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INVESTIGATION OF INCIDENCE ANGLE EFFECTS ON 2-D SOIL AMPLIFICATION OF BEDROCK SEISMIC MOTIONS: A DISCRETE TIME APPROACH

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Abstract: The problem of seismic soil amplification is of particular importance in relation to the assessment of local soil condition effects on seismic motions characteristics in soil deposits. Most of the practical studies have been limited to 1-D amplification of vertically propagating SH-waves considering essentially the problem in the frequency domain. In this paper, the approach is extended to an investigation of the effects of incident angles on 2-D amplification of *obliquely* propagating shear waves based on a discrete–time wave propagation formulation. Simple analytical models of seismic site amplification with respect to bedrock for *obliquely* propagating shear waves in both elastic and viscoelastic soil deposits overlying bedrock are derived. Emphasis is placed on the significance of incidence angle effects on seismic site amplification, considering different values of rock – soil impedance ratios. Sensitivity analyses are performed utilizing the soil profile model of the site of El–Asnam Cultural Center, in the El–Chellif region (North-Western Algeria) formulated on the basis of field test and laboratory investigations. Numerical results are discussed in terms of site amplification functions, spectral peak ratios and simulated surface acceleration time histories at the studied site.

Keywords: Non vertical incidence, Bedrock seismic motions, 2-D soil amplification, Discrete-time approach, El-Asnam Earthquake.

1Introduction

The problem of seismic site amplification has been the subject of considerable interest and research in recent yearse. g. (Trifunac and Todorovska, 2000; Zembaty, 2002; Zhao et al., 2006; Erdik and Toksöz, 2013).Site amplification is of importance in relation to assessment of the influence of local soil conditions on earthquake ground motions and site effectse. g.(Idriss, 1990), on ground motion modeling and simulation e. g.(Hammoutène et al., 1992; Stewart et al., 2012), and site classification to be used for microzonation studies and building codes development e. g**.** (Zhao et al., 2006). It can also be a critical factor influencingstructural response and the extent of damage on structures erected on soft soils e. g. (Navarro et al., 2012).

Various methods have been used to estimate seismic site amplification: a phenomenon the causes and effects of which are still not very well understood. A review of the generating mechanisms and models used to characterize site amplification from earthquake records, can be found ine. g. (Safak, 1997).

Extensive reviews on seismic site amplification, including experimental and empirical models, are available in the specialized literature e. g (Safak, 1995). However, most of the practical applications have considered the problem of vertically propagating shear waves(i. e. of 1-D soil amplification)e. g. (Ordonez, 2010). Further, almost all the previous investigations have considered the problem in the frequency domain.

In this paper, the approach is extended to an investigation of the effects of incident angles on site amplification of *obliquely* propagating shear waves (i. e. 2-D soil amplification) in both elastic and viscoelastic soil models overlying *bedrock*. Moreover, a discrete – time wave propagation formulation is utilized for this purpose. Emphasis is placed on the significance of incidence angle effects on seismic site amplification with respect to bedrock motions considering different rock / soil impedance ratios. Sensitivity analyses are performed in an attempt to predict soil amplification and surface motions at the site of El – Asnam Cultural Center in the El - Chellif region (North - Western Algeria), formulated on the basis of field test and laboratory investigations.Sensitivity of the numerical results to wave incidence angles is discussed in terms of site amplification functions, peak ratios and generated site specific surface motions.

2Discrete – time approach for site amplification of bedrock motions in elastic and viscoelastic media

In what follows, the discrete time wave propagation approach for seismic site amplification with respect to bedrock will be formulated first by considering vertically incident SH wave propagation through an elasticsoil layer overlying bedrock (subsection 2.1). The approach is then extended to the study of seismic site amplifications of *obliquely* incident SH waves in elastic (subsection 2.2) and viscoelastic (subsection 2.3) media. Multilayered soil media may be dealt with, in an approximate manner, using an equivalent soil layer model and applying the analytical models developed in this paper.

