16th World Conference on Earthquake, 16WCEE 2017

Santiago Chile, January 9th to 13th 2017

Paper N° 4838

Registration Code: S-C1459257731

A TIME-DOMAIN METHOD FOR RESPONSE-SPECTRUM-COMPATIBLE GROUND MOTIONS USING AUTOREGRESSIVE MODELS

Ayman Shama[(1)]

[(1)] *Director of Seismic Engineering, Ammann & Whitney Louis Berger's Long Span Bridge Division, 96 Morton Street New York, NY, 10014; Ashama@Ammann-Whitney.com*

Abstract

Seismic codes require scaling the design ground motion to be compatible to the target spectrum for a specific region. Current methods of spectral matching are based on wavelet analysis and require a great deal of computations. This paper presents a simplified methodology of adjusting earthquake accelerograms to match target design spectra. A stochastic harmonic model is used to represent the ground motion as a stationary random process. To account for the non-stationarity of earthquake ground motions, the known ground motion is subdivided into a sequence of time windows to represent its temporal variation. Each window is idealized as an autoregressive (AR) model to establish its power spectrum, which is scaled according to the ratio of the spectral amplitude of the design spectrum to the spectrum of the unscaled ground motion. The harmonic model was then used to simulate the scaled ground motion, which was expanded for each window into a set of discrete frequency components and the approach is repeated for the sequence of windows representing the entire record. The response spectrum of the scaled record is then compared to the target spectrum and the scaling process is continued iteratively to achieve an optimal convergence. The procedure is illustrated by scaling a suite of ground motions that were selected from different regions of the world to design spectra representing eastern, central, and western United States.

Keywords: Ground motion, Accelerogram, Spectral matching, Autoregressive model, Random process

1. Introduction

Earthquake design ground motions have several applications in the fields of geotechnical earthquake engineering and dynamics of structures. These applications include but not limited to ground response analysis; evaluation of geotechnical hazards such as liquefaction and slope failure; and in conjunction with the direct integration method of analysis for the seismic evaluation of structures.

Spectral matching is the process of modifying the frequency content of an actual earthquake ground motion record to match a target spectrum. Nowadays this approach of ground motion simulation is becoming very popular for the development of earthquake design ground motions due to the growth of the Global Seismographic Network (GSN), which is operated by the Incorporated Research Institutions for Seismology (IRIS), a consortium of United States of America and worldwide research institutions. This enables seismic engineers to select actual ground motion records that occurred at sites having characteristics that are representative of the seismic environment of the site of interest and adjust them to match the design response spectrum. In fact, it became a requirement by many design codes to use spectrum compatible earthquake design ground motions with the direct integration method of analysis. The American Association of State Highway and Transportation Officials (AASHTO), for example, defines the characteristics of the seismic environment to be considered in selecting time histories include but not limited to tectonic environment; earthquake magnitude; style of faulting; distance-to-source; and local soil conditions. AASHTO requires that the recorded time histories be scaled to the level of the design response spectrum in the period range of significance. Furthermore, ASCE-7-10 requires that the average of the square root of the sum of the squares (SRSS) spectra from all horizontal component pairs shall not fall below 1.3 times the corresponding ordinate of the design response spectrum for a more than 10 percent for this range of periods.

Spectral matching is achieved by modifying the frequency content of the selected earthquake record to be consistent with the design spectrum at a suite of spectral periods, i.e., the spectral amplitudes of the selected ground motion at all periods of range have to be scaled to match those of the target spectrum at the same periods. This process is done through either a frequency domain approach or a time domain approach. In the frequency domain, the Fourier amplitude spectrum of the selected ground motions is adjusted according to the ratio of the target response spectrum to the selected ground motion record response spectrum while retaining the Fourier phase spectrum unchanged. This approach usually does not lead to satisfactory convergence between the spectral ordinates. The time domain spectral matching approach, which is the approach required by most of the design codes, adjusts the selected ground motion record to match a design spectrum by using wavelet transform. The first optimized procedure for this type of time domain spectral matching was proposed by Kaul [1] and during the past years the continuous wavelet transform (CWT) has been used for this purpose. Lilhanand and Tseng [2] employed wavelets but used the response of elastic SDOF system rather than CWT. They extended their approach to simultaneously match spectra at multiple damping values through solution of a set of linear algebraic equations in matrix forms. Abrahamson [3] developed the program RspMatch that was based on the approach by Lilhanand and Tseng. The program has gone through several versions (Hancock et al. [4], Al Atik and Abrahamson [5]) Basus and Gupta [6] modified the Littlewood-Paley wavelet and used it for this application. Suarez and Montejo [7] developed the impulse response wavelet for the same purpose and claimed that they tried several types of wavelets for artificial earthquakes but with the exception of the Littlewood-Paley wavelet, all other wavelets were unable to achieve the objective. Shama [8] showed that this statement is inaccurate by employing the universal Morlet wavelet to develop spectrum compatible earthquake ground motions. The time domain spectral matching using CWT procedure is generally more complicated than the frequency domain approach and computationally demanding particularly when the number of spectral frequencies for comparison increases.

