



## Comparative seismic response of masonry buildings modelled by beam and solid elements

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### **Abstract**

The paper is aimed at evaluating the relative accuracy of beam element modelling and solid element modelling in assessing the seismic capacity of unreinforced low-rise masonry buildings. Three-storey buildings were analysed, assuming geometry and mechanical parameters to be representative of typical low-rise tuff masonry buildings, frequently found in seismic areas of Southern Italy. The most critical and controversial features of masonry building modelling by means of beam elements are initially presented and discussed. Subsequently, comparative pushover analyses, performed varying building geometry and modelling approach are presented and the quantitative (lateral strengths and displacements) and qualitative (failure mechanisms) differences are discussed. A major goal of this paper is to assess the influence of the modelling approach on code safety checks performed using the capacity spectrum method. The crucial points are emphasized and the feasible different assumptions for assessing the maximum building top displacement in both the considered modelling approaches are argued. Conclusive remarks show how the building modelling technique affects both the maximum building capacity in displacement and the earthquake displacement demand.

*Keywords: Masonry buildings, Masonry modelling, Pushover analysis, Capacity spectrum method.*

## 1. Introduction

The issuing of the Italian national seismic code in 2008 [1], containing detailed requirements on structural modelling and analysis of masonry buildings, stimulated the development of effective simplified nonlinear models and methods of analysis, since refined nonlinear finite element models based on solid elements were not yet considered as a suitable analysis tool in the everyday engineering practice.

As a matter of fact, Italian seismic codes issued until the Eighties did not explicitly require seismic-oriented calculations for masonry structures, but only to comply with some code requirements. For this reason, masonry structural design was very seldom supported by numerical analyses, which began to diffuse only after Friuli (1976) and Irpinia-Lucania (1980) earthquakes, even if initially limited to the retrofit of existing buildings.

In recent years, several models characterized by different levels of complexity were developed and proposed for masonry structures, based on elements with different dimensionality (beam, shell or brick type) and aimed at different analysis types [2, 3, 4, 5, 6, 7, 8, 9, 10].

Among the simplified models advised in literature, frame type models based on the assemblage of beam elements are increasingly diffusing also for unreinforced masonry buildings [11, 12, 13, 14, 15, 16]. The models derive from other materials beam systems and then their extension to masonry structures is unavoidably affected by approximations, whose reach is somehow tricky to assess. The simplicity of use, related to the familiarity in seismic analysis of r.c. and steel frame structures, is counterbalanced by topological and mechanical approximations that do not allow correctly envisaging influence and role of each resisting element, as well as of floors and wall-to-wall and wall-to-floor connections. The nonlinear pushover methods counterbalance to some extent the limits of linear analysis, and in some cases, especially for existing buildings, it is recommendable to use different modelling approaches for the same structure and critically compare the results [17]. Comparisons between capacity curves provided by finite element analysis and equivalent frames were carried out to assess the main differences [18, 19]. At the same time, the pushover accuracy dependence on force pattern and incremental control criterion of analyses was investigated [20].

This paper focuses the attention on some of the above issues in order to contribute to the development of a more reliable seismic analysis of masonry structures. Specifically, results from comparative seismic analyses of low-rise unreinforced tuff masonry buildings modelled by beam (masonry-type frame models) and by solid finite elements are presented and discussed. The comparison is primarily carried out to assess the differences in the safety verification performed according to the capacity spectrum method, resulting from the different modelling of the masonry structure.

The analyses concern the building overall response only, so the failure mechanisms depend merely on the in-plane response of walls. Failure mechanisms associated with out-of-plane wall response are not considered since they have to be verified through procedures (models and analysis methods) that are out of the purposes of this paper.

## 2. Remarks on frame system modelling of unreinforced masonry buildings

Surveyed damage patterns of masonry buildings after seismic events showed that major damages are localized in piers and spandrel beams, while their connecting regions were often only slightly interested by cracking phenomena. Such empirical observation led to the development of different simplified models, which consider each masonry wall as an assemblage of masonry panels, assuming the joint regions to be extremely resistant.

