



PROBABILISTIC CONNECTIVITY ANALYSIS FOR A ROAD NETWORK DUE TO SEISMICALLY-INDUCED DISRUPTIONS

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Abstract

In the immediate aftermath of a strong earthquake, the road networks play a crucial role in rescue and recovery operations. The damaged infrastructure may lose their transitability (fully or partially) leading to disruption of road links. The consequences in most cases go beyond the disruption of the road links. In fact, the disruption of a road link often influences the connectivity of the whole network. The connectivity (the connection between two specific points) is used herein in order to measure the post-event health status of a road network and to highlight its critical points.

Connectivity in its simplest form (from point A to point B) is used as an index of overall system performance. That is, the probability that the connectivity from point A to B is lost has been interpreted as a measure of system vulnerability (i.e., reduction in reliability). Moreover, the expected value of the number of alternative routes from point A to point B is a measure of system's redundancy or robustness. Defining the connectivity between two given points as a binary logical statement with logical values TRUE or FALSE, the network reliability is formulated as a standard link-set formulation. Such formulation, in the general case where the connectivity of the alternative routes are not independent cannot be easily solved; hence, a simulation based approach has been adopted.

In this work the disruption cause considered is due to an earthquake event and the network connectivity problem has been solved in a fully simulation-based manner. For a given earthquake scenario, a seismic intensity field has been generated taken into account spatial correlations in the residuals of the adopted ground motion prediction equation. On the other hand, also the seismic capacity of the vulnerable infrastructure given seismic intensity for a transitability limit state is simulated based on a joint probability distribution (considering the spatial correlations in the vulnerability of the infrastructure) and based on simplified working assumptions. Therefore, for each simulation, the ratio of the number of connected routes to total number of alternative routes is calculated. This results in information such as, the probability of the loss of connectivity between points A and B and the probability distribution (and statistics such as the expected value and the standard deviation) for the number of connected routes.

The entire methodology has been demonstrated as an application to a real case-study for the road network infrastructure in the Campania Region (Italy).

Keywords: road network connectivity, systemic reliability, seismic scenario, simulation-based reliability.



1. Introduction

In a Road Network (RN) vulnerability assessment, different disciplines are involved in order to define and solve the complex problems of network definition and resolution both from transport- and reliability-related points of view. The RNs are very complex system of roads, infrastructures, crossings, service structures, with a high grade of interconnections and dependences where, for each element, it is necessary to know a series of information related to the functionality (e.g. direction of travel, velocity, number of lanes, time of crossing intersection, traffic capacity, etc.). A RN may be investigated from different points of view. In transport-related terms, the network capacity quantifies the amount of traffic the RN can host before it become so congested that the entire circulation is halted [1], [2]. The choice of the routes in a network is an important aspect in order to study the potentially high-risk area in terms of traffic congestion. Different analytical models exist to evaluate the travelers route choice. Papola and Marzano [3] presented a probabilistic model (based on a generalized extreme value distribution) for route selection in the Italian Region of Lazio in a congested traffic assignment context. Several uncertainties encumber in a traffic assignment problem, both for route choice [4] and for travel time assessment. Uchida [5] proposed a method that take into account the uncertainties related to origin-destination traffic demand and traffic capacity of the arcs.

From the systemic reliability point of view [6], a RN may be mathematically modelled as parallel and series systems with possible redundancies [7] and adopting an adequate set of metrics in order to estimate the system's performance [8]. Recently, Taylor and Sekhar [9] have developed a procedure for recognizing the "weak spot" in a network (identified by the critical infrastructure whose failure has serious effects on access to specific location or overall system performance). The assessment of system performance of civil infrastructure systems under potential disruption scenarios is fundamental in order to minimize the recovery cost and time [9]. In the past years, many research efforts have been conducted in order to consider the impact of link (or group of links) failure in the generalized travel cost under localized [10] or extended area disruption [11].

Reliability assessment problems may be resolved efficiently with the Bayesian Network (BN) methodology. The general problem of RN functionality can be modeled analytically by identifying a minimum link-set or minimum cut-set [12]. Bensi, Kiureghian [13] proposed a BN framework for post-event risk assessment and decision making for infrastructure system taking into account the effects of spatial correlation of earthquake ground motion. Resilience of a network can also be considered an implicit measure of network reliability. For example, alternative metrics for measuring network resilience highlighting the probabilistic characterization of the different factors (e.g., uncertainty in seismic hazard, physical vulnerability, functional consequences, etc.) was proposed in [14, 15].

