontal Excitations

N.K. Hong[(1)], S.G. Hong[(2)]

[(1)] *Adjunct Professor, Seoul National University, namheek@snu.ac.kr*
[(2)] *Professor, Seoul National University, sglhong@snu.ac.kr*

Abstract

The construction of stone pagodas had been changed depending on design trend and material accessibility from low-rise to high-rise while keeping three major parts consisting of (1) upper part; (2) body part; and (3) base part. Each part takes on different function from both structural and architectural aspects. In order for such stone pagodas to represent their uniqueness by their geometric shapes, their deviation from symmetric and regular shapes has been pronounced. The problems related with the geometric deviation are apparent in the case of pagodas that greatly varied course size and pattern over heights. Therefore the effect of the geometric irregularity must be considered in the development of seismic vulnerability assessment of stone pagodas under horizontal excitations. The purpose of this study is to investigate the seismic performance of stone pagodas with vertical geometric irregular shapes by performing numerical analysis using the discrete element method for multi-level rigid blocks. First, major parameters that seem to affect seismic performance dominantly are identified based on the overview of Korean stone pagodas. And a baseline study is done for a single block to provide starting values for the numerical analysis for multi-level rigid blocks. Specifically, the frequency associated parameter and its related stiffness values are set through the baseline study by using Housner's simple rocking model that does not consider sliding motion. Upon the starting values, eigenvalue analysis is done using the DEM analysis by varying the size and the block segmentation. The main purpose of the eigenvalue analysis is to investigate the distribution of eigenvalues, which might have a relationship with motion changes of stone pagodas. Then, the parametric simulation has been performed for two-level block models to understand the vertical geometric irregularity in relation to the seismic performance under horizontal excitation. Five different cases including typical proportions of Korean stone pagodas and one case without vertical irregularity are simulated for numerical simulation. The numerical results of the parametric simulation have shown that the vertical geometric irregularity has much relevancy to the seismic performance. It is expected that this study will provide a starting point to consider the irregularity of stone structures under earthquake loadings.

Keywords: Stone pagoda, multi-level block, eigenvalue, vertical irregularity, seismic performance

1. Introduction

Earthquakes are a major destructive source that makes threats against cultural heritage structures. Especially, the 1906 San Francisco earthquake showed various damage patterns about stone structures including marble statues. In Korea, there are over 1,500 stone structures including stone or brick pagodas, stone bridges, stone caves and icehouses. And a great portion of the stone structures are stone pagodas. When an earthquake of magnitude 5.0 occurred in 1936, a five story stone pagoda of Sanggyesa was damaged. Nowadays, some researches have been directed to develop preventive measures appropriate for stone pagodas at government level [1]. In this relation, an overall understanding of seismic behavior of stone pagodas is needed to classify them considering the seismic vulnerability. However, it is very complicated to study the dynamic behavior of stone pagodas because there are numerous uncertainties from the level of material to a whole system.

In order to address the complexity of structural behavior of stone structures, the multi-level evaluation process has been proposed by the authors in reference and is still being refined. By adopting this process, the seismic performance of stone pagodas can be roughly evaluated at three different levels: (1) system; (2) sub-system; and (3) component. In the previous study [2], Housner's simple rocking model was used to evaluate overall rocking behavior of Korean pagodas in terms of a single rigid block. Housner's simple rocking model [3] that was developed without considering sliding motion provides an overall view for dynamic stability of relatively slender towers. Also, the sliding motion was also considered by establishing the relationship between the frictional coefficient with ground acceleration for a single block. In fact the overall stability for both rocking and sliding has been investigaed based on the limit theorem. Note that the stone pagodas that are strong in compression but very weak in tension carry their loads through thrust lines which must be provided inside the structural members to secure its safety. In this relation, the concept of limit theorem helps establish the possible ranges for the load path forming along the thrust lines. Upon the previous study, this study has focused on the study of seismic performanc of multi-story pagodas using the discrete element method (DEM) [4] to address the following issues: (1) the effects of vertical irregularity; (2) the effects of block segmentation of base part; and (3) the damage patterns for slender pagodas.