The CWT is carried out by breaking the ground motion time series into shifted and scaled versions of the original (mother) wavelet, and by using this procedure, the ground motion is decomposed into its wavelet components each at a certain frequency (period). Each wavelet component is then scaled according to the ratio of the target spectral amplitude to ground motion spectral amplitude at the corresponding frequency and by running inverse wavelet transform the scaled ground motion is developed from its scaled components. Following this concept, a ground motion time series can also be decomposed into a number of cosinusoidal waveforms depending on the duration and the sampling rate. Knowing the scale factor at each frequency, a spectrum compatible ground motion can be obtained. In fact, theoretical early methods of ground motion simulation [9] used superposition of sin waves with random phases and theoretical power spectra to generate ground motions that were modulated by intensity functions to simulate the real character of real earthquakes. In the present study, an exceptional improvement has been performed on this simple concept to modify real ground motion records so that their response spectrum become consistent with a target design spectrum.

The intent of the present study is to present an efficient approach for spectral matching that is illustrated and validated by scaling ground motions from different regions of the world to target response spectra. Auto-regressive models, which have been used successfully in the past two decades in the field of ground motion simulation, are combined with the harmonic model and combined together to present a computationally competent approach as good as traditional wavelet based spectral matching techniques.

2. Ground Motion Simulation Model

The harmonic process model as defined by Priestley [10] is used in the present study to idealize the ground motion accelerogram as a stochastic process being composed of a linear sum of harmonic components with certain amplitudes and frequencies:

$$a(i\Delta t) = \sum_{j=1}^{N_f} A_j \cos\left(\omega_j t + \phi_j\right) \qquad i = 0, 1 \ldots n \tag{1}$$

where n = the number of points of the ground motion, Δt= time step, A_j is the amplitude and the $\{\phi_j\}$, $(j=1,\ldots.N_f)$ are independent random phases each is assumed to have a uniform distribution on the interval $(0, 2\pi)$. The relationship between the amplitude of the stochastic process and the one sided local power spectrum can take the form [9]:

$$\frac{A_j^2}{2} \approx S_g(\omega_j)\Delta\omega \tag{2}$$

where $S_g(\omega_j)\Delta\omega$ is the contribution to the total power of the motion from the cosinusoid with frequency ω_j. Hence, by allowing the number of cosinusoids in the motion to become very large i.e. frequency approaches the nyquist frequency of the process, the discrete power will approach the continuous power spectrum curve.

By substituting equation (2) into (1) the ground motion acceleration is expressed:

$$a(i\Delta t) = \sqrt{2}\sum_{j=1}^{N_f} \sqrt{S_g(\omega_j)\Delta\omega}\,\cos(\omega_j t + \phi_j) \tag{3}$$

in which N_f = number of frequency intervals; $\Delta\omega = \omega_u / N_f$ with ω_u as cutoff frequency (nyquist frequency); and $\omega_j = j\ \Delta\omega$. Equation (3) is the general theoretical form of simulating an earthquake ground motion that can be based on a theoretical power spectrum such as the Kanai-Tajimi [11, 12] and a random generator for the phase angle of each contributing cosinusoid. This approach requires considerable modification to be applied in order to computationally reproduce a real earthquake time series in which frequencies and amplitudes are time dependent. Since the harmonic model assumes that the random process remains in equilibrium about a constant mean value, i.e., stationary random process, then it is impractical to hold this assumption true for the entire ground motion record. Nevertheless, the accelerogram can be subdivided into a number of segments that are convenient to satisfy stationarity and must contain sufficient data for the determination of the local power spectrum. By assuming erodicity of the ground motion random process, segments of a known accelerogram can be computationally simulated using equation (3) contingent evaluation of both the power and phase spectra of each segment precisely, which is outlined in the following section.

2.1 Evaluation of Power and Phase Spectra

Each segment of the ground motion time series is assumed as a stationary stochastic process with a zero-mean; hence, it can be described by its local power spectrum, which expresses the distribution of the variance with respect to frequency. The power spectrum for each segment was estimated using an autoregressive (AR) spectral estimation technique. The advantage of this approach is that the spectrum of the time series is determined directly as a function of the AR model parameters. AR methods are more favorable than other methods that are based on Fourier transform which require smoothing techniques to improve the spectrum estimator and reduce the variance and may introduce bias or distortion to the data. A time series AR model is employed since estimates of its parameters can be obtained as solution to linear equations, which can be handled by any programming algorithm. Also, in this context, AR models are more favorable when compared to other classes of time series models such as the moving average (MA), and the autoregressive moving average (ARMA) models that require more sophisticated calculations [13]. Knowing that the series conforms to an AR model, one can estimate its power spectrum by first estimating the parameters of the AR model from the data and then utilizing these estimates to obtain the theoretical continuous power spectrum. For an autoregressive process, each time series of interest (segment of the known accelerogram) is assumed to be a linear random process, wherein the current value of the process is expressed as a finite linear filter of previous values plus a white noise. Therefore assuming an AR model of order p the time series $x(t)$ can be obtained as:

$$x(t) = a + \sum_{j=1}^{p} \varphi_j x(t-j) + \varepsilon(t) \tag{4}$$

where a = constant; φ_j = autoregressive coefficient ; and $\varepsilon(t)$ = white noise contribution to $x(t)$. A key parameter for a successful simulation of an accelerogram is the exact determination of the model order p for each of its segments. Inaccurate estimation of the AR order for a segment may lead to poor correlation with the reference accelerogram in terms of frequency content and pattern. In the present study, the order p of the AR model for each segment was selected to have the lowest final prediction error (FPE) defined as [14]:

$$FPE(p) = \sigma^2 \left(\frac{n+p}{n-p} \right) \tag{5}$$

In which n = the number of data samples to which the model is fitted; and σ^2 = the residual variance of the AR model, computed as:

$$\sigma^2 = \frac{n-p}{n-2p-1} \left\{ \rho_0 + \phi_1 \rho_1 + \dots\dots\dots + \phi_p \rho_p \right\} \tag{6}$$

where ρ = the auto-covariance function of the random process at a certain lag. The autoregressive model parameters are then obtained by employing the Levinson-Dubrin recursive method for solving the Yule-Walker equations [15]. Knowing the order and parameters of the autoregressive model, its theoretical continuous power spectrum is then determined as:

$$S_g(\omega) = \frac{\Delta t \sigma^2}{2\pi \left| 1 + \sum_{j=1}^{p} \phi_j \exp^{-i\omega j \Delta t} \right|^2} \tag{7}$$

where, Δt = the sampling interval for the known accelerogram; and i = the complex value i.e. sqrt (-1). The phase spectrum for each window is obtained directly from the Fourier Transform of the record as:

$$\theta_j = \tan^{-1}\left(\frac{\mathrm{Im}\,(\omega_j)}{\mathrm{Re}\,(\omega_j)}\right) \tag{8}$$

in which, θ_i = the phase angle of the ith contributing cosinusoid with frequency ω_j; and $\mathrm{Im}\left(\omega_j\right)$, $\mathrm{Re}\left(\omega_j\right)$ are the imaginary and real parts of the Fourier amplitude of the ground acceleration at ω_j.

To illustrate the efficiency of the above approach, it was employed to simulate the ground motion record occurred at Gisborne, New Zealand, component N77E during the 1995 Off East Cape earthquake. The earthquake is strike-slip and has a magnitude of 7.4 and depth of 19.0 km. Fig. 1-a illustrates a comparison of the recorded and simulated ground motions and Fig. 1-b compares their response spectra. It is shown that the simulated ground motion followed satisfactorily the temporal variation and frequency content of the recorded ground motion.

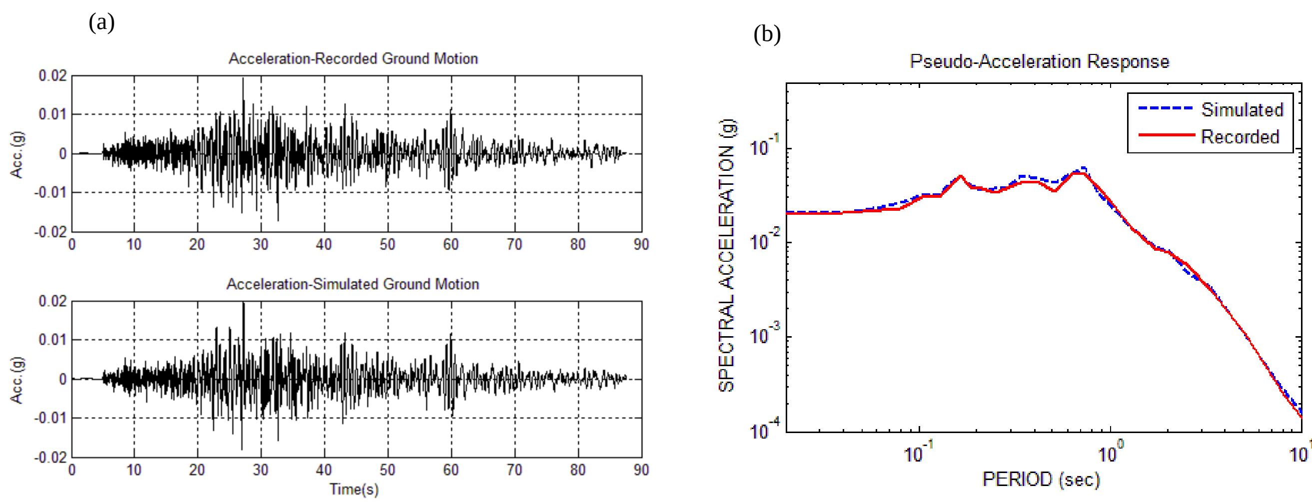

Fig. 1 – Comparison of recorded and simulated earthquake ground motion at Gisborne, New Zealand of the 1995 Off East Cape earthquake

2.2 Scaling Criterion

According to equation (3), the ground motion time series is basically sum of a number of cosinusoids each corresponds to a specific frequency/period. Therefore, the ground motion response spectrum can be calculated at each of these periods. Then the ratio between the target and calculated spectra at a period T_j is calculated as:

$$R_j = \frac{Sa(T_j)_{target}}{Sa(T_j)_{ground\ motion}} \tag{9}$$

The cosinusoids of the recorded signal are then multiplied by this ratio and a new signal is simulated based on the scaled harmonics. To ensure numerical stability of the velocity and displacement accelerograms, a four-pole Butterworth filter, with 0.35 Hz-40 Hz corner frequencies, was applied to each of the simulated ground accelerations. Once a simulated accelerogram is computed, its response spectrum is calculated and a set of new ratios R_j are calculated and multiplied by the cosinusoids of the recorded motion. This process continues iteratively and verification of convergence is established by means of the root mean square error as:

$$e(\%) = \sqrt{\frac{1}{N_f}\sum_{j=1}^{N_f}\left(\frac{S_a(T_j)_{target} - S_a(T_j)_{simulated}}{S_a(T_j)_{target}}\right)^2} \quad (10)$$

3. Verification of Proposed Method

To illustrate the procedure described above, three design response spectra were developed using the ASCE-7 requirements for Charleston, South Carolina, soil class C, peak ground acceleration (PGA) of 0.29g; Carbondale, Illinois, soil class B, PGA= 0.24g; and San Francisco, California, soil class D, PGA=0.42g. These sites are representative of eastern, central, and western United States. Seismicity of both Charleston and Carbondale is relatively higher than the rest of other regions in eastern and central US because Carbondale is located in southern Illinois within the New Madrid zone, which is characterized by very high seismicity. Also, Charleston has experienced devastating intraplate earthquakes in the past such as the one occurred in 1886 with an estimated magnitude of 7.0 and a maximum Mercalli intensity of X (Extreme). These response spectra are illustrated in Fig. 2 up to 3 seconds. Nevertheless, the spectral matching analysis was performed for a range of periods up to 10 seconds.

