



## WAVELET SPECTROGRAM ANALYSIS OF SURFACE WAVE METHOD FOR IN-SITU ATTENUATION AND DAMPING RATIO OF SOFT SOIL

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### **Abstract**

Soil attenuation and damping ratio are important parameters in soil dynamic analysis and design of structural dynamic. Both parameter can be obtained from the field measurement using the seismic measurement, e.g., the spectral analysis of surface wave (SASW). The SASW is one of seismic methods based on the dispersion of Rayleigh waves (R-waves) which is used to determine the shear wave velocity, shear modulus and depth of each layer of the soil profile. Seismic surface wave data detected in the seismic measurement are non-stationary in nature i.e. varying frequency content in time. Especially in the low frequency range measurement, i.e., in soft soil, the interested frequency of surface wave can be relatively low which can be less than 20 Hz. In these frequency values, the noisy signals may disturb in the identical frequency level of the surface wave signals generated from the source. In this paper, an improved technique of continuous wavelet transform (CWT) of derivative Gaussian function was employed in the seismic surface wave analysis in order to calculate the soil attenuation and damping of soft soils. The technique is then called as the wavelet spectrogram analysis of surface wave (WSASW). There are two procedures used in the analysis, i.e., first, time-frequency wavelet spectrogram was utilized to select the response spectrum of interest from recorded surface waves. Second stage is a time-frequency wavelet filtering which was used to remove noisy distortions in the selected spectrogram based on simple concept of wavelet filtering. Detected strong ground noises were able to be filtered out from the signals. Consequently, denoised signals representing the interested waveform of surface wave were reconstructed by inverse wavelet transform algorithm. Finally, clear pattern of phase spectrum can be easily calculated. Results showed that the wavelet spectrogram analysis of surface wave (WSASW) method is able to identify and reconstruct better spectrum seismic and phase velocity dispersion curve from surface wave measurement. Based on reconstructed seismic spectrum and velocity dispersion curve, soil attenuation and damping ratio can be obtained well. This technique can be applied to solve problems related to non-stationary seismic wave at soft clay soil.

*Keywords: damping ratio, attenuation, seismic, wavelet, surface waves*



## 1. Introduction

Soil attenuation is one of important parameters in geo-earthquake engineering problems associated with dynamic loading at low to moderate-strain levels, e.g., ground amplification during earthquake [1]. The attenuation parameter of soil can be either determined from the radiation and material damping of the soil structure. Attenuation in soil dynamics is a phenomenon that involves the interaction of several mechanisms that contributed to the energy dissipation of the seismic wave during dynamic excitation [2]. The parameter can be in situ evaluated by using seismic methods, i.e., measurement of wave velocities propagating through soil medium. The spectral analysis of surface wave (SASW) is one of common seismic techniques used for this purpose. Much of the basis of the theoretical and analytical work of this method for soil investigation has been developed. Current developments of the SASW method can be found in [3].

Many in situ and laboratory tests have been used to evaluate attenuation parameter. Rix et al. [2] investigated surface wave measurements to determine the attenuation and damping ratio of a layered soil deposit. In their studies, an attenuation curve was constructed from the observed spatial attenuation of Rayleigh wave amplitudes and then was inverted to obtain the material shear damping ratio. However, seismic data used in surface wave analysis are non-stationary in nature i.e. varying frequency content in time. Fourier transform that usually used by many researchers in seismic measurements works by expressing any arbitrary periodic function of time with period as sum a set of sinusoidal, thus some information of non-stationary seismic data in analysis maybe lost. In addition, the inability of conventional Fourier analysis to preserve the time dependence and describe the evolutionary spectral characteristics of non-stationary processes require tools which allow time and frequency localization beyond customary Fourier analysis. Wavelet analysis is becoming a common tool for analyzing localized variations of power within a time series. By decomposing a time series into time-frequency spectrum (TFW), one is able to determine both the dominant modes of variability and how those modes vary in time. The wavelet analysis has been used in numerous studies in geotechnical investigation, i.e., in situ shear modulus [3], phase velocity of soil structures [4], soil damping ratio [5]. The objective of this paper is to present the ability of wavelet spectrogram analysis of surface wave (WSASW) to evaluate in-situ attenuation factor and soil damping ratio at soft soil sites. Result and its application from field study carried out at soft soil site are also presented.