2. Major Parameters of Stone Pagodas

A pagoda generally refers to a tiered tower with multiple eaves common in many Asian countries. Early pagodas in Korea were constructed out of wood, but had been changed gradually to brick or stone ones for better protection against rot and fires. A considerable amount of such pagodas has been preserved and some of them were nominated as national treasures. Stone pagodas in Korea that were constructed using the dry construction method without mortar are uniquely identified in their shapes. Their shapes had been changed depending on design trend and material accessibility from low-rise to high-rise while keeping three major parts consisting of (1) top part; (2) body part; and (3) base part, as shown in Fig. 1. Each part takes on different function from both structural and architectural aspects and its design patterns are varied. Through the investigation of stone pagodas in relation with the seismic performance, some major factors are identified including the overall dimension, the geometric shapes, the degree of vertical irregularity, the material properties, and the stacking method. Then, a hierarchy for the multi-level seismic evaluation has been developed by using the dominant factors as criteria for the classification for the numerical analysis in a systematic way. Fig. 2 shows the hierarchy and the criteria are represented in terms of parametric variables whose values are simulated for the numerical simulation using the discrete element method.

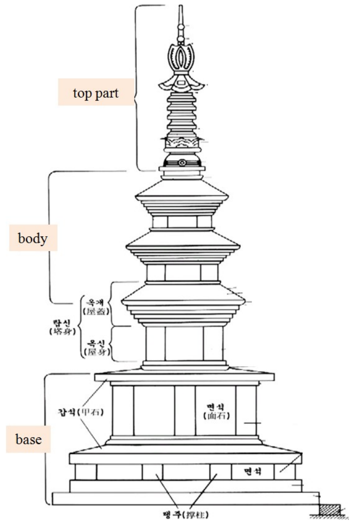


Fig. 1 – Basic components of Korean stone pagodas.

3. Baseline Study

3.1 Dynamic behavior of rocking model

Recently, many researches have shown that the DEM is effective to understand the seismic performance of masonry structures under horizontal excitations. The DEM enables us to observe the evolution of damage patterns considering both sliding and rocking. On the other hand, they also have raised the difficulty in determining material and physical properties because they behave very differently depending on the size and kind of material of each unit and on the pattern of the assemblage of the units. What is even worse is that such properties are very sensitive to the numerical response.

Prior to the DEM analysis, some major parameters that were considered in the Housner's simple rocking model (the leftmost figure in Table 1) have been studied. One of them is the frequency associated parameter which is defined as:

$$p = \sqrt{\frac{mgR}{I}} \qquad (1)$$

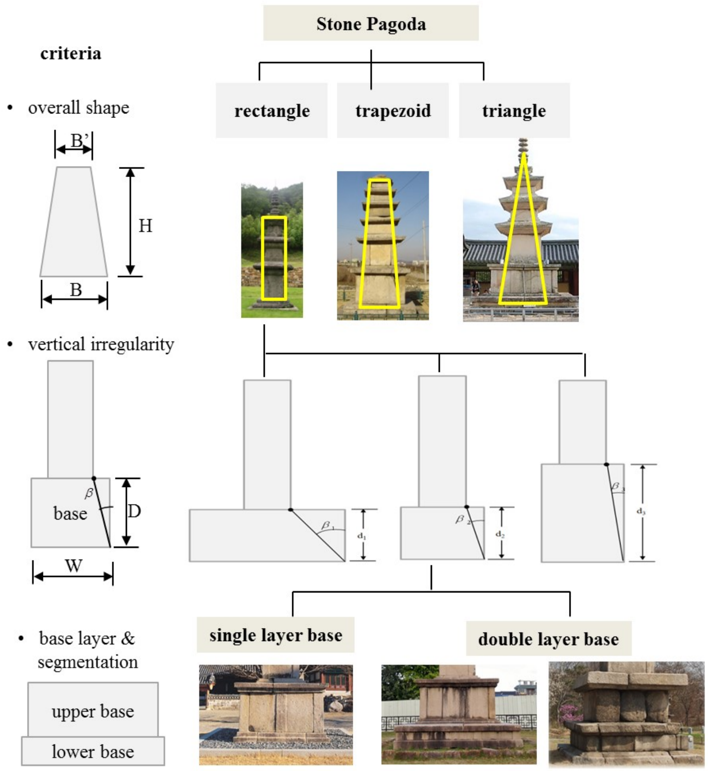
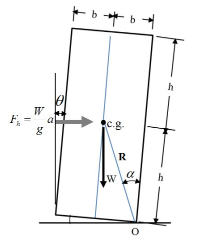
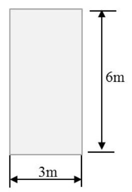
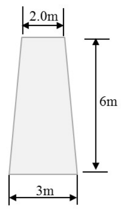
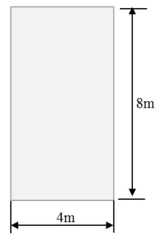