Tackling a network reliability problem for real multi-state complex systems usually involves considering components interdependencies [16] and is highly non-trivial to solve. This involves studying various aspects regarding to: the mathematical model adopted to represent the network and the interconnection between its components [17]; The choice of the metric used for representing system reliability (e.g., connectivity- or flow-based network reliability metrics [18]); the method adopted for solving the network reliability problem (e.g., the choice of a BN [19], simulation-based methods, or classical reliability methods); the application to complex systems such as electrical power networks [20, 21], gas pipelines networks [22, 23] and heating piping network [24].

This work focuses on the resolution of the road network reliability problem under a given seismic scenario considering connectivity as the reliability metric. It is assumed that the bridge infrastructure constitutes the only vulnerable points within the RN. That is, it is assumed that only the bridge collapse can lead to the loss of connectivity of the road segment on which the bridge is located. The seismic demand and capacity for the RN bridges are evaluated in a simulation based approach taking into account the spatial correlation in the residuals

of the ground motion prediction equation. A case study on a road network in Campania region (Italy) is illustrated.

2. Network connectivity analysis

A mathematical representation of RN can be constructed through the graph theory [25] whereby each point of discontinuity (e.g. intersections, modification of road’s property, etc.) is represented as a node and the links between the nodes are represented as oriented arcs. An example of RN-graph is reported in Fig. 1 where: nodes are labelled as N_i (i is an identification number), arcs are identified by a solid line if they allow the link in both directions or dashed line if they allow for only one direction with an arrow that identifies the direction. Arcs may be labelled with an identification number or with the start-end nodes. Each element of the graph embeds a series of information necessary for the network analysis (e.g. velocities, time, number of lanes, etc.).

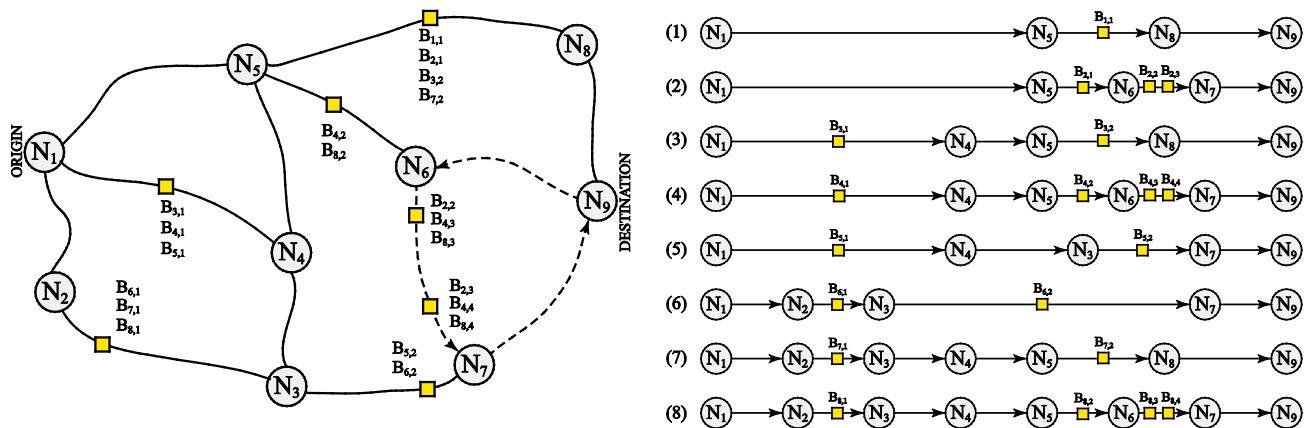


Fig. 1 - Example of road network in a graph-representation, alternative routes from N1 to N9

The information contained in the graph of a RN are not sufficient in order to assess the reliability of the system because there are different elements (e.g. bridges, galleries, retain walls, etc.) which, being weak- or interruption-points, may strongly affect reliability of the RN. For the reason just explained, RN-graph should be integrated with the position and vulnerability information (as explained in the next sections) of the vulnerable infrastructure. In this work, only bridges have been considered as weak-point along the RN-arcs and they are represented as an example in Fig. 1 as yellow squares labelled as $B_{i,k}$ (where i is the route index and k is the bridge index of the route i).