Piers and spandrel beams are frequently modelled as beam/column-type elements, with rigid end offsets to reproduce the stiffness of the joint region [11, 12, 13, 14, 15, 16]. This masonry buildings modelling technique is still a controversial topic, because of several unresolved issues, as described in the following.

(i) Piers are usually assumed to have nonlinear behaviour (frequently elastic-plastic with limited deformation), which can be associated to several failure criteria. Due to the lack of experimental researches, spandrels are commonly modelled in the same way as the piers, modifying only the strength values and the failure criterion for accounting the different axial force direction with respect to the mortar layers.

(ii) Whatever failure criterion is adopted, it is known that flexural and shear resistance of masonry elements significantly depends on the axial force  $N$ . Therefore, an erroneous assessment of  $N$  is considerably reflected on

the collapse mechanism and then on the evaluation of the structure seismic capacity. This is particularly remarkable in masonry-type frame models, since the exterior columns can be affected by large variation in axial forces when the frame structure is subjected to horizontal loads. In actual masonry buildings, instead, the connection between orthogonal walls can reduce this effect in outer piers, making the axial force variation less significant. Furthermore, numerical results remarkably depend on the value to which the axial force  $N$  in piers is set back as the overhanging spandrel beams reach a failure condition and then the spandrel coupling action ends up.

(iii) A further approximation affecting the masonry-type frame models can derive from vertical mass displacements produced by pier rotation if inelastic deformations take place, since such local displacements are not evaluated by one-dimensional pier elements. It ought to be additionally considered that the conventional cross-sections-remain-plane hypothesis may be invalid in many cases, particularly for squat masonry walls under in-plane loading.

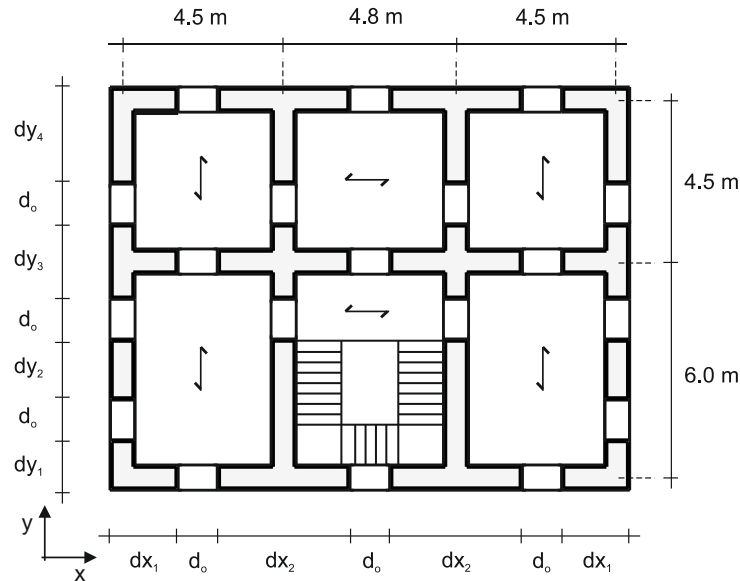
(iv) Finally, the frame models cannot accurately describe the actual behaviour of the load bearing piers, which frequently show complex cross-section shape. Indeed, in 3D building models it is customary to consider the piers at the wall intersection consisting of two independent piers having rectangular cross-section. That is, independent behaviour of piers in the two planes is assumed, and the interaction between in-plane and out-of-plane response of masonry wall portions is roughly modelled. Actually, the out-of-plane loaded wall portions bear a fraction of the lateral force, depending on the geometry of the pier portion in compression and on the interaction that might develop at the walls intersection. This assumption unavoidably leads to an approximate assessment of building self-weight, stiffness and pier capacity, and the influence on the building seismic response is not easy to quantify. Under the hypothesis that during building dynamic response no disconnection takes place among orthogonal walls, rigid offsets are commonly located to connect the upper nodes of piers at the floor level in order to reproduce the good arrangement in walls connection and to impose the continuity of displacements. Obviously, the rigid offsets notably influence the building response, so their introduction must be carefully evaluated.