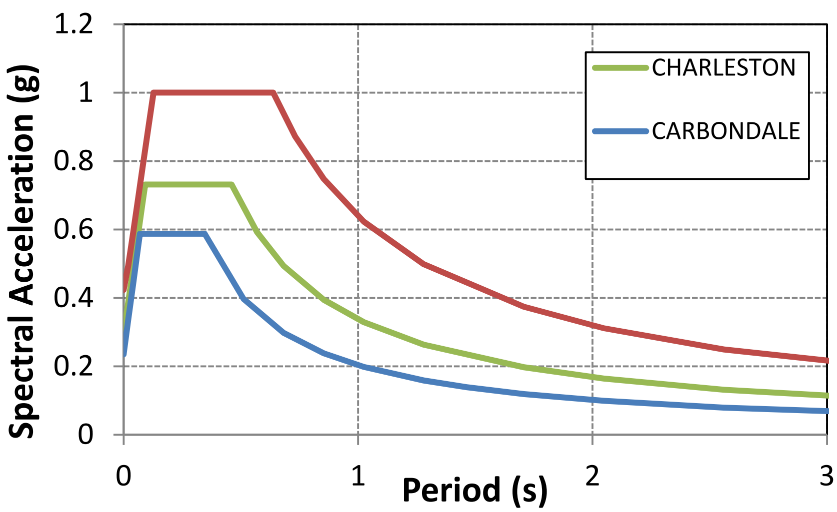

Fig. 2 – Design response spectra

Three recorded ground motions from different regions of the world were selected for scaling to each of the three design response spectra and displayed in Fig. (3). These ground motions are defined henceforth as “seed ground motions”. Each of the seed ground motions was subdivided into a number of segments. Each segment satisfied the zero-mean acceleration criterion. Table 2 summarizes the number of segments used for each ground motion and the autoregressive model order for each segment. Each segment was then simulated and scaled according to the scaling criterion outlined above.

Figures 4 through 6 display the response spectra of the seed ground motions before and after scaling to the target spectra. The target spectra are also displayed in the figures for comparisons. A clear spectral matching is shown for a range of periods up to 10 seconds. Figure 7 displays the scaled ground motions. It can be observed that the scaled ground motions maintained fairly well the basic characteristics of the seed ground motions with slight differences in amplitude timing.

Table 1 – Properties of recorded ground motions

Ground Motion #	Earthquake	Year	Country	PGA (g)	Station	Comp.	Mag.	Mechanism	Hypocentral Distance	Site Geology
1	Northridge	1994	USA	0.037	Wrightwood, CA	EW	6.7	Reverse	79 km	hard rock
2	Manjil	1990	Iran	0.18	BHRC, Qazvin	N66E	7.4	Strike slip	85.5 km	unkown
3	Off East Cape	1995	New Zealand	0.019	Gisborne	N77E	7.1	unknown	171 km	Interm. Soil