Connectivity analysis, seen as the exhaustive study of the alternative routes that enable the link between two different points of the RN, is the starting point for the reliability assessment of a RN. The knowledge of all the possible routes that connect two points of RN, combined with the vulnerability assessment of the weak infrastructures along them, is performed in order to estimate the probability of connection of RN. There are different kinds of connectivity problems: one-to-one, one-to-many, many-to-many. In this work the one-to-one problem is considered (together with an example of one-to-many connectivity problem) considering that one-to-many and many-to-many connectivity problem may be solved repeating the procedure defined for one-to-one connectivity. Defining R_i as the logical statement true if the route i is connected, the probability of connection may be written in the general form presented in Eq. (1), where N_r is the number of routes that allow the link between two different points of the RN.



$P_{c, network} = P(R_1 + R_2 + \dots + R_{N_r})$	(1)
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Where $P_{c, network}$ denotes the probability that the two points are connected and R_i is the statement that expresses the connectivity of route R_i . Eq. 1 states that the probability of having the two points connected is equal to the probability of having at least one of the routes between the two points connected. Eq. (1) highlights the importance of identification of alternative routes before estimating the reliability of RN. The algorithm proposed in this work, in order to find the all alternative routes between two point of RN, is based on the A* [26] search-on-graph using a simple heuristic [27] sort criterion during search process, which is function of the direction between the current node and the final destination node.

In order to assess the reliability of the RN, in this work, the vulnerability of the road-arc is assumed to be limited to the bridge infrastructures located along the arcs. For this reason, a route is considered interrupted only if one (or more) of its bridges reach the failure condition (i.e., lack of transitability).

The performance of each bridge is described using the performance variable Y defined in Eq. (2) where D and C represent respectively the demand and the capacity respect to a generic intensity measure. This work is focused on seismic-related disruptions so D and C are the seismic demand (hazard) and seismic capacity (vulnerability) in terms of Peak Ground Acceleration (PGA) for each bridge.

$Y = \frac{D}{C}$	(2)
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3. Seismic reliability of a portfolio of infrastructures

3.1. Scenario based demand PGA-field generation

The joint probability density function $f(\text{IM}|\text{EQ scenario})$ for the vector of Intensity Measure (IM) values at the location of each bridge of interest for a given earthquake scenario can be evaluated by employing a Ground Motion Prediction Equation (GMPE). It has been assumed that the vector IM is described by a joint Normal probability distribution with the mean of the logarithm (base 10) denoted as vector \mathbf{M} and covariance (of the logarithm) equal to matrix Σ . The components of the vector \mathbf{M} for a given earthquake scenario and site conditions are calculated according to [28] GMPE described above where the effect of the site amplification is accounted by using a function of the soil class defined in Eurocode 8 [29]. In the spatial correlation definition, both inter- and intra-event variability are considered. In particular, the covariance matrix, Σ , is defined as the sum of two inter-event and intra-event components:

$\Sigma = \sigma_{INTER}^2 \cdot \mathbf{e} + \sigma_{INTRA}^2 \cdot \mathbf{R}$	(3)
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where σ_{intra} represents the intra-event variability and σ_{inter} represents the inter-event variability (both parameters are tabulated in Bindi, Pacor [28]) in the residuals with respect to the GMPE; \mathbf{e} is the all ones matrix and \mathbf{R} is the matrix of correlation coefficients (with diagonal terms equal to unity and off-diagonals equal to ρ). The term ρ in the covariance matrix calculated according to the following formulation, proposed in Esposito and Iervolino [30]:

$\rho = \exp[-3 \cdot h / b(T)]$	(4)
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where h represents the distance between sites i and j and $b(T)$ is a coefficient equal to 10.8km. One of the specific characteristics of a joint Normal distribution for a vector of variables is that any given partition of the vector conditioned on the remaining components of the vector is still going to be a joint Normal distribution. With specific reference to the case of the vector of $\log_{10}\text{IM}$ values, let the vector of mean values \mathbf{M} and the covariance matrix Σ be partitioned as follows [31, 32]:



$$\mathbf{M} = \begin{bmatrix} \mathbf{M}_1 \\ \mathbf{M}_2 \end{bmatrix} \quad \Sigma = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{bmatrix} \quad (5)$$

where: \mathbf{M}_1 is the mean vector of PGA for the bridges of interest (according to the GMPE); \mathbf{M}_2 is the mean vector of calculated PGA for the stations within the area of interest (according to the GMPE); Σ_{11} is the covariance matrix for the calculated PGA for the bridges of interest; $\Sigma_{12}=\Sigma_{21}$ is the cross-covariance matrix for the PGA values calculated at the location of the bridges and those calculated at the location of the stations; Σ_{22} is the covariance matrix for the PGA values calculated at the stations. As described briefly above, the conditional distribution of the calculated PGA values given the registered PGA values at the stations is a joint Normal distribution with mean vector $\mathbf{M}_{1|2}$ and covariance matrix $\Sigma_{11|22}$:

$$\mathbf{M}_{1|2} = \mathbf{M}_1 + \Sigma_{12} \cdot \Sigma_{22}^{-1} \cdot (\mathbf{x}_2 - \mathbf{M}_2) \quad ; \quad \Sigma_{11|22} = \Sigma_{11} - \Sigma_{12} \cdot \Sigma_{22}^{-1} \cdot \Sigma_{21} \quad (6)$$

where \mathbf{x}_2 is the vector of the registered PGA values for the stations.