The aforesaid issues evidence that seismic analysis of masonry buildings can be affected by several uncertainties if piers and spandrels are modelled as column and beam type elements respectively, instead of shell or solid elements. The main resulting approximations are examined in this paper.

Lastly, it should be remarked that the usual option of contemplating both soft and strict hypotheses, frequently assumed in structural modelling, is not relevant in the seismic analysis of masonry buildings, since it can lead to notably different seismic capacity, which could not be acceptable in historical building retrofit.

### 3. Analyzed buildings and modelling assumptions

Three-storey unreinforced masonry buildings were analysed (Figure 1). Geometry and mechanical parameters of masonry are representative of typical low-rise tuff masonry buildings, frequently found in seismic areas of Southern Italy. The floor height and the wall thickness were constantly assumed to be respectively equal to 3.5 m and 0.60 m. The pier width and the window geometry were varied in order to analyze buildings with piers of different slenderness, whilst maintaining the same plan configuration of Figure 1. Only results obtained assuming the windows width do equal to 1.20 m ("Building A") and 1.80 m ("Building B") are presented in this paper, since they are representative of two limit patterns. Initially, single walls (identified as "Wall A" or "Wall B") were extracted from the 3D buildings and analyzed alone to better understand the influence of the wall modelling on the seismic response. It has to be emphasized that the choice of simple geometrical schemes allows to introduce only a limited number of parameters, so that the numerical results are more easily controlled and discussed. Additional analyses on building schemes having different and not aligned wall openings will be performed in future phases of the research.



Plan A and B

Fig. 1 – Plan-wise distribution of buildings analyzed

The weight density of masonry was assumed equal to  $16 \text{ kN/m}^3$ . In addition to the self-weight, extra loads were considered acting at the floor levels: presuming timber floor structures, dead loads equal to  $4 \text{ kN/m}^2$  and live load equal to  $2 \text{ kN/m}^2$  were considered. Tributary areas at each floor were assumed to contribute to the vertical loading of each wall according to the floor beams direction illustrated in Figure 1. In the beam element modelling, the masses were inevitably lumped at the story levels and summed to the masses associated to floors weights and loads, whereas in the solid modelling approach they were distributed along the height of the building, so as actually occurs. This implies a further influence on the results of non-linear static analyses, which was considered necessary to accept to make the comparison consistent with the analyses usually performed in daily practice.

### 3.1 Modelling using solid elements

Eight node solid elements of size of about  $0.3 \times 0.3 \times 0.3 \text{ m}$ , with isotropic behaviour, were used for modelling the masonry structure (Figure 2a). The model of the 3D building is composed of 17736 and 15684 elements, respectively for the Building A and B.

The material parameters were selected according to the Italian National Building Code [1] and on the basis of experimental tests and reference values: bricks having tensile strength  $f_b = 0.3 \text{ MPa}$ , Young's modulus  $E_b = 1800 \text{ MPa}$ , Poisson's ratio  $\nu_b = 0.2$ , and mortar joints having  $f_j = 0.05 \text{ MPa}$  and  $E_j = 300 \text{ MPa}$  were assumed.

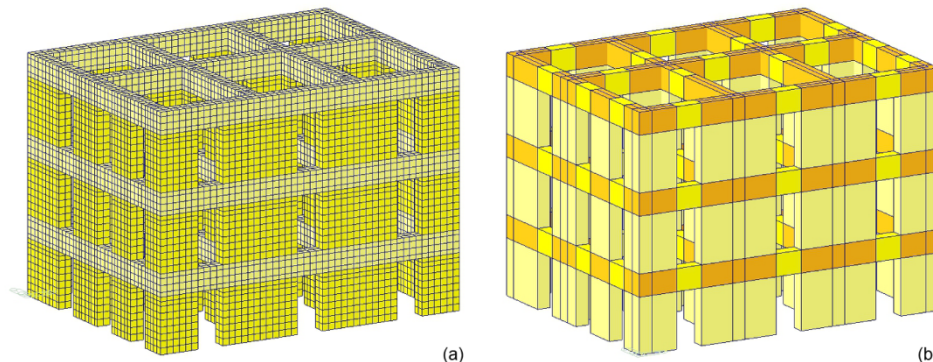


Fig. 2 – Building modelling approach: solid element (a) and beam element (b)