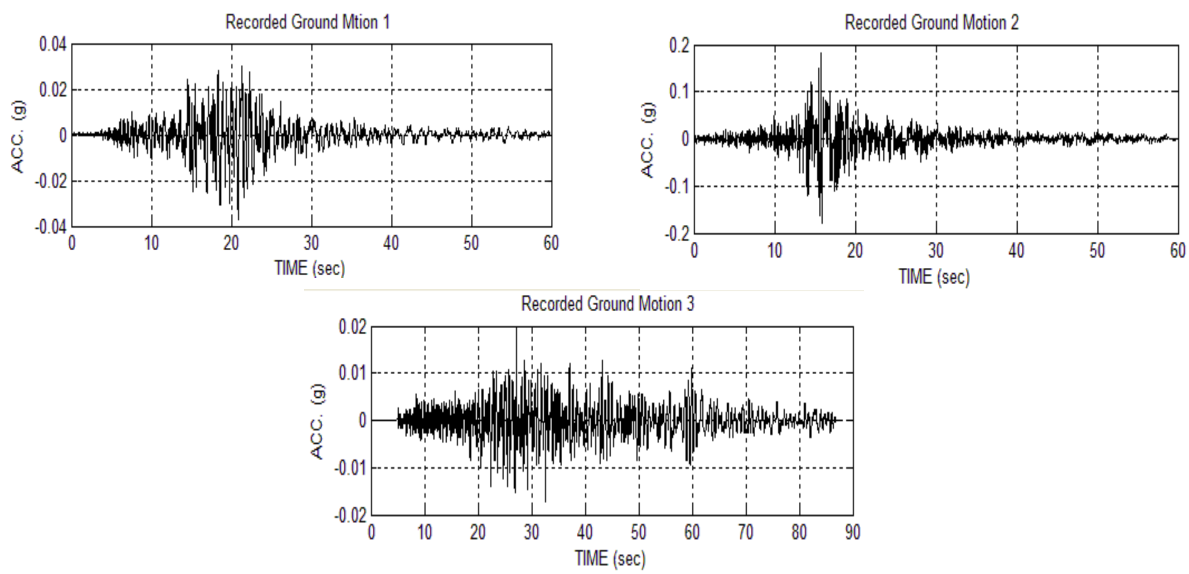

Fig. 3 – Recorded ground motions employed for scaling

Table 2 – Properties of ground motion segments to establish the power spectra

Ground Motion Number	Number of Segments	Segment size (seconds) / Autoregressive Model Order 1	2	3	4	5	6	7	8	Total Duration (seconds)	Time Step (seconds)
1	8	5.12/20	5.12/26	10.24/18	10.24/17	20.48/31	5.12/16	2.56/11	1.13/8	60.01	0.01
2	8	5.12/22	5.12/5	10.24/23	10.24/15	10.24/25	10.24/24	5.12/16	4.09/18	60.41	0.01
3	7	5.12/4	10.24/23	10.24/22	10.24/26	10.24/12	10.24/7	31.38/32	-	87.7	0.02