## 2. Research Methodology

### 2.1 Wavelet Analysis

A wavelet can be defined as a function of  $\psi(t) \in L2(\mathfrak{R})$  with a zero mean localized in both time and frequency. By dilating and translating the wavelet  $\psi(t)$ , a family of wavelets can generally be produced as:

$$\psi_{\sigma,\tau}(t) = \frac{1}{\sqrt{\sigma}} \psi\left(\frac{t-\tau}{\sigma}\right) \quad (1)$$

where  $\sigma$  is the dilation parameter or scale and  $\tau$  is the translation parameter ( $\sigma, \tau \in \mathfrak{R}$  and  $\sigma \neq 0$ )

The continuous wavelet transform (CWT) is then defined as the inner product of the family wavelets  $\Psi_{\sigma,\tau}(t)$  with the signal of  $f(t)$  which is given as:

$$F_W(\sigma,\tau) = \int_{-\infty}^{\infty} f(t) \frac{1}{\sqrt{\sigma}} \overline{\psi}\left(\frac{t-\tau}{\sigma}\right) dt \quad (2)$$



where  $\bar{\psi}$  is the complex conjugate of  $\psi$  and  $F_W(\sigma, \tau)$  is the time-scale map. The convolution integral from equation 2 can be computed in the Fourier domain. In order to reconstruct the function  $f(t)$  from the wavelet transform, Calderon's identify [6] can be used and is obtained as:

$$f(t) = \frac{1}{C_\psi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F_W(\sigma, \tau) \bar{\psi}\left(\frac{t-\tau}{\sigma}\right) \frac{d\sigma}{\sigma^2} \frac{d\tau}{\sqrt{\sigma}} \quad (3)$$

$$C_\psi = 2\pi \int \frac{|\hat{\psi}(\omega)|^2}{\omega} d\omega < \infty \quad (4)$$

where  $\hat{\psi}(\omega)$  is the Fourier transform of  $\psi(t)$ . The integrand in equation 4 has an integrable discontinuity at  $\omega = 0$  and implies that  $\int \psi(t) dt = 0$ .

The total energy contained in a signal,  $x(t)$ , is defined as its integrated squared magnitude as follows:

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt \quad (5)$$

The relative contribution of the signal energy contained at a specific  $\sigma$  scale and  $\tau$  location in the CWT is given by the two-dimensional wavelet energy density function:

$$P_{F_W}(\mu, \xi) = |F_W(\sigma, \tau)|^2 \quad (6)$$

A plot of  $P_{F_W}$  is known as a scalogram which is analogous to the spectrogram and the power spectrum density (PSD) surface of the STFT (short-time-Fourier-transform). The scalogram can be integrated across  $\sigma$  and  $\tau$  to recover the total energy in the signal as follows:

$$P_{F_W} = \frac{1}{C_g} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} P_{F_W}(\sigma, \tau) \frac{d\sigma}{\sigma^2} d\tau \quad (7)$$

where  $C_g$  is the admissibility constant of the wavelet function  $\psi(t)$ .

## 2.2 WSASW Procedure for Attenuation and Soil Damping Ratio Analysis

A proposed procedure used in attenuation analysis of soil structures is described in Figure 1 as follows.

1. Select the wavelet function and a set of scale,  $s$ , to be used in the wavelet transform. The different wavelet function may influence the time and frequency resolution. In this study, a Morlet wavelet function was selected as a mother wavelet in the CWT analysis.

A commonly used wavelet in CWT is the Morlet wavelet where its shape is a Gaussian-windowed complex sinusoid. It is defined in the time and frequency domains as follows:

$$\Psi_0(t) = \pi^{-1/4} e^{imt} e^{-t^2/2} \quad (8)$$



$$\hat{\psi}_0(s\omega) = \pi^{-1/4} H(\omega) e^{-(s\omega - m)^2 / 2} \quad (9)$$

where  $m$  is the wavenumber, and  $H$  is the Heaviside function. In this study, a Morlet wavelet is within an adjustable parameter  $m$  of 7 which is used. This parameter can be used for an accurate signal reconstruction of seismic surface waves in low frequency. In addition, the Gaussian's second order exponential decay used in time resolution plot results in the best time localization.