The analyses were performed using the "strumas" model of the computer program MidasGen®, specifically developed for masonry structures. This a "micro-macro" modelling based on the hypothesis of homogeneous equivalent material [21], which considers the characteristics of the three constituents (bricks, bed and head mortar joints). The technique of homogenization is the one proposed by Pande et al. [22] and based on the equality of the deformation energy. The properties of the equivalent masonry material depend, therefore, on the size of blocks, the thickness of bed and head mortar joints, the Young's modulus, the Poisson's ratio and the tensile strength of blocks and mortar joints. The model assumes an indefinitely elastic behaviour in compression. The analytical procedure is linear in each step, but if the principal tensile stress exceeds the strength of a constituent, its contribution to the new stiffness matrix of the homogenized material is reduced or cancelled. The reduction depends on a parameter of stiffness abatement, assumed to be equal to  $10e-4$ , which corresponds to nearly elastic-plastic behaviour [23]. The regular Newton-Raphson was chosen as iteration method.

### 3.2 Modelling using beam elements

The masonry structure was considered as an assemblage of column-beam elements, according to customary routines and requirements of most seismic codes (Figure 2b). The rigid offset lengths at pier and spandrel ends were simply assumed equal to the half of the spandrel height and of the pier width respectively.

Masonry with compression strength  $f_{mc} = 1.5$  MPa, shear strength  $f_v = 0.03$  MPa, Young's modulus  $E_m = 1100$  MPa and Poisson's ratio  $\nu = 0.2$  were taken into account.

Piers and spandrel beams were assumed to have elastic-plastic behaviour based on plastic hinge concept. Hinges are located at both ends of each element for the bending component and at the center for the axial and the shear components; the strength threshold values were derived by both flexural and shear failure mechanisms. During analyses, the model was updated each time an element achieved a limit condition (resisting bending moment, shear strength, axial strength).

### 3.3 Loading and analysis procedure

In the pushover analysis the loads were imposed onto the structure in a two-step sequence. Firstly, the vertical loads were applied and subsequently the lateral loads were monotonically increased. The height-wise lateral load distribution proportional to the product of the displacements of the first vibration mode times the floor tributary masses (distribution d1) and the invariant height-wise lateral load distribution (d2) were assumed. No accidental eccentricity was considered in performing the pushover analysis, that is the lateral forces were applied at the mass center of the floors. The mass center at the roof was assumed as control point. The safety verification was performed according to the capacity spectrum method, applying the response spectrum shaped for Ariano Irpino at the life safety limit state, which has one of the highest peak ground acceleration within the Campania Region (soil type C, topographic category  $T_1$ ,  $V_n$  nominal life  $V_n = 50$  years, class of use  $C_u = 2$ , peak ground acceleration  $a_g = 0.351g$ ,  $F_o = 2.304$ , soil coefficient  $S = 1.215$ ,  $TB = 0.196$  sec,  $TC = 0.589$  sec,  $TD = 3.002$  sec).

The safety verification of buildings modelled by beam elements was performed according to the failure criteria advised by the Italian seismic code [1], that is when the horizontal load capacity of building is decreased to 80% of the maximum one or when any further increment in lateral load is impossible. For buildings modelled by solid elements, considering the greater difficulty in identifying the failure state, the safety verification was carried out both at the achievement of the maximum drift in piers and at evident failure state of the building structure, such as the attainment of maximum shear strength in a floor of the building.

### 3.4 Periods of vibration

Preliminarily, the fundamental periods of vibration were computed to assess possible differences owing to the building modelling. Table 1 contains the computed values, showing that there are no major differences between the two modelling approaches. (the maximum difference was computed for the Wall B in about 8%). Therefore, an appropriate correlation among dynamic properties of buildings differently modelled was assumed to be achieved, making feasible to compare results from pushover analyses.

