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A Bernstein polynomial model in Bayesian inversion of surface-wave dispersion for earthquake site response in British Columbia, Canada

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Abstract

Knowledge of near-surface properties of the soil column, in particular, the shear-wave velocity (Vs) profile over the upper 10s of meters, is important for characterizing the expected ground response to earthquake shaking at a specific site. Non-invasive and passive methods based on recording ambient seismic noise are increasingly popular for estimating Vs structure with minimal cost and site disruption. This paper applies a fully nonlinear Bayesian inversion methodology, based on parallel tempering, to estimate Vs profiles and uncertainties using surface-wave dispersion data processed from passive seismic array recordings. In the inversion, the Vs profile is parameterized using a Bernstein polynomial basis, which efficiently characterizes general depth-dependent Vs gradients in the soil column. Bernstein polynomials provide a stable parameterization in that small perturbations to the model parameters (basis function coefficients) result in only small perturbations to the Vs profile. Shear-wave velocity profiles and uncertainties are obtained from the velocity-depth marginal posterior probability density. This methodology is applied to passive seismic array recordings collected at several sites in Kitimat, British Columbia, Canada, a region where earthquake site response is of significant interest. The probabilistic Vs profile results ultimately provide probabilistic estimates of site response factors such as peak ground velocity/acceleration and Vs_{30}.

Keywords: Bayesian Inversion; Passive Seismic Array; Surface-wave Dispersion, Seismic site assessment

1. Introduction

It is well known that the level of shaking experienced during an earthquake is strongly dependent upon the local, near-surface geology. Specifically, as seismic waves transition from a medium of high seismic impedance (velocity times density) to a medium of low seismic impedance, conservation of energy requires the wave amplitude to increase. Sites located on soft, unconsolidated sediments experience higher-amplitude shaking than sites located on stiffer material or bedrock (other factors being equal). For this reason, determining properties of the near-surface geology, in particular the shear-wave velocity (Vs), is important for characterizing the expected ground response to earthquake shaking at a specific site.

This study focuses on the use of passive seismic recordings, or ambient vibrations, to infer near-surface geology at a specific site [1-5]. Passive techniques are increasingly popular because they often have lower demands in cost and logistics compared to other methods. The technique used here analyses two-dimensional (2-D) array recordings of ambient seismic noise to estimate the phase velocity of fundamental-mode Rayleigh waves at different frequencies. The relationship between phase velocity and frequency characterizes the dispersion of Rayleigh waves and is characteristic to the depth-dependent Vs structure. The earth structure at a particular site can therefore be inferred from the Rayleigh wave dispersion curve. This study applies an advanced Bayesian inversion methodology in which the posterior probability density (PPD) of the depth-dependent Vs structure models is estimated at a site. This provides an estimate of the most-probable Vs structure as well as a quantitative measure of the model uncertainty.

The theoretical earthquake response, and associated seismic hazard, at a site can be estimated given an earth-structure (Vs) model. Probabilistic inference of the Vs structure model can be extended to the probabilistic estimation of site response and seismic hazard categorization [6]. The inversion results presented in this study are used to provide probabilistic estimates of linear amplification factors of peak ground velocity/acceleration and Vs_{30} (the time-averaged velocity over the upper 30 meters of the subsurface Vs model). The study area for this work is the town of Kitimat, British Columbia, Canada (Fig. 1). Kitimat, and the surrounding region, have received a significant number of large infrastructure development proposals. Thus, estimating earthquake site response in this region is of particular interest.