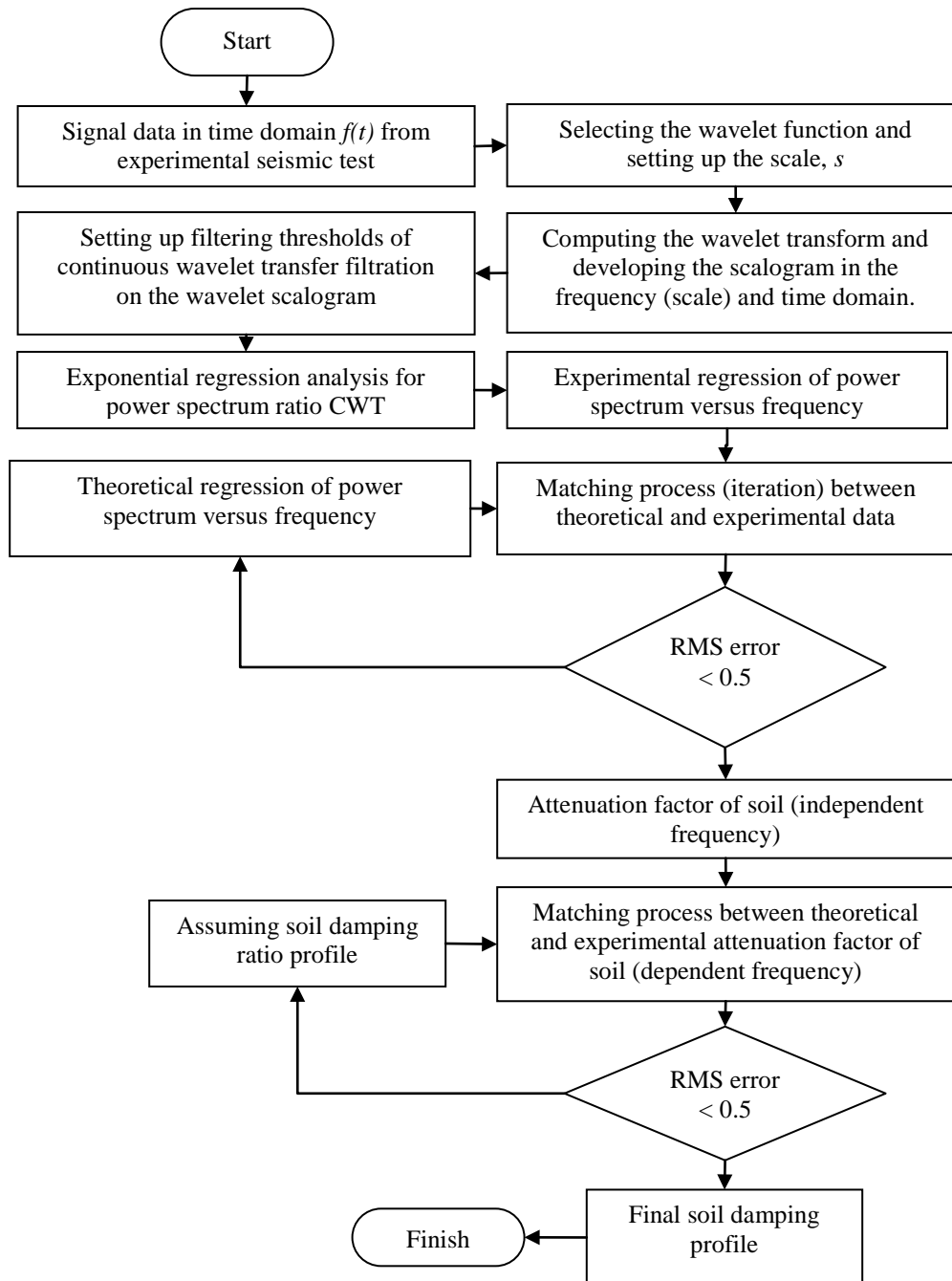


Fig. 1 – Flow chart of attenuation and soil damping ratio calculation in WSASW technique



2. Develop the wavelet scalogram by implementing the wavelet transform (Eq. 2) using computed convolution of the seismic trace with a scaled wavelet dictionary. Wavelet scale is calculated as fractional power of 2 using the formulation [7]:

$$S_j = S_0 2^{j\delta_t}, j = 0, 1, \dots, J \quad (10)$$

$$J = \delta_j^{-1} \log\left(\frac{N\delta_t}{S_0}\right) \quad (11)$$

where,  $s_0$  is smallest resolvable scale =  $2\delta_t$ ,  $\delta_t$  is time spacing, and  $J$  is largest scale. Both equations proposed by Torrence and Compo [7] were used to generate scalogram containing wavelet energy spectrum.