2.1 Seismic site amplification of vertically incident SH waves ($\alpha_2 = \alpha_1 = 0$) in an elastic medium

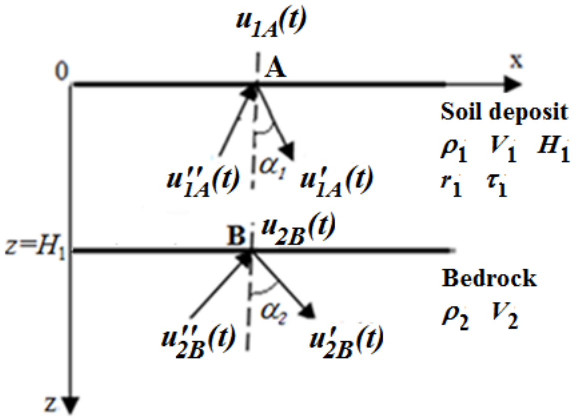

Fig. 1: Incident and reflected waves in an elasticsoil layer overlying bedrock

Figure 1 above shows incident and reflected waves in an elastic soil layer overlying bedrock. Vertically up-going SH waves ($\alpha_2 = \alpha_1 = 0°$)from soil-bedrock interface are filtered by the soil layer, transforming the bedrock motion $u_{2B}(t)$ into surface vibration$u_{1A}(t)$.The soil bedrock interface is located at depth $z = H_1$. The double prime ($''$) and single prime ($'$) notations are respectively associated with the incident and the reflected waves. Subscripts 1 and 2 of angles of incidence α, mass density ρ and shear wave velocity V, refer to soil layer and rock respectively.

Considering the transmission of waves through the soil layer shown in Fig. 1 and applying the rules of transmission and reflection of vertically propagating waves (see e. g. (Safak, 1995; Aki and Richards, 2002)), one can define the bedrock motion as follows:

$$u_{2B}(t) = u''_{2B}(t) + u'_{2B}(t) + (1-r)u'_{1A}(t) \qquad (1)$$

where,

$$u'_{2B}(t) = r u''_{2B}(t) \qquad (2)$$

in which $r = \frac{\rho_2 V_2 - \rho_1 V_1}{\rho_2 V_2 + \rho_1 V_1}$ is the reflection coefficient of the vertically up-going wave and $(1-r)$ the transmission coefficient of the down-going wave. Equation (1) states that the movement at the top of the bedrock is the result of the upgoing wave at the top of the bedrock, plus its reflected portion, andthe transmittedpart of the downgoing wave at the bottom of the soil layer.

One can also write the following set of equations in order tolink the bedrock motion $u_{2B}(t)$to the surface vibration $u_{1A}(t)$utilizing the down-going reflected component$u'_{1A}(t)$, the refracted (up-going) transmitted component $u''_{1A}(t)$and the incident wave $u''_{2B}(t)$:

$$u''_{1A}(t) = -r u'_{1A}(t-\tau) + (1+r) u''_{2B}(t-\tau) \qquad (3)$$

$$u'_{1A}(t) = u''_{1A}(t-\tau) \qquad (4)$$

$$u_{1A}(t) = 2u''_{1A}(t) \qquad (5)$$

in which $(1+r)$ is the transmission coefficient of the up-going waves and $\tau = H_1/V_1$ is the one way wave propagation time in the soil layer. Equation (3) states that the upgoing wave at the top of the layer equals to the reflected portion of the downgoing wave at the bottom of the layer, plus the transmitted part of the incident wave from the bedrock. Equation (4) shows that the downgoing wave at the bottom of the layer is the upgoing wave reflected by the free surface. Equation (5) assumes that the reflection coefficient at the free surface is two. This being the case, we can re-write Eq. (3) as follows:

$$u''_{1\mathrm{A}}(t) = -u'_{1\mathrm{A}}(t-\tau) + (1-r)u'_{1\mathrm{A}}(t-\tau) + \quad u''_{2B}(t-\tau) + r u''_{2B}(t-\tau) \tag{6}$$