Fig. 2 – Seismic performance parameters of stone pagodas.

where m is the mass, g the acceleration of gravity, I the moment of inertia about the axis of rotation $I = \left(\frac{4}{3}\right)MR^2$, and R is defined as $R = \sqrt{b^2 + h^2}$. For rectangular blocks p^2 reduces to $3g/4R$. The frequency associated parameter is varied depending on the block size even with the same geometric proportion.

Another important parameter related to the overturning of a slender block is the geometric angle of the block which is defined as $\alpha = \tan^{-1}\left(\frac{b}{h}\right)$. This geometric block angle is termed as slenderness angle in this study. It is noted that the slenderness angle is totally dependent on the geometric aspect ratio. However, the slenderness angle does not differentiate scaled blocks at all. That is, the block angle is the same for all different blocks of the same geometric proportion. As seen in Table 1, the slenderness angle of the block of 3m x 6m is the same as of 4m x 8m. Therefore, it should be noted that the slenderness angle must be used as a part of measure for the rocking stability for a single block.

Table 1 – Major parameters of rigid block analysis

			
p (rad/sec)	1.481		1.283
slenderness angle (rad/degree)	0.464/ 26.579		0.464/ 26.579

3.2 Dynamic behavior of DEM model

The DEM has been adopted in this study to study seismic response of stone pagodas in terms of an assemblage of rigid blocks. There are two types of polyhedral block that can be modeled by 3DEC: rigid blocks, which have six degrees-of-freedom (three translational and three rotational); and deformable blocks, which are subdivided internally into tetrahedral that have three translational degrees of freedom at each vertex (or node). For the DEM analysis, it is important to establish appropriate values for some parameters including weight density, axial and shear stiffness of block elements, joint material properties, and damping parameters. However, it is not easy to obtain those values without experimental tests. As a first step, the values for the frequency associated parameter for the rectangular blocks in Table 1 were used as reference targets to tune the other properties except the geometric values. The material properties that were selected as appropriate for further numerical analysis are provided in Table 2.

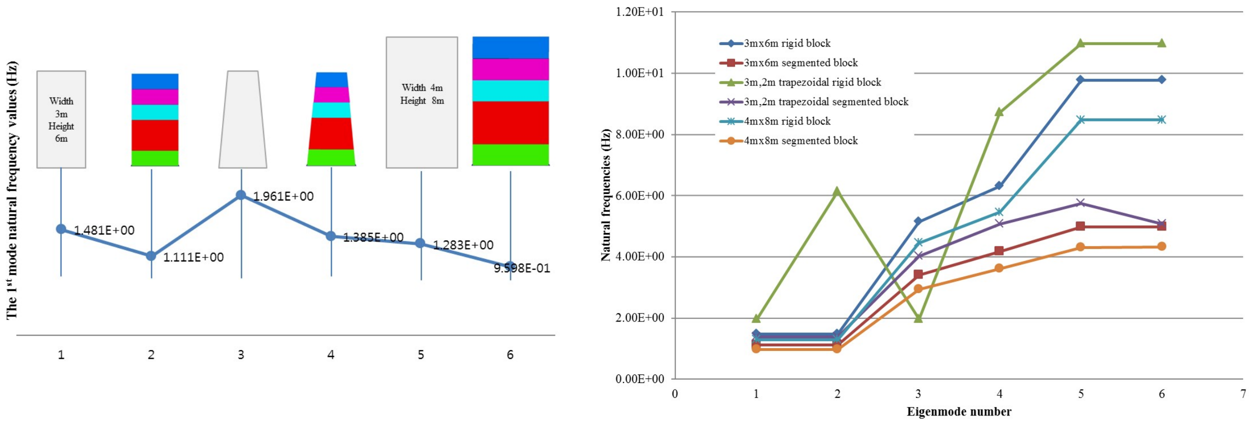
Table 2 – Material properties for the DEM analysis