3. Convert the scale dependent wavelet energy spectrum (scalogram) or power spectrum density of the signal to a frequency dependent wavelet energy spectrogram in order to compare directly with Fourier energy spectrum.
4. Perform the CWT filtration on the wavelet spectrogram by obtaining the time and frequency localization thresholds. In this study, the CWT filtration was developed by a simple truncation filter concept which only considers the passband and stopband. Threshold values in time and frequency domain are then set as the filter values between passband and stopband. It allows a straight filtering in each of the dimensions of times, frequencies and spectral energy. The noisy or unnecessary signals can be eliminated by zeroing the spectrum energy and consequently, they are fully removed when reconstructing the time domain signal. Thus, the interested spectrum of signals is to be passed when the spectrum energy is maintained in original value. A design of the CWT filtration is proposed by [3].
5. Reconstruct the time series of seismic trace using equation 3 and generate the power spectrum density or spectrogram from denoised signals.
6. Generate PSD ratio from both signals as experimental power spectrum ratio versus frequency using the linear regression.
7. Generate the theoretical regression of power spectrum density (amplitude) ratio versus frequency using following equation [5]:

$$\ln\left[\frac{W_f^{R_2}(u,s)}{W_f^{R_1}(u,s)}\right] = \ln\left[\left\{\frac{R_1}{R_2}\right\}^n \{G(R) \cdot G(I) \cdot K(R)\} e^{-\alpha(f)(R_1-R_2)}\right] \quad (12)$$

$$\ln\left[\frac{W_f^{R_2}(u,s)}{W_f^{R_1}(u,s)}\right] = k - \alpha(f)(\Delta R) \quad (13)$$

where,  $R_1$  and  $R_2$  = geophones distance from the sources (if using two geophones),  $G(R)$  = geometric spreading factor,  $G(I)$  = instrumentation correction factor and  $K(R)$  = correction for refracted and transmitted waves,  $\alpha$  = independent frequency-attenuation factor. Power density spectrum equation was proposed by Rosyidi and Taha [5] to calculate the power spectrum ratio in each frequency of seismic waves generated from received signals.

8. By matching the data of theoretical to the experimental regression line, the attenuation factor of soil structures can be obtained. By repeating the procedure outlined above the attenuation factor corresponding to each wavelength is subsequently generated.

9. An effective soil damping ratio of R-wave in layered medium can be defined from the attenuation analysis and the value is frequency dependent. Its value may become very high for the first few modes of vibration. Finally, to obtain the soil damping ratio profile from the attenuation curve, the inversion analysis by using the procedure proposed by [8] and [9] was performed.

### 2.3 Field procedure of surface wave measurement

In this study, the multi-channel of WSASW was employed to collect the seismic surface wave data for soil dynamic evaluation. A configuration set up on the WSASW measurement is shown in Fig. 2. An impact source of 8 to 12 kg and 20 to 25 kg dropped weight was used to generate seismic waves. These waves were then received using two 1-Hz frequency natural vertical geophones. Thus, they were recorded by using a set of spectrum analyser for processing (Fig. 2).