which by considering Eqs. (1), (4) and (5), leads to

$$u_{1A}(t) + \ u_{1A}(t-2\tau) = 2u_{2B}(t-\tau) \tag{7}$$

Equation (7) is a recursive filter for calculating the surface motions at the soil site from the bedrock motions at the soil-bedrock interface, assuming no damping in the soil. Equation (7) uses only one parameter, τ, and is exact for an elastic, homogeneous soil layer over bedrocksubjected to vertically propagating waves. Taking the Fourier transform of both sides of Eq. (7) and denoting the Fourier transforms of $u_{1A}(t)$ and $u_{2B}(t)$by $U_{1\mathrm{A}}(f)$ and $U_{2\mathrm{B}}(f)$respectively, leadsto the soil transfer function $H_\mathrm{B}(f)$with respect to bedrock,assuming no damping:

$$H_\mathrm{B}(f) = \frac{U_{1\mathrm{A}}(f)}{U_{2\mathrm{B}}(f)} = \frac{2\, e^{-i2\pi f\tau}}{1 + e^{-i4\pi f\tau}} \tag{8}$$

which, after some algebra, leads to

$$H_\mathrm{B}(f) = \frac{1}{cos(2\pi f\tau)} \tag{9}$$

the amplitude of which is:

$$|H_\mathrm{B}(f)| = \frac{1}{|cos(2\pi f\tau)|} \tag{10}$$

To determine the resonant frequencies, we make the derivative of$|H_\mathrm{B}(f)|$ with respect to $f$equal tozero leading to$cos(2\pi f\tau) = 0$and hence to the resonant frequencies $f_\mathrm{k} = k/(4\tau) = kV_1/(4H_1)$ where k = 1, 3, 5, etc.…which shows that the parameter τdetermines the location of the spectral peaks of the site amplification function.

It is also important to note that the transfer function$H_\mathrm{B}(f)$does not depend on the properties of the underlying bedrock. It corresponds to the assumption of a rigid based displacement imposedat the bedrock level similar to the movement induced by a shaking table.

2.2 Transfer function of *obliquely*incident SH waves in an elastic medium

It can be shown from (Kausel E and Roêsset, 1984) that the solution of the 2-D SH-wave propagation problem with non vertical incidence is identical to the 1-D solution with equivalent shear wave velocity $V_{\mathrm{e}1}$ in the z direction:

$$V_{\mathrm{e}1} = \frac{V_1}{\sqrt{1-\left(\frac{V_1}{V_2}\right)^2 sin^2\alpha_2}} \tag{11}$$

Hence, the one way wave propagation time in the same direction in the elastic soil layer with equivalent properties is given by:

$$\tau_\mathrm{e} = \frac{H_1}{V_{\mathrm{e}1}} \tag{12}$$

It follows from Eq. (9)that the transfer function of obliquely incident SH waves can be expressed as:

$$H_{\mathrm{B},\alpha}(f) = \frac{1}{cos(2\pi f\tau_e)} \tag{13}$$

the amplitude of which is

$$\left|H_{\mathrm{B},\alpha}(f)\right| = \frac{1}{|cos(2\pi f\tau_e)|} \tag{14}$$

with resonant frequencies given by

$f_k(f) = k\frac{V_{e1}}{4H_1}$, where k = 1, 3, 5, …. (15)

Moreover, continuity requirements for the shearing stresses across the soil – bedrock interface result in the following relationships:

$$\mu_{e1} = \mu_1 \tag{16}$$

$$\rho_{e1} = \rho_1\left(1 - \left(\frac{V_1}{V_2}\right)^2 sin^2\alpha_2\right) \tag{17}$$

where μ_{e1}, ρ_{e1} are respectively the equivalent shear modulus and mass density associated with the 1-D equivalent soilprofileand μ_1, ρ_1 those associated with the 2-Dwave propagation model with non vertical incidence. Thus, the discrete-time wave propagation formulation for vertically incident SH waves can be used to study the site amplification of a soil profile subjected to *obliquely* incident SH waves. It suffices to replace in the soil layer, the actual properties by the equivalent soil properties as defined above. It must be noted that although this method is simple, it is *exact*.