Weight density	Normal stiffness	Shear stiffness	Friction angle	Tensile strength
2,650 kg	1.66e7 Pa/m	1.66e7 Pa/m	30°	0 MPa

Upon setting the material values, the same block models of Table 1 are simulated for the parametric simulation by changing geometric shapes and the block segmentation. Specifically an eigenvalue analysis has been done using the DEM analysis to obtain the natural frequency values for the following 6 eigenmodes:

Mode 1-1st bending mode
Mode 2- mode shape orthogonal to mode 1
Mode 3-2nd bending mode
Mode 4- mode shape orthogonal to 2nd bending mode
Mode 5-1st torsional mode
Mode 6- 3rd bending mode

Fig. 3 shows the results that were obtained of using an eigenvalue analysis for the DEM models. It is noted that the frequency values for the two rectangular blocks are the same as those obtained based on the Housner's simple rocking model as intended. It is also understood that the natural frequency values are varied depending on the shape, size and segmentation. The reduction of mass increases frequency values while the increase of the size and the segmentation reduces frequency values. Moreover, the distribution of higher frequencies is sensitive to rigid blocks. That is, the contribution of dominant frequencies will be much for the rigid blocks while the contribution of higher modes will be much for the segmented blocks.

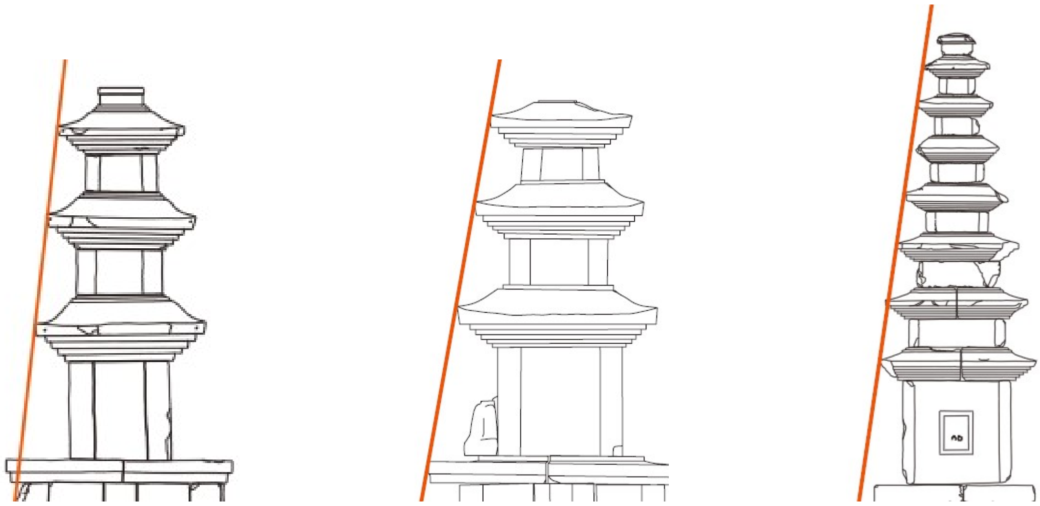


a. for the first modes b. for 6 eigenmodes

Fig. 3 – Natural frequency values obtained from the DEM analysis

4. Multi-level Rigid Block Models

The structural silhouette of stone pagodas had been designed to satisfy the two important aspects of stability and elegancy harmoniously. Fig. 4 shows some example that describes the slopes of visual inclined lines. It is noted that the geometric proportion of each part has been kept within certain ranges of the slope angle for a whole structure.



a. slope angle of 83˚ a. slope angle of 80˚ a. slope angle of 82˚

Fig. 4 – Slope of visual inclined lines of Korean stone pagodas

4.1 Natural frequency values of two-block model

To understand the effect of the vertical geometric irregularity of Korean stone pagodas, two-block models are simulated using the DEM analysis. Based on the review of existing multi-story stone pagodas in Korea, five cases have been selected for the parametric simulation. The detailed dimensions and the total weight of each model are provided in Table 3. The slope angles of visual lines are ranged in certain ranges between 80˚ and 83˚ while the variation of base dimensions is made to be appropriate for 3 story stone pagodas. Note that the case 4 with no vertical irregularity is also included in the parametric simulation for the comparison of the analysis results. Moreover, the two-block models are further refined by varying the degree of the segmentation from rigid block to the 1st segmentation and to the 2nd segmentation, as shown in Table 4.