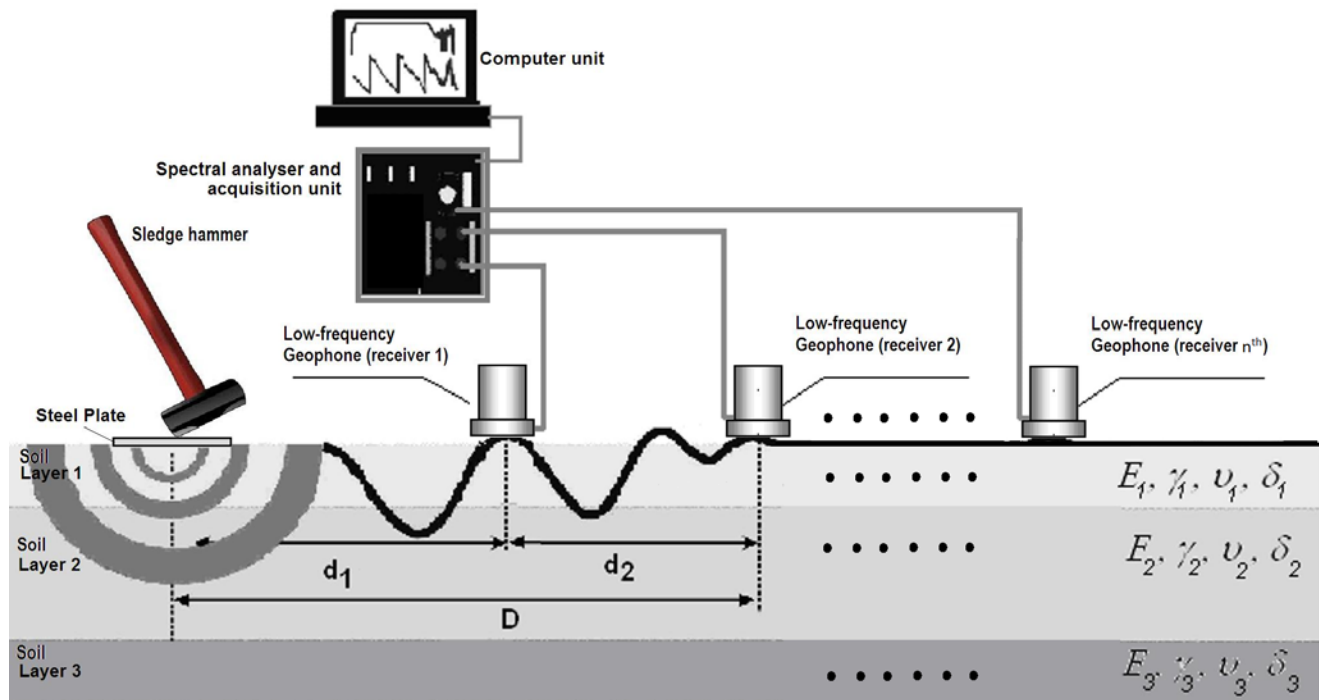


Fig. 2 – WSASW field measurement set up using multi-channel geophones on the soil sites

In order to collect seismic data in the field measurement, multi-channel geophones configuration of 1 and 2 m of the receiver spacings were required to sample different soil depths. The short receiver spacings with a high frequency source may be used to sample the shallow layers of the soil profile while the larger receiver spacings with a set of low frequency sources can be employed to sample the deeper soil layers.

## 3. Result and Discussions

### 3.1 Soil Properties of Test Site

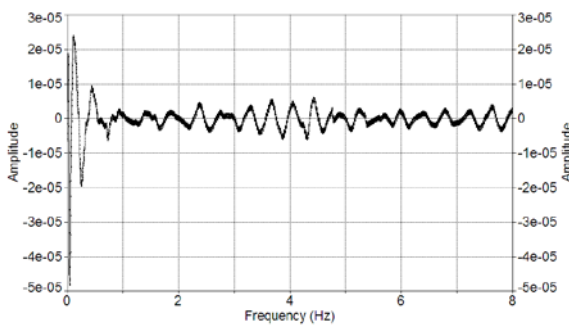
The soil tests were conducted in Kelang, Malaysia. From the site investigation, it was shown that the soil type is greyish clay with decayed wood at most of the soil layers of the subsoil stratum. The site was geologically classified as recent quaternary of dominantly alluvial deposits of soft marine clay with traces of organics.

### 3.2 Response Spectrum and Power Spectrum Density

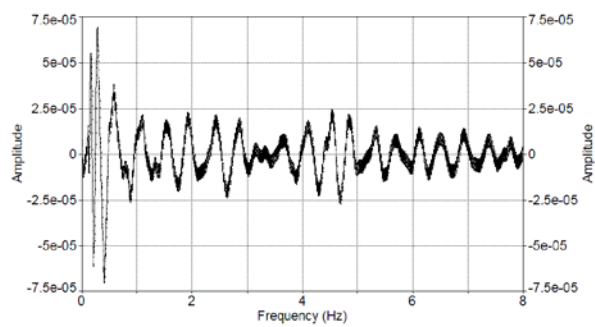
Fig. 3 shows the recorded signals from multiple impacts of a source. The seismic data was recorded using field configuration of 8 m receivers (geophones) spacing. From the recorded signals, it can be recognized that higher

amplitude is measured for first mode of R-wave amplitude. It is also noted that the decreasing signal magnitude is identified as the R-wave attenuation in the soil layer which is an important characteristic for energy decrement. The waveform of seismic signal recorded in measurement is transient and non-stationary event. Weak recorded signal of seismic wave particularly in channel 2 is also identified as an effect of environmental noise which maybe produced from ground noise and man-made vibration. This means that either the input signals or behaviours of system at different moments in time were not identical. When the signals were transformed into frequency domain (Figure 4), time-dependent behaviour of the seismic waves and noise events may be lost. In the energy content which these events present at different times and frequency, would not be picked up by a conventional Fourier analysis. It also cannot instantly separate the event of true seismic waves from noise signals. Consequently, it is difficult to interpret the correct energy of waves in both signals.