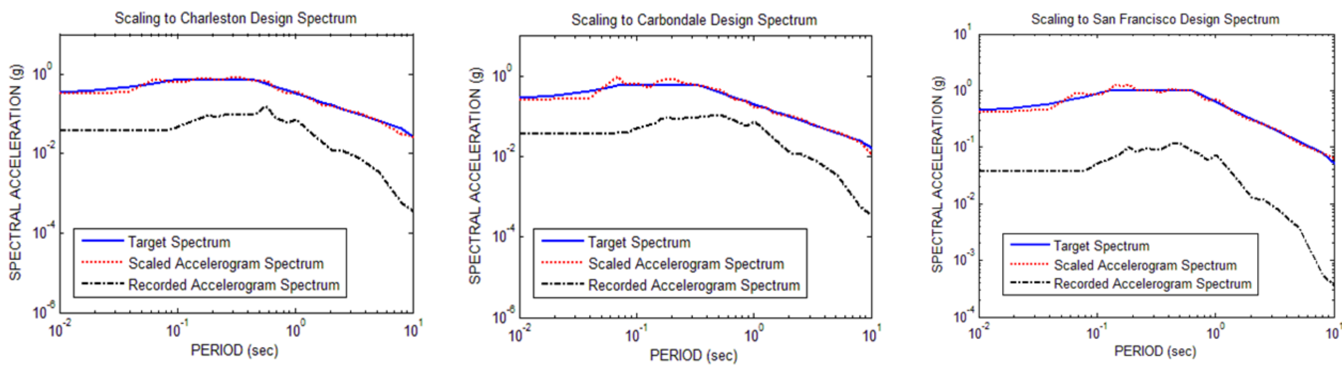

Fig. 4 – Scaling of ground motion 1 to the target spectra

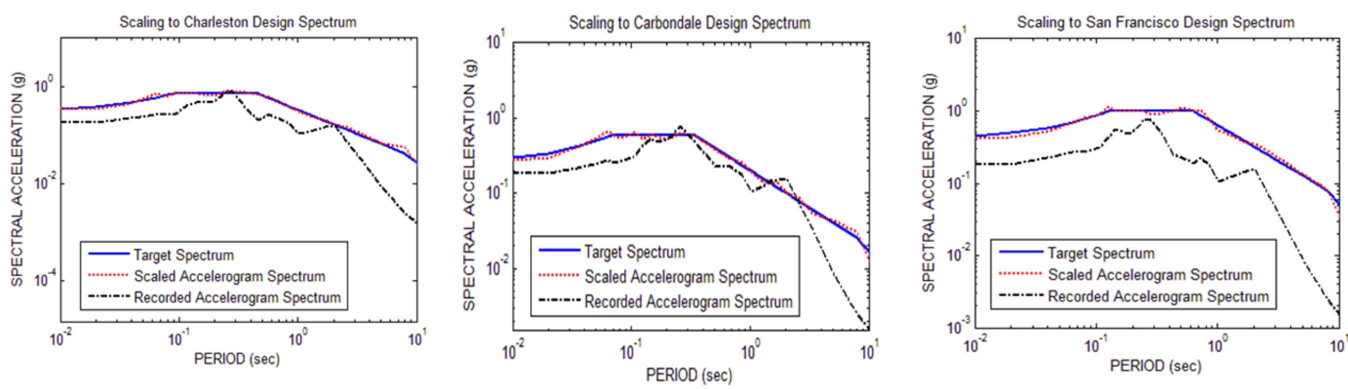

Fig. 5 – Scaling of ground motion 2 to the target spectra

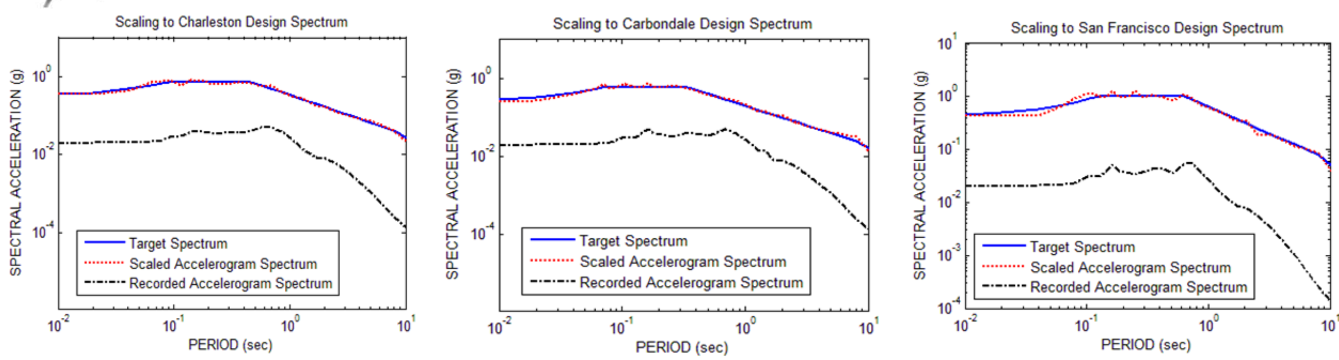

Fig. 6 – Scaling of ground motion 3 to the target spectra

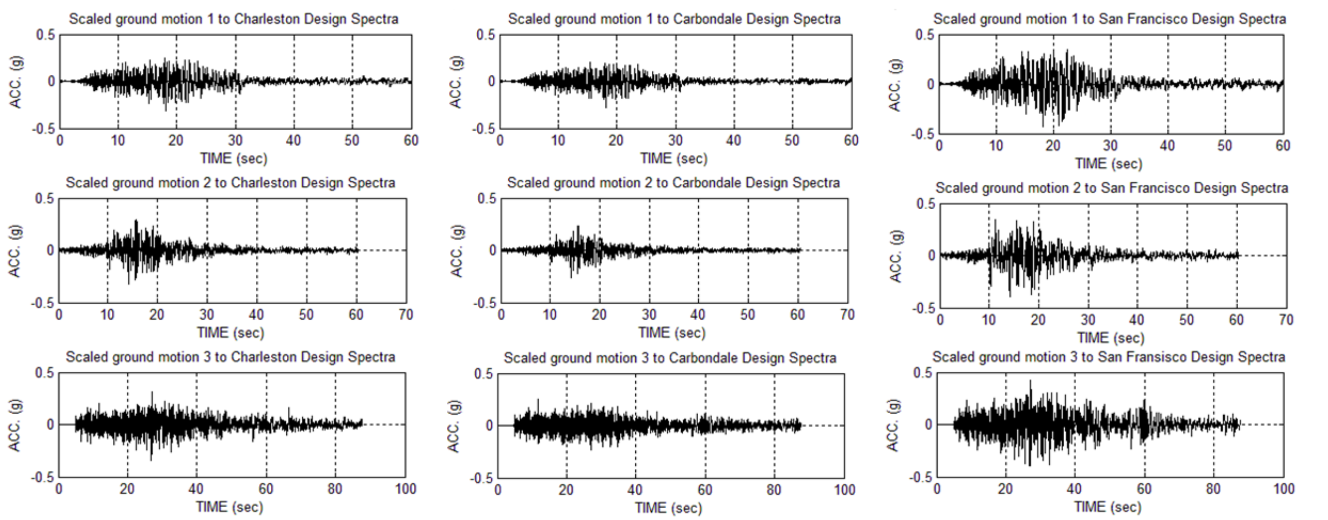

Fig. 7 – Scaled ground motions

The present study demonstrates a significant advantage of the spectral matching approach. The selection of target spectra and seed ground motions in terms of site properties was done deliberately haphazard to show that the spectral matching process will adjust for the differences in frequency content on the two sites. As an example, the design spectrum for Carbondale was selected rock Class B. The seed Off East Cape earthquake ground motion was selected at a site with intermediate soil properties. Nevertheless, the ground motion was adjusted to be compatible with the rock design spectrum as displayed in Figures 5 and 7. It is recommended, however, for practical applications, that the seed ground motion must have the same seismic environment as the target site, for which scaling is achieved. The seismic environment includes but not limited to earthquake magnitude, distance, style-of-faulting, directivity conditions, and site conditions. AASHTO emphasizes the significance of the tectonic environment i.e., plate boundary versus shallow crustal faults; earthquake magnitude; source-to-site distance; and local site conditions. AASHTO also recommends that the overall shape of the spectrum of the seed ground motion be consistent with the shape of the target design response spectrum and that the seed ground motion be initially scaled so that its spectrum is at the approximate level of the target spectrum before spectral matching.

For three dimensional analyses ASCE 7-10 requires that the average of the square root of the sum of the squares (SRSS) spectra from all ground motions considered for analysis does not fall below 1.3 times the corresponding ordinate of the design (target) spectrum for the range of periods of interest by more than 10%. This rule was applied to the ground motions and the three design response spectra considered in this study. The results are shown in Fig. 8, which displays the three target spectra and their 1.3 scaled values. The spectra of the adjusted ground motions and their SRSS spectrum are also displayed for comparisons. It is observed that the ASCE-7-10 scaling rule has been satisfied.