3.2. Vulnerability assessment of a class of bridges

In this work, the collapse limit state for the bridge infrastructure is consistent with those defined by Noto and Franchin [33] and Borzi, Ceresa [34] with the restrictions imposed by Eurocode 8 Part 3 [35]. In particular, based on the distributions of $f(\text{IM})$ and $f(\text{PGA}_{\text{collapse}})$, for each bridge of the portfolio and for each simulation, is possible to achieve two different conditions (no failure and failure):

$$I(\text{Bridge} | \text{PGA}, \text{PGA}_{\text{collapse}}) = \begin{cases} 0 & \text{no failure} \quad , \text{PGA} < \text{PGA}_{\text{collapse}} \\ 1 & \text{failure} \quad , \text{PGA} \geq \text{PGA}_{\text{collapse}} \end{cases} \quad (7)$$

where I is an indicator function which is equal to one in the case of failure and equal to zero in the case of no failure; PGA is the IM value at a particular bridge location and $\text{PGA}_{\text{collapse}}$ is the capacity value for the collapse limit state for the same bridge. In this work the portfolio is considered homogenous and perfectly correlated so that the PGA capacity values for the collapse limit state are equal for all the bridges in the portfolio.

An established Bayesian updating scheme in closed-form is employed herein [32] in order to update the distribution of the collapse limit state capacities for the portfolio of bridges. This section lays out briefly the components of this updating scheme. Let us assume that $\lambda = \eta_{\text{PGA},c}$ denotes the median PGA value marking the onset of the collapse limit state for the portfolio under consideration. Arguably, this statistic corresponds to a distribution reflecting the uncertainty due to both the record-to-record variability in bridge capacity for a given bridge and the bridge-to-bridge variability in the capacity over the entire portfolio. Assuming that λ takes into account only the first source of uncertainty (record-to-record) mentioned above, a prior probability distribution for λ can depict the uncertainty due to bridge to bridge variability. If the prior probability distribution for λ is Log Normal and that the likelihood function is expressed by a Log Normal probability density, it can be shown (e.g., Singhal and Kiremidjian [36]) that the posterior distribution of the parameter λ , $f(\lambda)$, is also Log Normal with mean value $\mu''_{\log \lambda}$ and standard deviation $\sigma''_{\log \lambda}$:

$$\mu''_{\log \lambda} = \left[\mu'_{\log \lambda} \cdot \left(\frac{\zeta_p^2}{n_p} \right) + \mu'_p \cdot \sigma_{\log \lambda}^2 \right] / \left[\left(\frac{\zeta_p^2}{n_p} \right) + \sigma_{\log \lambda}^2 \right] \quad ; \quad \sigma''_{\log \lambda} = \left[\left(\frac{\zeta_p^2}{n_p} \right) \cdot \sigma_{\log \lambda}^2 \right] / \left[\left(\frac{\zeta_p^2}{n_p} \right) + \sigma_{\log \lambda}^2 \right] \quad (8)$$

where: $\mu'_{\log \lambda}$ is the prior mean of $\log \lambda$; μ'_p is the mean of the natural logarithm of the $\lambda = \eta_{\text{PGA},c}$ values obtained as data; $\sigma'_{\log \lambda}$ is the standard deviation of $\log \lambda$; ζ_p is the standard deviation of the natural logarithm of the of the $\lambda = \eta_{\text{PGA},c}$ values obtained as data; n_p is number of $\lambda = \eta_{\text{PGA},c}$ values obtained as data.



The fragility curves derived by Borzi, Ceresa [34] and Noto and Franchin [33] for a very large portfolio of highway RC girder bridges in Italy have been used herein in order to calculate the prior statistics $\lambda = \eta_{PGA,c}$. Moreover $\mu'_{\log\lambda}$ has been calculated as the natural logarithm of the median

PGA_{DS} value for the median fragility curve reported by Borzi, Ceresa [34] and $\sigma'_{\log\lambda}$ has been calculated as the logarithmic standard deviation of the median PGA_{collapse} values for the individual fragility curves reported by [37].