The time-frequency (TF) analysis of CWT was then employed to overcome the identification problem of spectral characteristic of non-stationary seismic wave signals and conduct the filtration analysis to reconstruct the interested signals from measurement. Filtration technique used in this analysis was recommended by [3] which is based on time-frequency thresholds. The technique can identify and remove the noise spectrum from the recorded signals. Denoising and cleaning noise signals are possible to improve the clarity of response spectrum analysis.

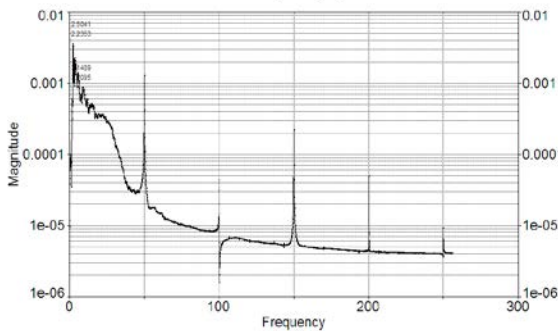


(a). Signal from Geophone 1

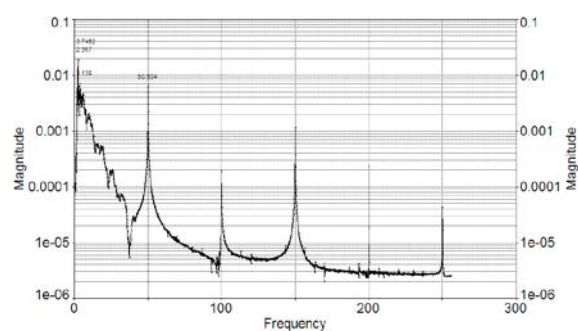


(a). Signal from Geophone 2

Fig. 3 – Seismic surface wave signals recorded from two channels of geophone with 8 m geophone spacing



(a). FFT of signal from Geophone 1



(b). FFT result of signal from Geophone 2

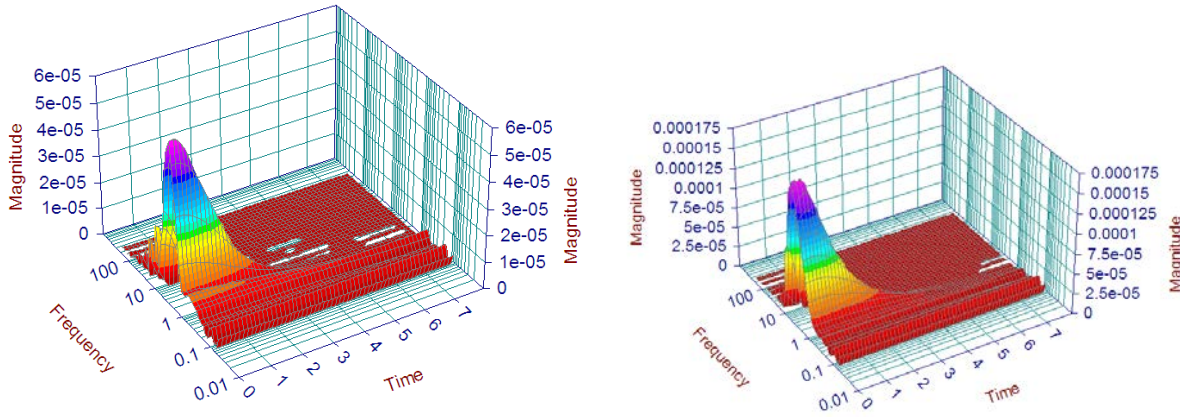
Fig. 4 – Fourier amplitude from both signals

There is a primary step to set the thresholds for wavelet filtering. It is to define a region of time-frequency space. This is mainly used to select appropriate signal energy event and to reconstruct signal components. The time and frequency fields define limits in spectrogram filtering. It means that the noise signals are removed from the spectrogram and only seismic wave signals of interested exist. The inverse wavelet transform then gives back a denoised seismic signal. The power spectrum density of wavelet or spectrogram for both denoising signals is





shown in Fig. 5. The spectrum range of the seismic waves of interest was found in the range of 5 to 30 Hz and 5 to 35 Hz for signals recorded on channel 1 and 2, respectively. The energy attenuation is also visibly identified from both spectrums.