Table 1 – Fundamental vibration period in x direction

	Beam Elements [s]	Solid Elements [s]	Difference [%]
Wall A	0.195	0.208	6.7
Wall B	0.237	0.256	8.0
Building A	0.220	0.223	1.4
Building B	0.268	0.261	-2.6

## 4. Results from nonlinear static analysis

### 4.1 Dependence on walls modelling

Figure 3 compares the capacity curves of single walls in X-direction of buildings A and B computed by both considered modelling. The pushover curves of beam element modelling are unavoidably characterized by the intrinsic approximation due to the concentrated plastic hinge approach, whereas the results of solid element modelling depend on the characteristic of the model itself, which does not account for real crack opening and sliding owing to its homogenized nature. The drops in the capacity curves of the wall modelled by beam elements identify the loss of strength due to elements failure.

Figure 3 shows how the beam element modelling underestimates the lateral capacity of single wall with respect to solid element modelling. But the noteworthy outcome is the considerable influence of wall geometry (pier width and spandrel length) on the comparison between modelling. Specifically, larger the pier width  $L_p$ , more the modelling by beam elements underestimates the wall lateral strength. The above differences are obviously due to the already highlighted intrinsic inaccuracy of beam elements in reproducing the in-plane seismic behaviour of masonry walls that have large pier width.

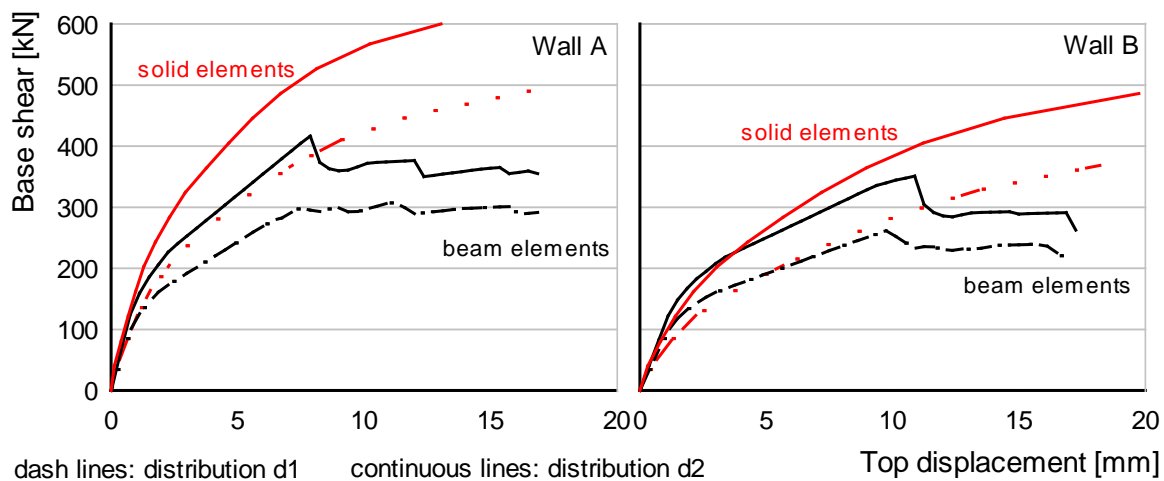


Fig. 3 Capacity curves of single walls in X-direction computed by both modelling approaches