It is important to note that all the scaling performed were achieved for periods up to 10 s. Including high periods (5 s to 10 s) may result in velocity and displacement amplitudes for the scaled ground motion that are significantly different from those of the seed ground motion. To illustrate this effect, we display the Fourier spectrum of the seed ground motion 2 in Fig. 9 It can be observed that this ground motion lacks cosinusoids for frequencies less than 1.57 rad/s i.e., periods longer than 4 seconds. Scaling this ground motion to the target spectrum for periods up to10 seconds has introduced to the scaled ground motion additional cosinusoids corresponding to periods from 4 s to 10 s that do not exist in the seed record. These additional cosinusoids are responsible for the amplitude and timing shifts in the velocity and displacement time histories of the scaled motion. Fig. 10 displays the time histories of Acceleration, velocity, and displacement for ground motion 2 and those of scaled ground motion to San Francisco Design Spectrum for two cases: periods range up to 10s (case 1); and periods range up to 4 s (case 2). It is observed that the displacements amplitudes for case 1 are almost 2.5 the displacement amplitudes for case 2, which is due to the contribution of the accumulated amplitudes of the 4 s to 10 s cosinusoids. It is very imperative to determine the natural period of the structure for which the ground motion will be employed for its seismic analysis. The frequency content of the spectrum compatible ground motion must include the first natural period of the structure. For rigid and high rise flexible buildings, period ranges up to 5 s are quite enough for the purpose of the seismic analysis without changing the frequency content of the seed ground motion extensively. In this context, ASCE-7-10 requires that the range of periods for building structures must be between 0.2 T and 1.5 T, where T is the natural period of the structure. For other structures such as cable structures, cable stayed, and suspension bridges that possess natural periods in the range of 4 s to 10 s, it is essential to include these periods in the spectral matching process. The range of periods of interest must be included in the selected seed ground motion, which can be achieved by examining the Fourier amplitude spectrum of the seed ground motion.

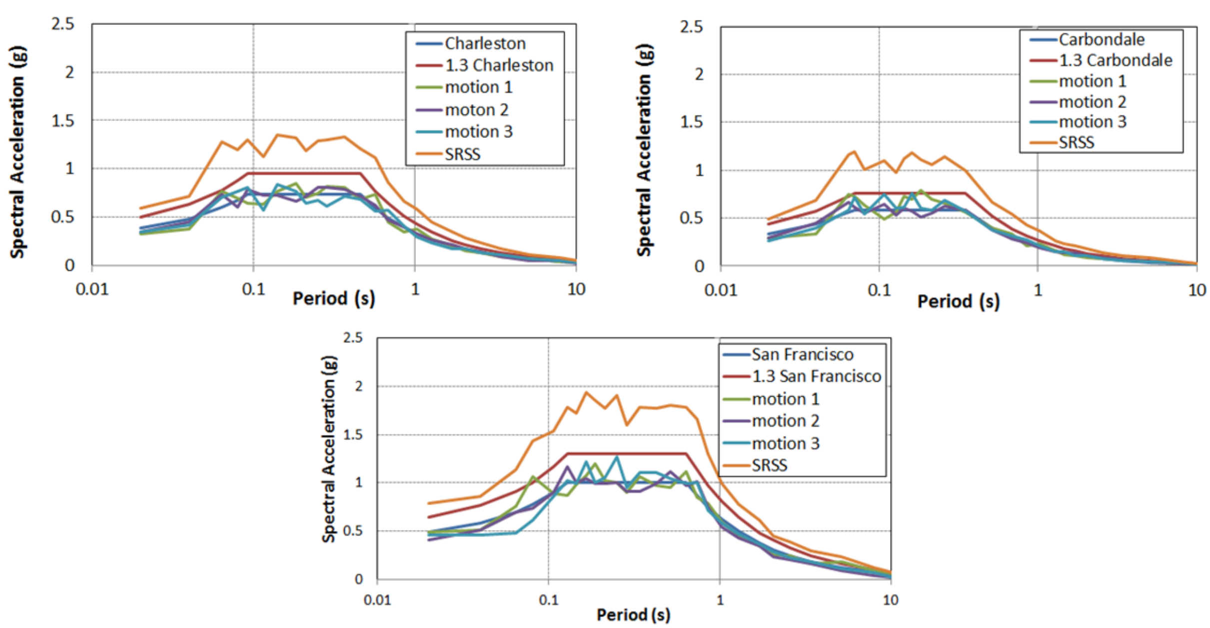

Fig. 8 – Comparison of the scaled target spectra to the SRSS spectrum

3. Conclusions

This study idealized the ground motion accelerogram as stationary random process composed of a linear sum of harmonic components. To account for its temporal variation, the entire time history of the seed ground motion was subdivided into several segments to satisfy stationarity and contain sufficient data for calculating the local power spectrum. The local power spectrum for each segment was calculated by assuming each segment as a time series and the AR model was employed to determine the local power spectrum. The order of each AR model was selected to have the Akaike's lower final prediction error and the parameters were obtained by solving the Yule-Walker equations. The simulated ground motion is then adjusted iteratively to be compatible with the target spectrum according to the ratio of its response spectrum at all periods of range to the target response spectrum at the same periods. Based on this study, the following concluding remarks can be drawn:

- The proposed approach proved to be as good as other approaches in the literature but simpler and can be handled easily using any programming algorithm.
- It is preferred that the seed ground motion be consistent with the site of interest in terms of the seismic environment and local soil conditions.
- It is very significant to know the natural period of the structure for which the ground motion will be employed for its seismic analysis prior to conducting the spectral matching process.
- The range of periods of interest for scaling must exist in the selected seed ground motion. This can be achieved by examining the Fourier amplitude spectrum of the seed ground motion.