(a). Signal from channel 1 (b). Signal from channel 2

Fig. 5 – Power spectrum density of wavelet or spectrogram from both denoised signals

### 3.3 Attenuation analysis and soil damping ratio

From Fig. 5, an experimental data trend of power spectrum ratio between both signals from logarithmic natural ( $\ln$ ) function of spectrogram ( $W_f^{R_2}$ ) over the first signal magnitude ( $W_f^{R_1}$ ) versus frequency can be obtained. This ratio represents as the decay factor curve of frequency dependency from the R-wave motion (Figure 6). A simple linear regression analysis is subsequently performed on the experimental data of decay factor curve. The experimental regression equation is produced as:

$$\ln \left[ \frac{W_f^{R_2}(u, s)}{W_f^{R_1}(u, s)} \right] = -0.0244(f) + 2.3025 \quad (14)$$

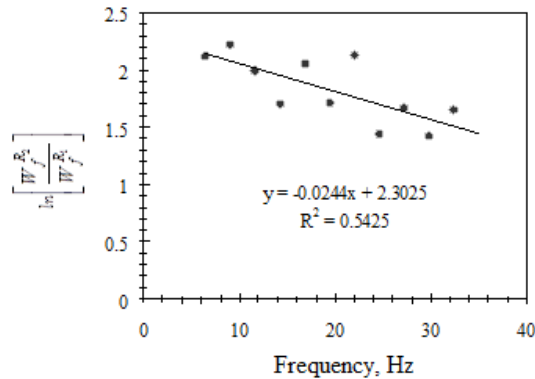


Fig. 6 – Regression analysis of attenuation coefficient of the soil from Fig. 5.

Coefficient of determination ( $R^2$ ) was obtained from the plot is 0,5425, it shows the analysis of variance between ratio of energy spectrum versus frequency. It also presents the proportion of the total variation in energy





spectrum accounted for by the regression model. It means ratio of spectrum accounts for 54,25 % of the total variation in frequency. This means that other 55,75% of the variation may be due to inherent variability in frequency or to other factors, i.e., effect of geometric and disturbance of low frequency noises. The theoretical regression analysis of attenuation derived from Eq. 13 can then be written as:

$$\ln \left[ \frac{W_f^{R_2}(u, s)}{W_f^{R_1}(u, s)} \right] = -\alpha(f)(\Delta R) + k \quad (15)$$

$$= -8\alpha_0(f) + k$$

The best-fit curve is then established between the decay factor of the experimental data (Eq. 14) and the theoretical regression analysis equation (Eq. 15) by trial and error for different values of  $\alpha_0$  from visual best-fit evaluation of the two curves. The best-fit value of frequency-independent attenuation coefficient of the soil is calculated as  $3.05 \times 10^{-3}$  s/m at frequency of 5 to 35 Hz. The root-mean-square (RMS) error for this fitting curve is found to be 0.2.

By repeating the procedure for attenuation analysis in each frequency value for all seismic data, the experimental attenuation curve is subsequently generated. An example of attenuation versus frequency curve at the soil site is presented in Fig. 7. By knowing the experimental attenuation profile, the shear damping ratio can be obtained by inversion process. In the inversion analysis, the soil model is typically assumed as the homogeneous linear elastic layers over a half-space with model parameter of shear wave velocity, shear damping ratio and thickness for each layer. Due to shear damping ratio is unknown data and no prior information is provided from the previous field or laboratory soil data, therefore, the soil model for inversion process should be assumed with rational values for soil model parameter. These values in soil model is then automatically adjusted during the inversion process.