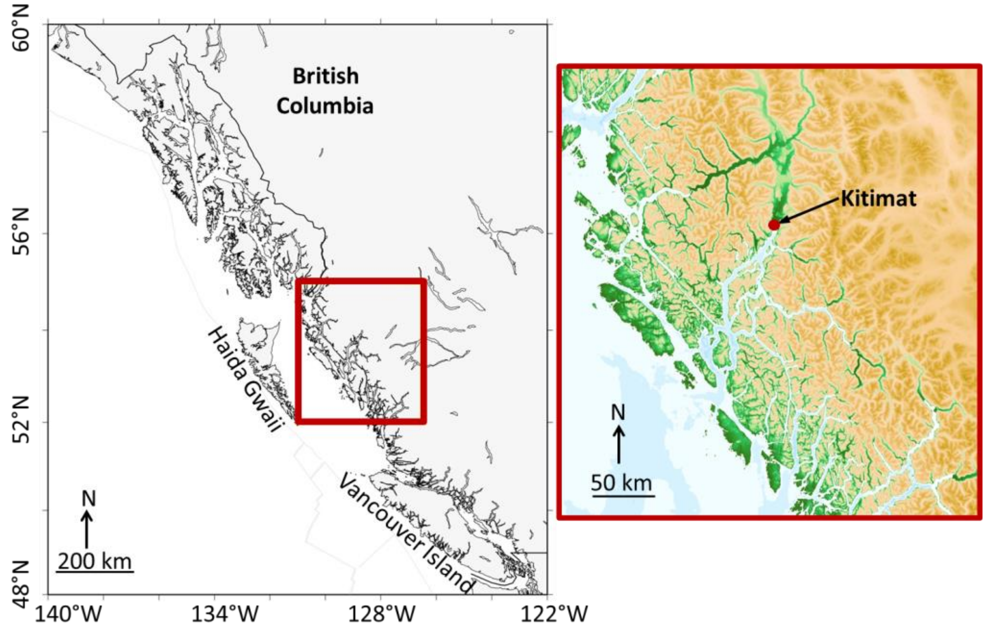

Fig. 1 – Location of Kitimat, British Columbia, Canada.

2. Methods

2.1 Estimating dispersion curves

This study applies the technique of estimating the dispersive characteristic of fundamental-mode Rayleigh waves from passive seismic recordings. Passive seismic recordings were conducted at three sites in Kitimat, Canada. At each site, five seismographs were deployed in a cross formation to simultaneously record approximately one hour of ambient seismic noise (Fig. 2). This procedure was repeated four times at each site, each time with a different instrument-separation distance.

The 2-D array recordings of seismic noise were processed using frequency-wavenumber (*f*-*k*) analysis [2, 3]. In *f*-*k* analysis, recordings from each instrument in the array are processed simultaneously to determine the direction and phase velocity of the dominant coherent signal in the noise field (assuming the dominant signal is produced by the fundamental-mode Rayleigh wave). Each frequency of interest is processed individually. At a given frequency, recordings are first sectioned into time windows of frequency-dependent length. For each time window, a brute-force search method is used to find the maximum coherency of the array in 2-D wavenumber space. The location of this maximum provides an estimate of the direction and, more importantly, the wavenumber vector magnitude of the Rayleigh wave for that time window (the wavenumber magnitude allows direct calculation of phase velocity). This process is repeated for all time windows to build up a histogram of phase velocity at a given frequency. The median of the histogram is taken as the estimated phase velocity (the median is used rather than the mean as the distributions are often asymmetrical). This process is then repeated for every frequency of interest. Only a small frequency band of the dispersion curve can be sampled using recordings from a particular array size due to resolution and aliasing limitations [3, 4]. By performing *f*-*k* analysis on the four different sets of recordings (with different instrument separation distance) for a given site, the dispersion curve is estimated over a larger frequency band. Fig. 2 shows the array configurations for the four sets of recordings, as well as the estimated dispersion curve at a site in Kitimat.