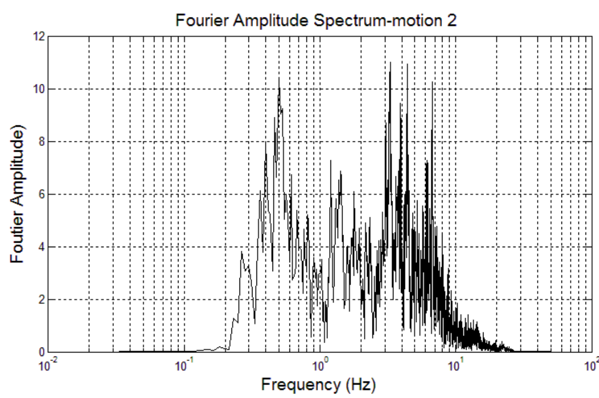

Fig. 9 – Comparison of the scaled target spectra to the SRSS spectrum

5. References

[1] Kaul M (1978): Spectrum-consistent time-history generation, *ASCE J. Eng. Mech.* EM4, 781–788.

[2] Lilhanand K and Tseng W (1988): Development and application of realistic earthquake time histories compatible with multiple damping response spectra. *Ninth World Conf. Earth. Engin. Tokyo, Japan,* (2), 819–824.

[3] Abrahamson N (1992): Non-stationary spectral matching. *Seismol. Res. Lett.* 63, 30.

[4] Hancock J, Watson-Lamprey J, Abrahamson N, Bommer J, Markatis A, McCoy E, and Mendis R (2006): An improved method of matching response spectra of recorded earthquake ground motion using wavelets. *J. Earthquake Eng.* 10, 67–89.

[5] Al Atik L and Abrahamson N (2010): An improved methodology for nonstationary spectral matching. *Earthquake Spectra,* 26 (3), 601–617

[6] Basu B and Gupta V (1998): Seismic response of SDOF system by wavelet modeling of nonstationary processes. *J. Eng. Mech.*, 12410, 1142–1150.

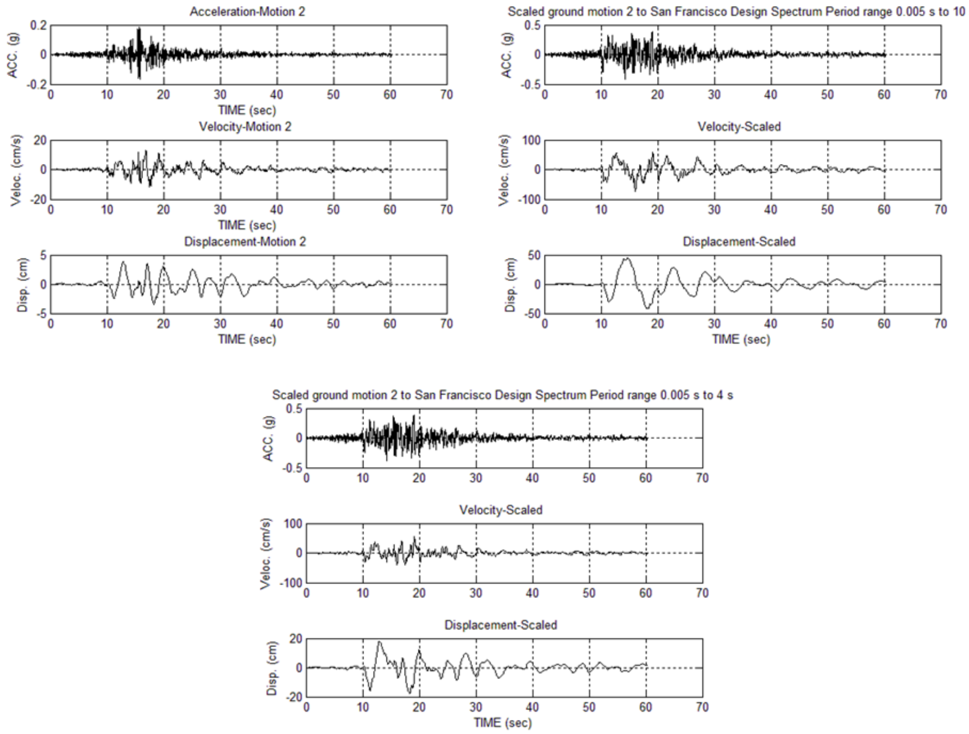

Fig. 10 – Comparison of the scaled target spectra to the SRSS spectrum

[7] Suarez L and Montejo L (2005): Generation of artificial earthquakes via the wavelet Transform. *Int. J. Solids Struct.* 42, 5905–5919.

[8] Shama A (2012): Spectrum Compatible Earthquake Ground Motions by Morlet Wavelet. *20th Analysis & Computation Specialty Conference. 163-172*

[9] Gasparini D and Vanmarcke E (1976): SIMQE: A Program for Artificial Motion Generation. *Department of Civil Engineering, MIT.*

[10] Pristley MB (1981): Spectral Analysis and Time Series. *Elsevier Academic Press, London, UK.*

[11] Kanai K (1957): Semi-Empirical Formula for the Seismic Characteristics of the Ground. *Univ. Tokyo Bull. Earthquake Res. Inst., (35), 309-325*

[12] Tajimi H (1960): A Statistical Method of Determining the Maximum Response of a Building Structure during an Earthquake. *Proc. 2nd World Conf. Earthquake Eng. Tokyo, (II), 781-798*

[13] Marple Jr SL (1987): Digital Spectral Analysis with Applications. *Prentice Hall; NJ*

[14] Akaike H (1974): a New Look at the Statistical Model Identification. *IEEE Transactions on Automatic Control,* (AC-19), 6, 716-723

[15] Box GE, Jenkins GM, Reinsel GC (1994): Time Series Analysis Forecasting and Control.3rd ed., *Prentice Hall, NJ*