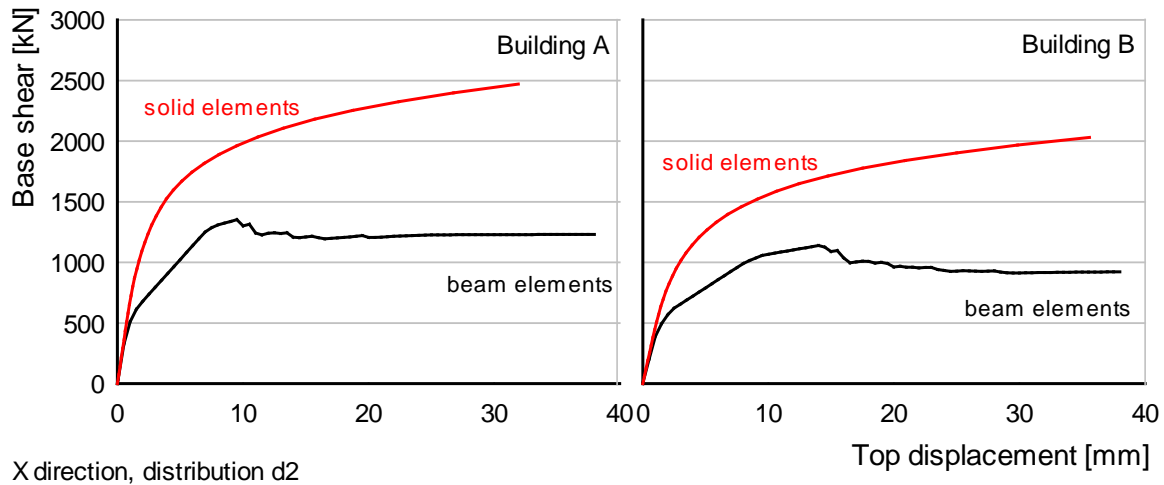


Fig. 4 Capacity curves of 3D buildings loaded in X-direction

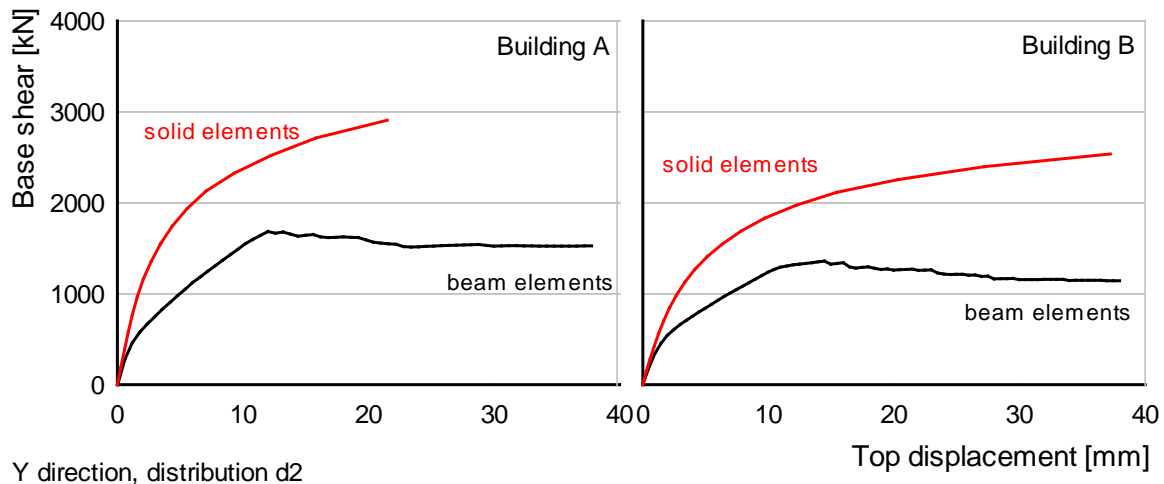


Fig. 5 Capacity curves of 3D buildings loaded in Y-direction

Figures 4 and 5 compare the capacity curves of 3D buildings loaded in X and Y-direction under the invariant height-wise lateral load distribution d2. The pushover curves confirm the conclusions highlighted by single walls, but with greater differences in capacity between beam and solid element modelling.

The comparison in terms of damage and failure mechanisms provided by the two examined modelling approaches is partially hindered by the substantial dependence of the failure of beam element modelling on the spandrels behaviour. In fact, the wall failure is usually due to the shear failure of spandrels if the behaviour is elastic-brittle and to the achievement of the maximum provisional wall drift if the spandrels have large inelastic behaviour. Nevertheless, it can be stated that the sequence of pier and spandrel failures in beam element modelling depends on the pier width with respect to the spandrel length. If the pier width is small, the wall achieves the resisting bending moment in piers before the spandrels fail in shear, whereas if the pier width is larger the spandrel shear failure precedes the pier failure. Conversely, the solid element modelling sometimes provides shear failure of piers that precedes or is contemporary with the spandrels failure.