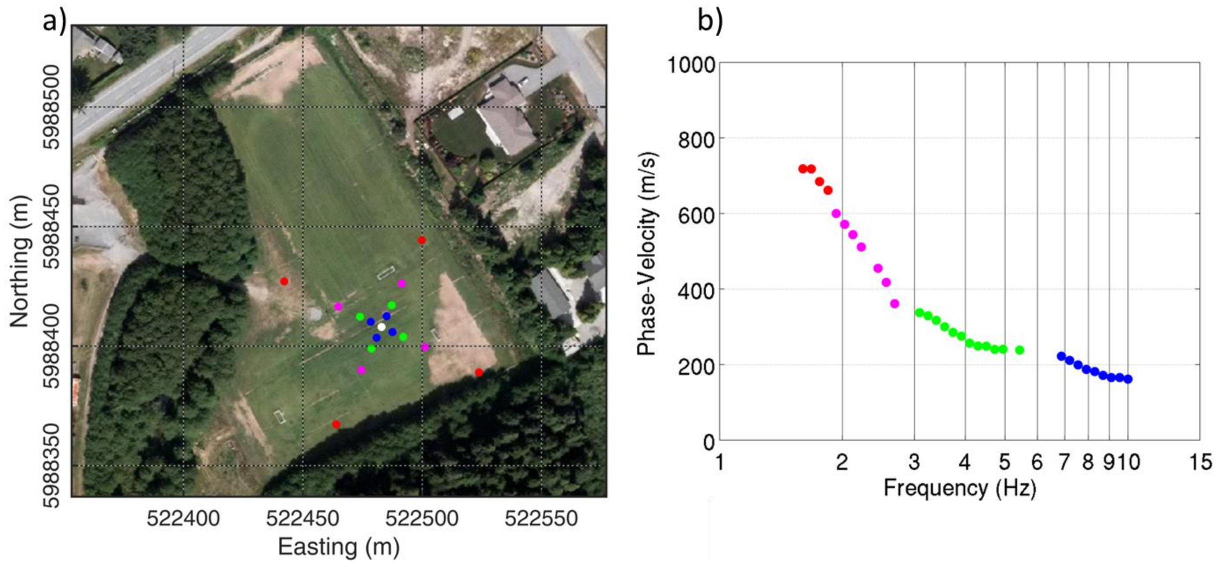

Fig. 2 – (a) Seismograph array configurations, and (b) estimated dispersion curve at Kitimat site 1. Colours indicate the correspondence between array configurations in (a) and portions of the dispersion curve in (b). The centre point was common for all array configurations.

2.1 Bayesian inversion and Bernstein polynomial representation

This study applies non-linear Bayesian inversion to estimate depth-dependent Vs structure from dispersion data. In Bayesian inversion, the parameters which describe the model are treated as random variables constrained by prior information and data. The solution to Bayesian inversion is described by properties of the PPD of the model parameters. Specifically of interest in this study is the Vs-depth marginal density. In this way, the most probable Vs profile can be determined from the PPD as well as a quantitative measure of the uncertainty in the Vs profile. Properties of the PPD are estimated by numerical integration. Typically, Markov-chain Monte Carlo (MCMC) sampling methods, such as Metropolis-Hastings sampling, are used to draw unbiased (dependent) samples from the PPD [7-9]. This study applies parallel tempering, within a Metropolis-Hastings framework, to efficiently search/sample the parameter space [10, 11]. Prior information on the model parameters is treated as uniform distributions, bounded within geologically reasonable values. Wide prior bounds are chosen to allow the data information, as opposed to the prior information, to generally constrain the inversion results. The inversion results are represented by 2 million model samples (after discarding initial burn-in samples). Data uncertainties are treated in terms of slowness (the reciprocal of phase-velocity) as errors in slowness scale linearly with errors in the wavenumber vector magnitude (the output of *f-k* analysis). For similar slowness error over frequency, this has the effect of increasing the phase-velocity errors at low frequencies. This is believed to be a physically reasonable representation of the frequency dependence of the data errors, and will be discussed in the subsequent section.