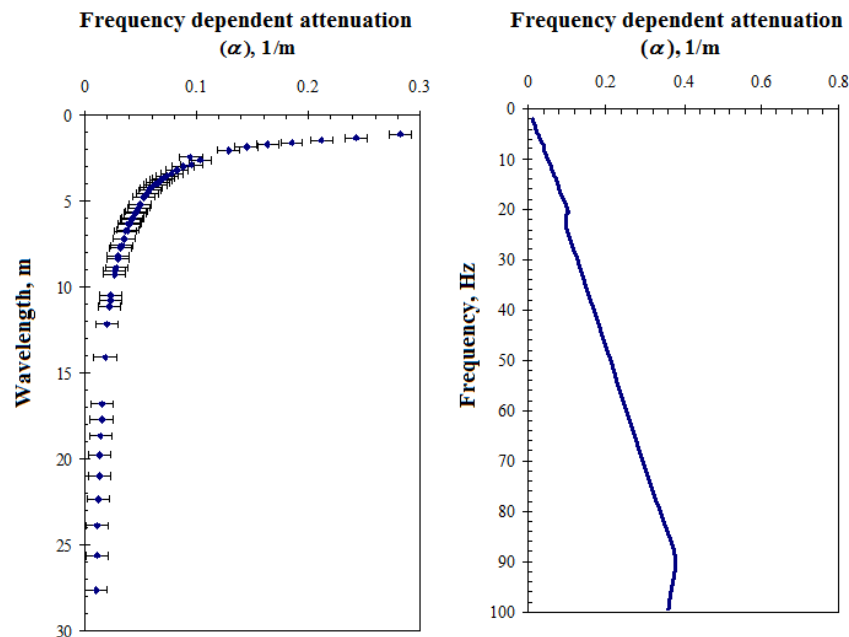


Fig. 7 – Frequency dependent attenuation profile in wavelength and frequency domain

The inversion analysis is processed by using Herrmann (1994) [11] code based on a weighted, damped, least-squares algorithm. Fig. 8 shows comparisons between the experimental attenuation curves and the

theoretical attenuation data. Iteration processes were conducted to match between both experimental and theoretical curves. Fig. 8 presents the final profiles of shear damping ratio for the last iteration of the inversion process with lowest RMS error.

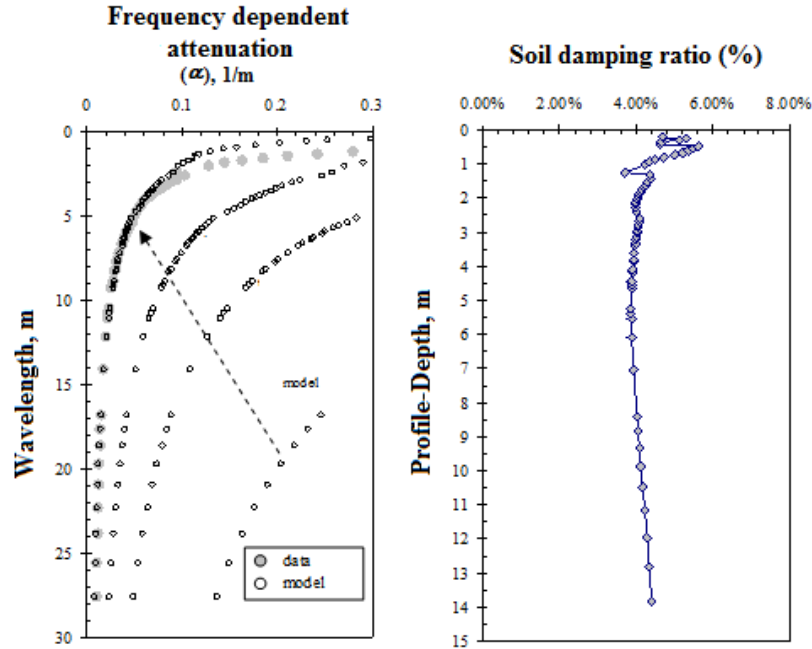


Fig. 8 – Soil damping ratio based on frequency dependent attenuation profile

#### 4. Conclusion

In this paper, an improved seismic method of the wavelet spectrogram analysis of surface waves (WSASW) technique for measurement of the soil dynamic properties at soft soil site is presented. The denoising and reconstruction technique of the response spectrum from surface wave propagation on a soft soil using time-frequency spectrogram analysis of continuous wavelet transforms is also proposed. The 2-D wavelet spectrogram is able to clearly identify the various events of interest of the seismic surface waves and noisy signals. Based on the generated spectrogram, the thresholds for CWT filtration could be easily obtained. Consequently, the denoised signals of the seismic surface waves were able to be reconstructed by inverse wavelet transform considering the thresholds of the interested spectrum. Finally, the WSASW is also able to evaluate the soil dynamic properties, i.e., soil attenuation and damping ratio properties at soft clay soil site as performed in this study.

#### 5. Acknowledgements

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