4. The path-set approach for connectivity assessment between two points

In this section, the NR reliability is expressed in terms of the connectivity between two points “origin” and “destination” as also illustrated in Figure 1. Suppose that a set of N_r alternative routes are identified between points of origin and destination. Connectivity P_c can be expressed as the probability that at least one of the identified alternative routes is viable and is equal to the complement of the probability P_{nc} that none of the alternative routes are viable:

$P_c = 1 - P_{nc} = 1 - P(\overline{R}_1 \cdot \overline{R}_2 \cdots \overline{R}_{N_r}) = P(R_1 + R_2 + \cdots + R_{N_r})$	(9)
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Where statement R_i , $i=1:N_r$ is a logical statement with TRUE value when the i^{th} route is viable and with FALSE value when the i^{th} route is not viable and \overline{R}_i is the negation of R_i (i.e., \overline{R}_i is TRUE when R_i is FALSE). The product sign can be read as AND (logical product) and sum sign can be read as OR (logical sum). Assuming independence between the connectivity of alternative routes (not true in general) the above term can be written as:

$P_c = 1 - P_{nc} = 1 - \prod_{i=1}^{N_r} P(\overline{R}_i) = 1 - \prod_{i=1}^{N_r} (1 - P(R_i))$	(10)
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As mentioned before, it has been assumed that the connectivity of each route can be lost if and only if at least one bridge infrastructure on the route collapses. Denoting the collapse of bridge infrastructure with statement B_{ij} where $i=1:N_r$ and $j=1:N_{br,i}$ where $N_{br,i}$ is the number of bridges on route i , B_{ij} is TRUE when bridge j of route i does not collapse, $P(R_i)$ can be expressed as following:

$P(R_i) = P(\overline{B}_{i1} \cdot \overline{B}_{i2} \cdots \overline{B}_{iN_{br,i}})$	(11)
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Assuming independence between collapse of bridges located on the same route (a simplifying assumption, is not true in general), the probability that route i remains connected can be calculated as:

$P(R_i) = \prod_{j=1}^{N_{br,i}} (1 - P(B_{ij}))$	(12)
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5. Simplified procedure for estimating the number of connected routes

Let the Bernoulli variable S_i , $i=1:N_r$ represent the connectivity of each route. Recall that a Bernoulli variable can assume only two numerical values; herein, it has been assumed that $S_i=1$ for when the route is connected and $S_i=0$ for when the route is not connected. The statistics (expected value and standard deviation) for the Bernoulli variable S_i are:



$E(S_i) = P(R_i)$ $\sigma(S_i) = \sqrt{P(R_i)(1 - P(R_i))}$	(13)
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The total number of routes that remain connected $R_{connected}$ can be expressed as:

$R_{connected} = S_1 + S_2 + \dots + S_{N_r}$	(14)
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It can be shown that the expected value and standard deviation for the number of connected routes is (assuming independence between the routes):

$E(R_{connected}) = \sum_{i=1}^{N_r} P(R_i)$ $\sigma(R_{connected}) = \sqrt{\sum_{i=1}^{N_r} P(R_i)(1 - P(R_i))}$	(15)
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As a result, the probability that at least R routes remain connected can be approximated by a standard Gaussian complementary cumulative distribution:

$P(R_{connected} > R) = 1 - \Phi\left(\frac{R - E(R_{connected})}{\sigma(R_{connected})}\right)$	(16)
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6. The simulation-based approach for connectivity assessment between two points

As a result of performing a connectivity analysis between two or more points, it is possible to know: (a) the number of alternative routes; (b) the bridges located on each route. All the bridges identified, represent the portfolio of infrastructures for the reliability analysis. In a Monte Carlo Simulation MCS framework it is possible to generate a certain number of scenarios in terms of seismic hazard (see the section 3.1) and vulnerability (see the section 3.2) for the portfolio of infrastructures. In particular, for each simulation a specific value (for each bridge) of seismic demand (D) and seismic capacity (C) is extracted so a specific value of the performance variable (some kind of critical demand to capacity ratio) Y can be calculated. Herein, two different interpretations of the performance variable Y are adopted (Eq. (17)):

$Y_{wl,s} = \max_i \left(\max_j Y_{i,j,s} \right) \quad ; \quad Y_{ls,s} = \min_i \left(\max_j Y_{i,j,s} \right)$	(17)
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Where Y_{wl} and Y_{ls} represent the weakest-link and link-set definitions of the critical systemic demand to capacity ratio, respectively; s is related to the s-th simulation, j is the index of the bridge of the route i. The results of the simulation process with regards to the network performance can be synthesized through the probability distributions reported in Fig. 2:

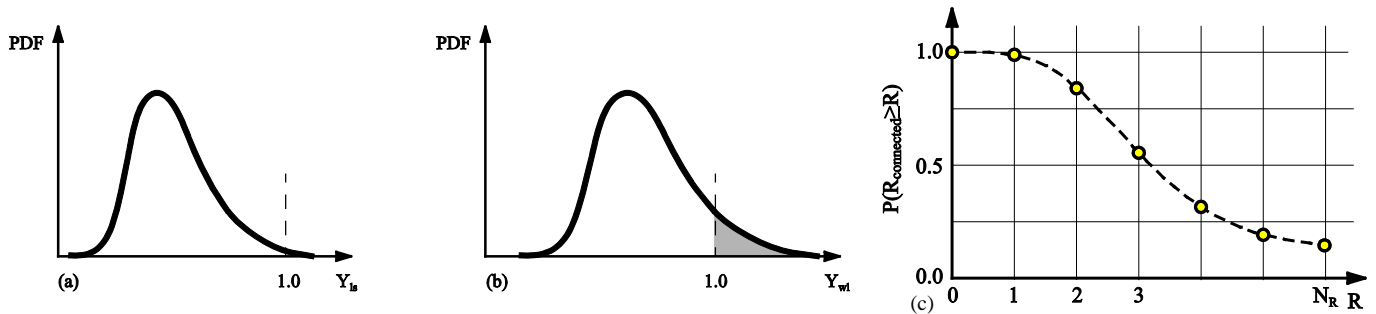


Fig. 2 - a) probability density for Y link-set; b) probability density for Y weakest-link; c) probability of having at least R connected routes.



In other words, $P(Y_{wl} \leq 1)$ expresses the probability that all of the routes within the network remain connected and $P(Y_{ls} \leq 1)$ expresses the probability that at least one of the routes within the network remains connected. Another way of expressing the network connectivity is to calculate –based on simulations– the probability of having at least R routes connected (Fig 2c).

7. Application to a road network

The methodology described herein has been applied to consider the connectivity between three different starting points (Mercato San Severino, Lioni and Pratola Serra) to the main Hospital located in the city of Avellino (in Campania region, Italy). The hypothesized scenario is the one occurred during the Irpina earthquake of the 23th of November 1980 ($M_w = 6.9$). The road network considered in this example is composed of only the main roads; that is, highways, expressways, and provincial roads. The connectivity analysis is performed neglecting the traffic condition. The vulnerability of each route is assumed to be represented by the vulnerability of the bridges located along the route. In Fig. 4a, a representation of the considered road network has been reported. The starting points (Mercato San Severino, Lioni and Pratola Serra) are indicated with cross marker and the Hospital is identified by the H marker. In the picture, the roads that shape the network are represented with thick lines using different colors for the three starting points (yellow for Mercato San Severino, blue for Lioni and red for Pratola Serra). The bridges involved in the connectivity analysis are identified on the map with circular markers. Also the projection of the Irpinia earthquake fault is reported in the map and it is possible to see that Lioni is located within the projection of the seismic source. The connectivity analysis has been performed on the described road network and the analytical results are reported in Table 1, (for each individual route and for all them as a whole) are reported: the number of alternative routes identified, the length of the shortest route, the mean length of the link, the shortest travel time, the mean travel time and the number of bridges involved in the link.

For the portfolio of bridges identified, a seismic reliability analysis has been performed with 1000 simulations. In Fig. 3, two examples of generated demand PGA are reported for the portfolio of infrastructure. It has been assumed that the infrastructures have the same capacity expressed in terms of PGA. The capacity of the portfolio is simulated from a fragility curve characterized by median equal to 0.602g and a logarithmic standard deviation equal to 0.095.

Table 1 – Analytical results of connectivity analysis.