In many geophysical inverse problems, earth-structure profiles are modelled as a stack of homogeneous layers. This form of model parameterization is typically ineffective at resolving smooth, depth-dependent gradient structures, and can often introduce non-physical discontinuities in the inversion result. This study implements a Bernstein polynomial (over a half-space) representation of the Vs profile in the inversion to efficiently model general depth-dependent gradients [12]. The Bernstein polynomial is defined by the sum of Bernstein basis functions, weighted by corresponding coefficients [13, 14]. The Bernstein basis functions are pre-defined for a given polynomial order, and so the inversion estimates the coefficients which weight the individual basis functions in the sum. In this way, general Vs structure is represented using fewer parameters than a classical layered model parameterization. The distinctive advantageous property of the Bernstein polynomial, over other polynomial forms, is its stability. Specifically, small perturbations to the model parameters (basis-function coefficients) result in only small, localized perturbations to the Vs profile [12].

3. Results and Discussion

Fig. 3 shows the results of the Bayesian inversion (with Bernstein polynomial representation) of the data shown in Fig. 2. Specifically, Fig. 3(a) shows the Vs-depth marginal PPD, where warmer colours (red) represent higher probability and cooler colours (blue) represent lower probability (note the PPD is normalized independently at each depth to highlight structure over the entire profile). The inversion result shows that velocity increases gradually with depth down to approximately 100 m. There exists a discontinuity to significantly higher velocity at this depth, where the model transitions from a Bernstein-polynomial representation to a uniform underlying half-space. This velocity discontinuity is believed to be the transition between soft, unconsolidated Holocene sediments and harder basement material (bedrock or older, harder Pleistocene material) and is comparable to observations made in other, similar depositional environments along the west coast of British Columbia [4]. The width of the probability density indicates that velocities in the shallow sediments are well constrained relative to the velocity of the underlying basement material. In general, uncertainties increase with depth. Fig. 3(b) shows the mean of the distribution of predicted data generated from the MCMC sampling, as well as one-standard deviation error bars. This figure shows the inversion results is are good agreement with the measured dispersion data. The predicted data have larger standard deviation at low frequencies than at high frequencies. This is likely due to the treatment of data errors (as explained in the previous section). Note also that the width of the Vs marginal profile increases with depth, and the velocity in the basement is poorly constrained, consistent with a loss of resolution with depth of surface-wave dispersion data (the case with most geophysical techniques).