## 4.2 Verification by capacity spectrum method

The maximum displacement capacity of masonry elements is a crucial parameter as significantly constrains the ULS safety verification in non-linear static analyses [24]; it mainly depends on type of masonry, failure mode (shear failure mechanisms generally provide minor deformation capacity) and level of axial load (in general, the greater the axial load, the lower is the capacity in displacement of walls).

Nowadays, the check is always performed using the capacity spectrum method comparing the capacity of a structure with the demands of earthquake ground motion on it. In this context, the computed failure mechanism is definitely a discriminating factor in the displacement capacity of structures.

In the beam element modelling, the failure criteria provided in the Italian seismic code [1] were considered: accordingly, the displacement capacity corresponding to the ultimate limit states was evaluated on the force-displacement curve as the displacement corresponding to a force degradation by not more than 20% of the maximum one; the flexural and shear failure of each pier was assumed to be achieved when the drift of the deformable pier height  $h_p$  is equal to 0.6% and 0.4% respectively.

In the case of solid element modelling, the identification of the building displacement capacity involves more difficulty, both because different assumptions are feasible both because the model provides hardening force-displacement curves for behavior its intrinsic characteristic (and therefore degradation in strength equal to a predetermined percentage cannot be evaluated). The building displacement capacity could be identified based on the achievement of maximum drifts in walls or maximum equivalent (effective) stresses, but it was deemed more appropriate to identify it as that corresponding to the achievement of the maximum load capacity in a masonry pier. Specifically, the flexural failure was recognized comparing the bending moment obtained integrating the stresses acting on the cross-section with the ultimate one, whereas the shear failure was assumed coinciding with the achievement of the shear strength computed according to the equation proposed by Turnsek, Cacovic and Sheppard (1981).

Table 2 – Safety indices IS for Building A (larger piers width)

	Beam Elements				Solid Elements			
	Drift [%]	Demand [mm]	Capacity [mm]	$I_s$	Drift [%]	Demand [mm]	Capacity [mm]	$I_s$
Drift 0.40% $h_p$	0.40	51.19	40	0.78	0.40	34.02	30.06	0.88
Drift 0.60% $h_p$	0.60	51.21	60	1.17	0.60	35.60	43.46	1.22
NTC 2008 failure	0.68	51.21	68	1.32	-	-	-	-
Pier failure	-	-	-	-	0.64	36.07	46.76	1.30

Table 3 – Safety indices IS for Building B (smaller piers width)

	Beam Elements				Solid Elements			
	Drift [%]	Demand [mm]	Capacity [mm]	$I_s$	Drift [%]	Demand [mm]	Capacity [mm]	$I_s$
Drift 0.40% $h_p$	0.40	56.42	40	0.71	0.40	45.84	59.48	1.30
Drift 0.60% $h_p$	0.60	56.42	60	1.06	0.60	49.21	83.03	1.69
NTC 2008 failure	0.68	56.42	69	1.22	-	-	-	-
Pier failure	-	-	-	-	0.44	46.85	65.03	1.39

On the basis of performed analyses, it could be stated that the examined structures have a typical shear behavior, with most of damage concentrated at the lowest floor and a larger capacity in y-direction.



Table 2 and Table 3 contain the safety indices  $I_s$  in x-direction, computed as the ratio of capacity to demand, obtained for both the element modelling under the previously introduced seismic input. In addition to the values of  $I_s$  obtained by applying the two collapse criteria introduced for the two modelling, values of  $I_s$  computed at the achievement of maximum drifts in piers are also provided.

Tables 2 and 3 show that the verification performed according NTC for beam element modelling leads to results similar to the ones provided for solid element modelling by the pier failure, especially for Building A. Nevertheless, in the Building A (which has larger piers and therefore more squat) the ultimate condition identified by both modelling corresponds to the attainment of a drift of 0.6% of  $h_p$ , while in the Building B (which has slender piers) the failure provided by the two modelling corresponds to different situations: the verification according NTC in the beam modelling corresponds to a drift of 0.68% while in the solid modelling to a drift of 0.44%.

It must also be pointed out that a small variation in the evaluation of the maximum building capacity in displacement affects a far greater the safety verification in solid element modelling compared to that in the beam modelling. In fact, the force-displacement curve is more hardening than that of beam modelling (which is almost elastic-plastic) and the equivalence with SDOF systems which is based on the capacity spectrum method depends mainly by the ultimate displacement (Figure 6).