	M.S. Severino to Hospital	Lioni to Hospital	Pratola Serra to Hospital	All to Hospital
Alternative routes (number)	5	14	3	22
Shortest route distance	24 km	51 km	10 km	10 km
Mean distance	44 km	71 km	46 km	62 km
Shortest route travel time	21'	44'	8'	8'
Mean travel time	38'	61'	39'	53'
Bridges involved (number)	32	75	49	77

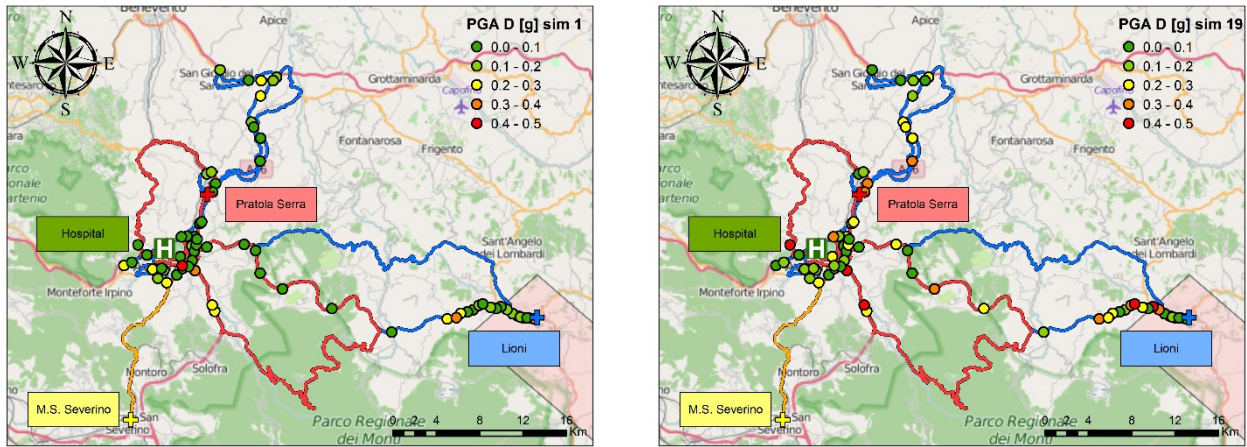


Fig. 3 - Demand PGA for the portfolio of bridges for two simulations (left: Sim. #1; right: Sim. #19).

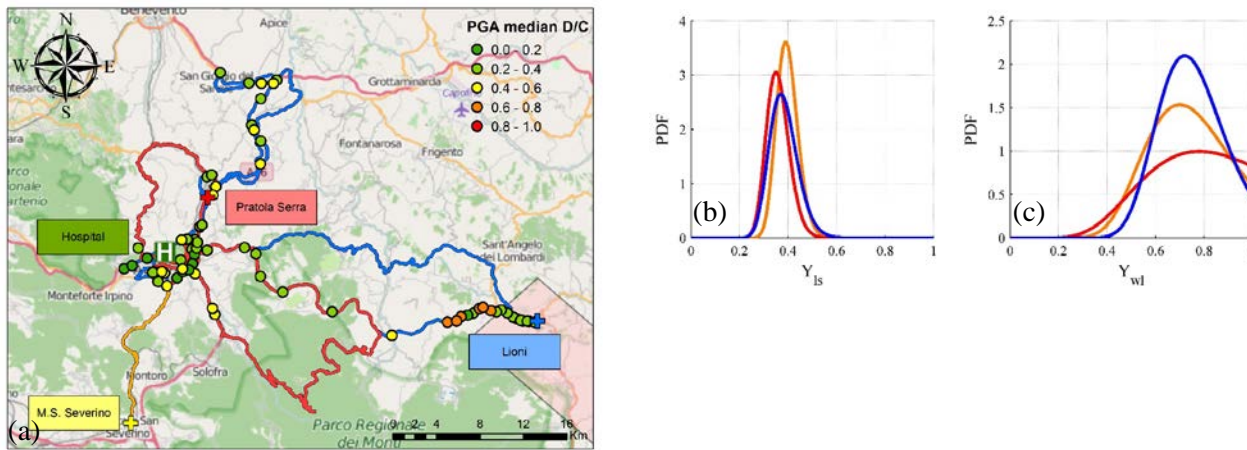


Fig. 4 - (a) Map of median D/C PGA ratio for the portfolio of bridges; (b) PDF of Y in the link-set approach; (c) PDF of Y in the weakest-link approach. Red curve represents Pratola Serra – Hospital; Blue curve represents Lioni – Hospital; Yellow curve represents M.S. Severino – Hospital.

The distributions of the performance variable Y (both from link-set and weakest-link point of view) are reported in Fig. 4(b, c). As may be seen, the probability that the performance variable Y in a link-set approach is less or equal to 1 is of 100% for all of the three case of connectivity considered (that is, with 100% probability at least one of the routes remain connected). The weakest-link performance variable distribution gives, instead, information about the worst route in the simulation process. The probability that Y in weakest-link approach is greater than 1.0 is equal to the probability to lose the connectivity of at least one route. In Table 2, for the three chosen places, the numerical values for the above-mentioned connectivity metrics are reported.