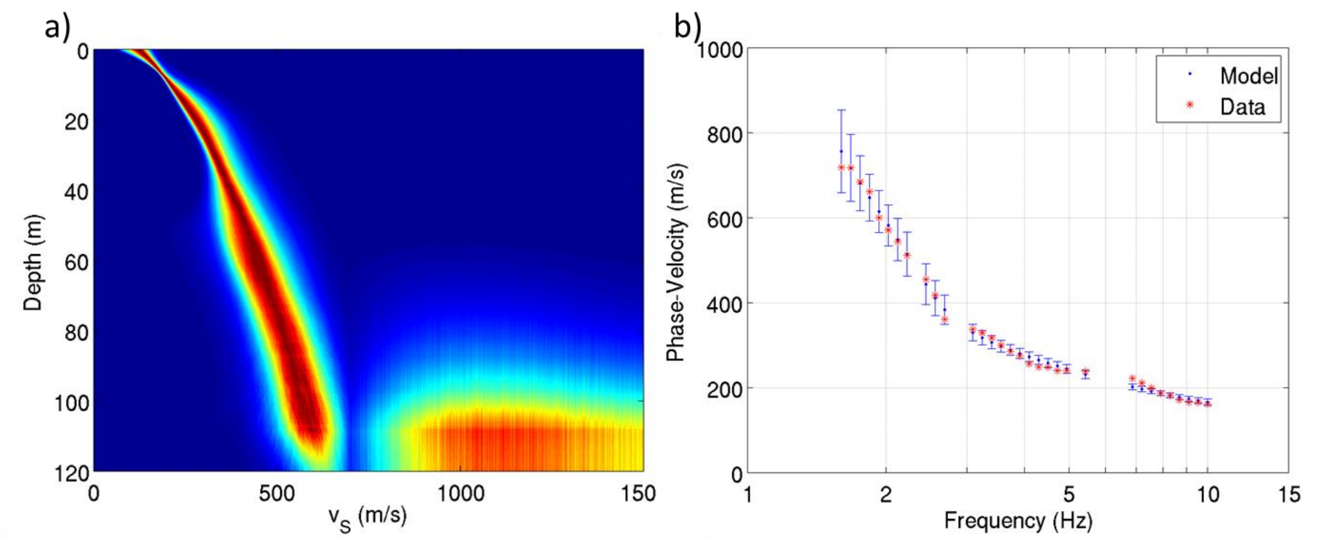

Fig. 3 – a) Velocity-depth marginal distribution, and b) fit to data of Bayesian inversion result for dispersion data collected at Kitimat site 1.

Given a particular earth-structure model, the theoretical earthquake response at the study sites can be estimated. The inversion results presented here allow for probabilistic estimation of site response and seismic hazard categorization [6]. Currently, many building codes and ground motion prediction equations (GMPE) rely on a parameter called Vs_{30} to classify seismic hazard and predict site amplification. Vs_{30} is defined as the time-averaged shear-wave velocity to 30 m depth. From this definition, Vs_{30} is weighted more heavily toward low-velocity material, which generally has a greater impact on seismic site amplification. Fig. 4(a) shows the probability density of Vs_{30} calculated from the MCMC samples of Vs profiles. Fig. 4(b) and (c) shows the linear site amplification term for peak ground acceleration and velocity from Boore and Atkinson's GMPE [15] computed for the Vs_{30} distribution in Fig 4(a). The National Building Code of Canada [16] adopts the site classifications of the National Earthquake and Hazards Reduction Program. Sites are classified into six different groups (A-F) based primarily on ranges of estimated or measured Vs_{30} values. Fig 4(a) shows 100% of the calculated Vs_{30} distribution falls within site classification D, or stiff soil.

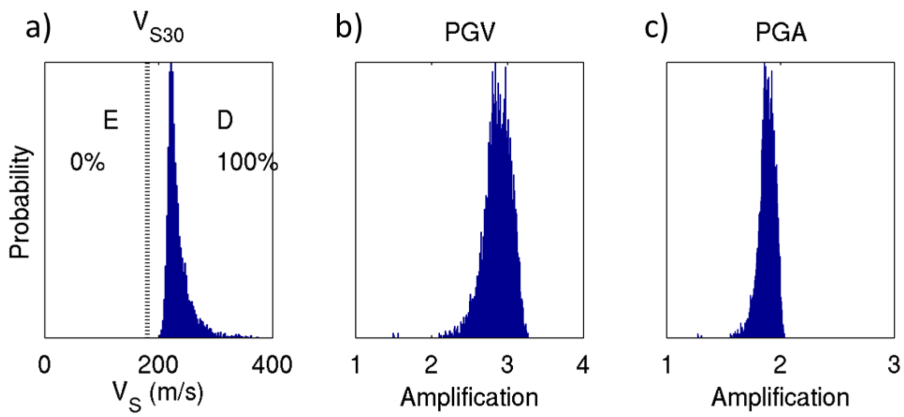

Fig. 4 – Probability densities for (a) Vs_{30}, (b) peak ground velocity, and (c) and peak ground acceleration.

4. Conclusions

This study applies an advanced Bayesian inversion methodology to estimate Vs profiles using surface-wave dispersion data processed from passive seismic-array recordings. The Bernstein polynomial model effectively represents the gradational trends in the Vs profile and estimates the total depth of the soft-sediment column. The Bayesian inversion scheme allows for quantitative assessment of model uncertainties and the extension to uncertainties in characterizing the expected ground response to earthquake shaking at sites in Kitimat, Canada. The ability to quantify the confidence in the estimated earthquake site response, and earthquake site classification, is valuable information for developers, planners, and public officials in the region.

5. Acknowledgements

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