Table 3 – Description of two-block models for the parametric simulation

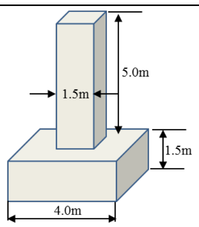
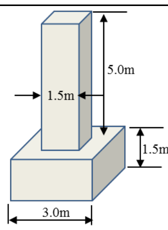
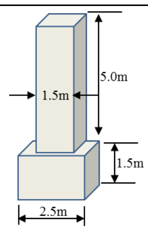
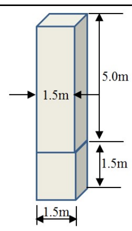
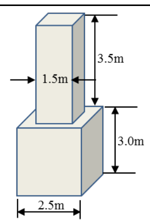
	Case 1	Case 2	Case 3	Case 4	Case 5
Dimen sions					
W_{total} (kg)	93,413	6,5588	54,656	38,756	70,556

Table 4 – Two-block models with different segmentations

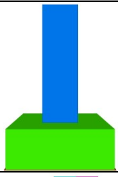
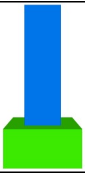
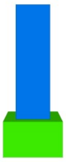
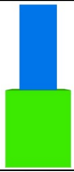
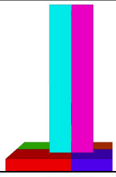
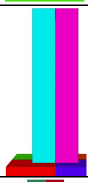
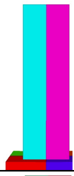
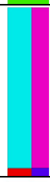
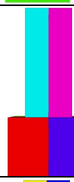
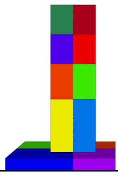
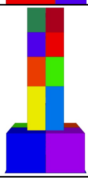
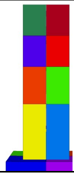
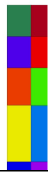
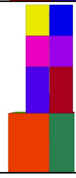
	Case 1	Case 2	Case 3	Case 4	Case 5
rigid block					
1st segmentation					
2nd segmentation					

Table 5 provides the natural frequency values for the five cases that are obtained upon the eigenvalue analysis using the DEM analysis.

Table 5 – Natural frequency values

	Without segmentation				
	Case 1	**Case 2**	**Case 3**	**Case 4**	**Case 5**
1st	**8.639e-01**	**8.363e-01**	**8.087e-01**	**5.721e-01**	**1.205e+00**
2nd	8.639e-01	8.363e-01	8.311e-01	5.721e-01	1.287e+00
3rd	4.796e+00	4.582e+00	4.401e+00	3.832e+00	3.671e+00
4th	5.919e+00	5.805e+00	5.765e+00	4.693e+00	4.124e+00
5th	7.268e+00	6.311e+00	5.900e+00	4.790e+00	4.934e+00
6th	7.268e+00	6.311e+00	6.422e+00	4.790e+00	6.797e+00
	With 1st segmentation				
1st	**7.029e-01**	**8.701e-01**	**7.221e-01**	**4.935e-01**	**9.576e-01**
2nd	7.190e-01	8.701e-01	7.414e-01	4.935e-01	1.014e+00
3rd	4.358e+00	4.857e+00	4.051e+00	3.545e+00	2.925e+00
4th	4.741e+00	5.677e+00	4.310e+00	3.877e+00	2.837e+00
5th	6.416e+00	5.699e+00	4.497e+00	3.982e+00	4.241e+00
6th	6.419e+00	5.677e+00	4.850e+00	3.982e+00	5.077e+00
	With 2nd segmentation				
1st	**6.400e-01**	**7.665e-01**	**6.538e-01**	**4.738e-01**	**9.231e-01**
2nd	6.401e-01	7.664e-01	6.676e-01	4.738e-01	9.720e-01
3rd	3.440e+00	3.416e+00	2.991e+00	2.581e+00	2.568e+00
4th	3.442e+00	3.422e+00	3.067e+00	2.580e+00	2.660e+00
5th	3.454e+00	3.836e+00	3.484e+00	2.976e+00	4.012e+00
6th	3.574e+00	3.938e+00	3.544e+00	3.382e+00	4.401e+00

Fig. 5 plots the first mode natural frequency value vs. the total weight for the five rigid block models. It can be understood that the geometric shape has something to do with its stiffness. Figs. 6, 7 and 8 show the first mode frequency values for the five cases with different segmentations. As the degree of segmentation increases, the natural frequency values become lower. Note that the eigenvalues of the case 5 are greatly deviated from those of the case 4 of regular system.