Table 2 - Analytical results of connectivity analysis.

	Pratola Serra to Hospital	Lioni to Hospital	M.S. Severino to Hospital
Probability of all routes connected $P(Y_{wl}) \leq 1$	0.58	0.95	0.79
Probability of all routes not connected $P(Y_{ls}) > 1$	0.0	0.0	0.0



The probability distributions, in terms of CCDFs, of the connected routes are plotted in Fig. 5. In thick continuous line the results of the simulation based method are reported while in thin dashed lines the CCDF's obtained by following the simplified procedure in Section 5 (assuming that the connectivity of alternative routes are independent) are depicted. The statistics of the distribution are synthetized in Table 3.

Table 3 – Statistics of the distributions

	Pratola Serra to Hospital	Lioni to Hospital	M.S. Severino to Hospital
Simulation based method [median; log-std]	[2.62; 0.20]	[9.14; 0.22]	[3.89; 0.24]
Path-set approach method [median; COV]	[2.96; 0.38]	[9.82; 0.29]	[4.41; 0.38]

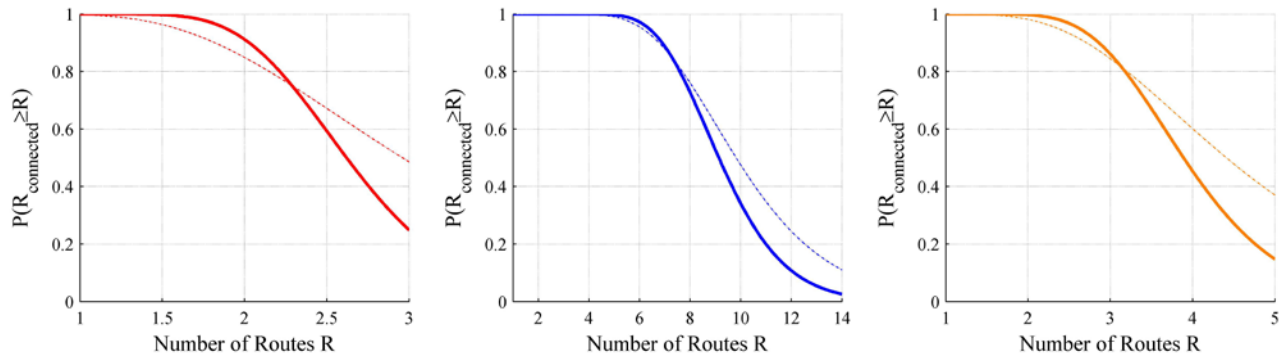


Fig. 5 - Probability to have at least R connected routes for: (left) Pratola Serra to Hospital; (centre) Lioni to Hospital; (right) M.S. Severino to Hospital. In thick continuous line the simulation-based results; in thin dashed line the simplified approach.

A comparison in terms of number of connected routes can be made by observing the curves reported in Fig. 5. It can be observed that the simplified approach, based on the hypothesis that the connectivity of alternative routes of statistically independent, leads to larger dispersions with respect to the simulation-based results.

8. Conclusions

The purpose of this paper is to propose a probabilistic method for road-network systemic analysis due to seismically-induced disruptions using connectivity as the response parameter of the network. The network connectivity analysis is described with particular attention to the algorithm used to find the alternative routes. The search-on-graph is performed with the A* algorithm using a directional heuristic function to find quickly the main alternative routes of a given network. A simulation-based approach is implemented in order to assess the seismic reliability of a portfolio of infrastructures in network connectivity terms. The scenario-based ground shaking map in terms of peak ground acceleration was obtained considering the spatial correlation between the residuals of the ground motion prediction equation. The collapse limit state capacity for the portfolio of infrastructures is assumed to be identically distributed and fully-correlated (i.e., the seismic vulnerability respect to the collapse limit state is represented by one fragility curve). A case study is presented for a road-network within the Italian region Campania where the connectivity between three different starting points and the main hospital in Avellino city has been investigated. A comparison between the simulation-based approach and the simplified analytic solution highlights the difference in the probability distribution of the number of connected routes for the three-to-one connectivity problem.



The methodology proposed in this work can be useful in providing the network manager with an instrument that provides information about: (a) the consequences induced by eventual collapse of the bridges after a strong seismic event; (b) the safest routes (after a seismic event) to be used to coordinate the relief operations; (c) the most fragile bridges (and routes) given a possible seismic scenario.

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