The seismic demand can also significantly vary with the modelling technique of the structure when the capacity spectrum method is used, and the variability is even greater in the solid element modelling. In fact, the displacement demand significantly depends on the initial stiffness of the equivalent bilinear SDOF system (especially when the demand spectrum is intercepted, as often happens, on the plateau); being determined by the equivalence of areas, the initial stiffness is affected to a greater degree by the variation of the ultimate displacement when the force-displacement curve is hardening (Figure 6).

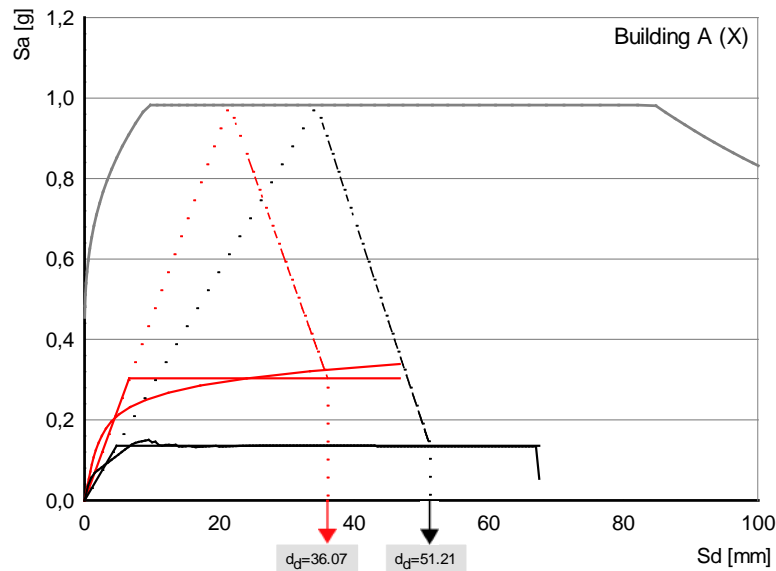


Fig. 6 Code verification of Building A in X-direction by the capacity spectrum method

## 5. Conclusive remarks

The relative accuracy of beam element modelling and solid element modelling in assessing the seismic capacity of unreinforced low-rise tuff masonry buildings has been assessed in the paper. Comparative nonlinear static analyses have shown quantitative (lateral strengths and displacements) and qualitative (failure mechanisms) differences, strongly dependent on the walls geometry and on the adopted strategy in wall modelling.

The pushover curves have confirmed the expected great differences in capacity between beam and solid element modelling, because of the unavoidable intrinsic approximation due to the concentrated plastic hinge approach of the beam modelling approach. Conversely, the results of solid element modelling depend on the characteristic of the model itself, which does not account for real crack opening and sliding owing to its homogenized nature. The comparison in terms of damage and failure mechanisms provided by the two examined modelling approaches is also partially hindered by the substantial dependence of the failure of beam element modelling on the spandrels behaviour.

The analyses have evidenced that the larger the pier widths, the more the beam element modelling underestimates the building lateral strength with respect to the solid element modelling. In this regard, it has to be evidenced that underestimations of lateral strength may lead to unreasonable over-dimensioned retrofit of existing buildings, whose safety level could be intrinsically higher than the one computed.

All the above remarks highlight that a crucial point is the assessment of the maximum top displacement, which excessively depends on both wall modelling and spandrel behaviour and holds a decisive role in seismic code analyses, especially when performed by the capacity spectrum method. Beam and solid element modelling of masonry walls can provide different maximum top displacements, partially due to the intrinsic peculiarity of each modelling strategy: the considered solid FEM approach cannot deal with increasing deformation due to severe crack opening, whereas the beam FEM approach can develop very large displacements on account of the spandrel behaviour assumption. It follows that upshots of nonlinear static analyses and safety verification performed according to code procedures can be consequently different. Obviously, this is unacceptable since the conclusive outcome of nonlinear static analysis should be mildly, and not notably, dependent on the structural modelling.

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