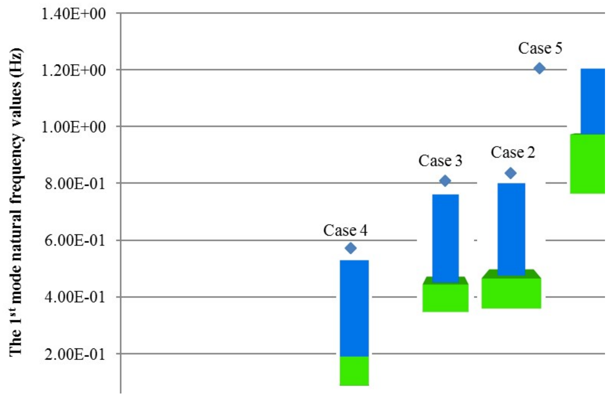


Fig. 5 – The first mode frequency value vs. total weight

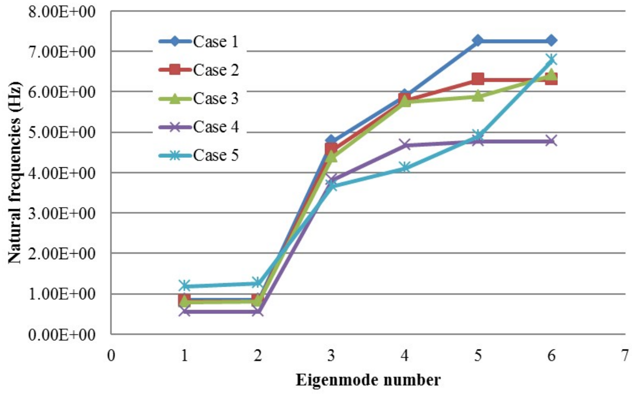


Fig. 6 – The first mode frequency values of rigid blocks

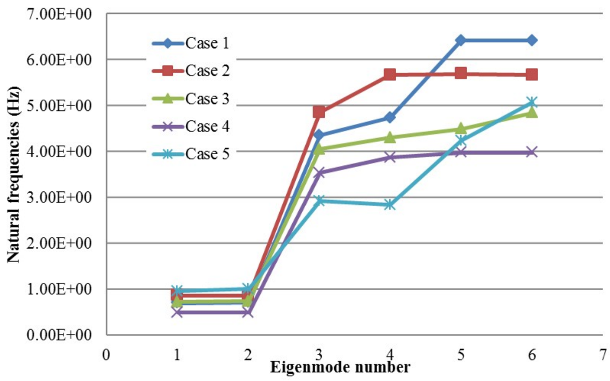


Fig. 7 – The first mode frequency values with 1st segmentation

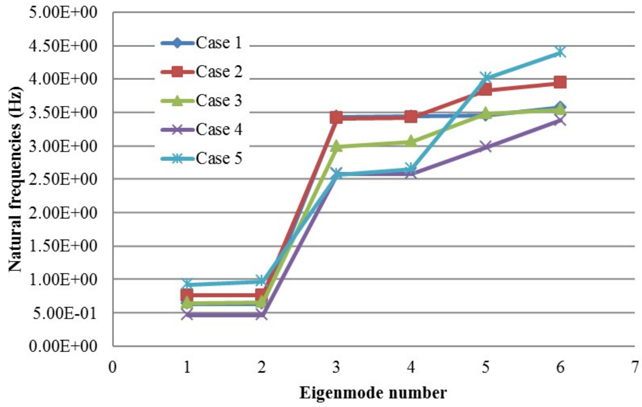


Fig. 8 – The first mode frequency value with 2nd segmentation

4.2 Dynamic behavior of two-block models

The dynamic response of Korean stone pagodas are investigated by plotting horizontal displacement versus time for the two-block model contacting on a rigid base under impulse with no damping. The required impact to initiate rocking is first obtained through an equivalent static analysis carried out on the monolithic structure subjected to a constant horizontal acceleration and the gravitational acceleration. The amplitude and the duration of the impulse are determined by repeating the dynamic analysis with incremental steps. Fig. 9 shows the horizontal displacements at the top level over the time along with the final failure modes. As expected for the structures with the geometric irregularity, the worst case regarding the top displacement is seen in Case 5 with much irregularity. However, it is not possible to conclude that the adverse effect is caused by the geometric vertical irregularity only. Note that there are many different types of irregularity including vertical geometric irregularity, vertical stiffness irregularity, mass irregularity, strength irregularity, and torsional irregularity.

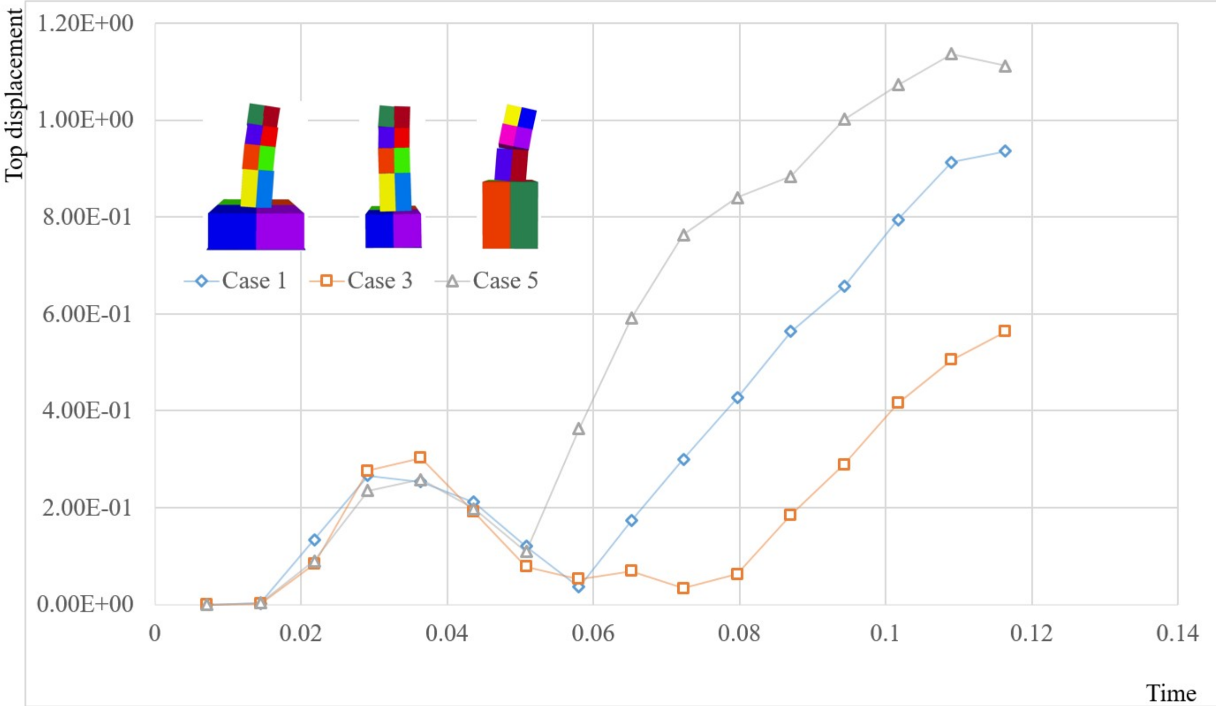


Fig. 9 – Dynamic Behavior of two-block models under impulse loading

5. Conclusions

Visual silhouettes of Korean stone pagodas are uniquely identified in many ways including the construction method, overall size, geometric shape. Among them, the problems related with the geometric deviation are apparent in the case of pagodas that greatly varied course size and pattern over heights. Therefore the effect of the geometric irregularity has been considered in this study toward the development of seismic vulnerability assessment of stone pagodas under horizontal excitations. To understand the effect of the vertical geometric irregularity of Korean stone pagodas, the parametric simulation was done for the two-block models using the DEM analysis. The effect of the vertical geometric irregularity was seen in the results of the parametric simulation. Even though this study deserves further study, it serves a starting point to consider the irregularity of stone structures under earthquake loadings.

Acknowledgements

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