It should also be noticed that in general, $\frac{V_1}{V_2} < 1$ so that $\rho_{e1} \leq \rho_1$, i.e. the equivalent soil profile is lighter and the amplification peaks for non vertical incidence will evidence a shift towards the higher frequencies of the order of V_{e1}/V_1. It follows that:

- For the soft bedrock case, $V_1 < V_2$, so that $V_{e1} \neq V_1$, and a slight frequency shift should be observed.
- For the rigid bedrock case, $V_1 \ll V_2$, so that $V_{e1} = V_1$, and no frequency shift could be observed.

This being the case, it should be pointed out that the antiplane soil motion $u(x,z,t)$caused by an obliquely propagating plane wave of frequency *f*= 2*π*/*ω*and unit amplitude is of the form:

$$u(x,z,t) = \left[A_1 \exp(-i\frac{2\pi f}{V_{e1}}z) + A_2\exp(+i\frac{2\pi f}{V_{e1}}z)\right]\exp(-i\frac{2\pi f\ sin\alpha_1}{V_1}x)\exp(i\ \omega\ t) \tag{18}$$

which characterizes a problem of 2-D wave propagation.Since$\left|\exp(-i\frac{2\pi f\ sin(\alpha_1)}{V_1}x)\right| = 1$, it is seen that the 2-D site amplification problem with non-vertical incidence is identical to the 1-D site amplification with equivalent shear wave velocity. However, it should be emphasized that the exponential factor in 'x' introduces additional phase shifts in the generatingof site specific ground motionscorresponding to different values of the angle of incidence (thus leading to different times of occurrence of their PGA'sas will be shown later in Section 4).

2.3Extension to seismic siteamplification of *obliquely* incident SH waves in a viscoelastic medium

Effect ofattenuation in the soil transfer function can be introduced via the quality factor$Q = 1/(2\beta)$(see e.g. (Aki and Richards, 2002)) and recalculating the equivalent wave propagation time τ_e (Eq. (12)), using the complex shear wave velocity

$$V_{e1}^Q = V_{e1}(1 + i\beta) \tag{19}$$

where β is the damping ratio.

With this assumption, a new complex S-wave propagation time in the viscoelastic soil profile with equivalent properties

$$\tau_e^Q = \frac{H_1}{V_{e1}^Q} \tag{20}$$

can be evaluated. It follows, by using Eqs. (19), that Eq. (20)can be approximatedas:

$$\tau_e^Q = \frac{2Q}{2Q + i}\tau_e \cong (1 - \frac{i}{2Q})\,\tau_e \tag{21}$$

Theviscoelastic transfer function $H_{B,\alpha}^Q(f)$of *obliquely* incident bedrock motionscan thus be evaluated after substituting τ_e^Q for τ in Eq. (9) and then using the approximation (21), as follows:

$$H^{Q}_{\mathrm{B},\alpha}(f) = \frac{2\, e^{-i2\pi f\left(1-\frac{i}{2Q}\right)\tau_e}}{1 + \mathrm{e}^{-i4\pi f\left(1-\frac{i}{2Q}\right)\tau_e}} \tag{22}$$

the amplitude of which is expressed again in terms of τ_e:

$$\left|H^{Q}_{\mathrm{B},\alpha}(f)\right| = \frac{2}{\left|e^{2\pi f\frac{\tau_e}{Q}} + 2\cos(4\pi f\tau_e) + \mathrm{e}^{-\,2\pi f\frac{\tau_e}{Q}}\right|^{1/2}} \tag{23}$$

For illustration purposes, consider now the soil amplification functions shown in Fig. 2, derived for the site of El-Asnam Cultural Center (North-Western Algeria) assuming elastic and viscoelastic soil layer models overlying bedrock.

The characteristics of the soil layer and the bedrock, formulated on the basis of field tests and laboratory investigations, are respectively ρ_1 = 1.65 g/cm3, V_1 = 300 m/s, H_1 = 11 m (leading to the one way wave propagation time τ = 0.037 s ≅ 0.04 s) fo the soil deposit ; and ρ_2 = 2.4 g/cm3, V_2 = 900 m/s for the bedrock. The quality factor for the viscoelastic soil model is estimated to be equal to 12. The results are presented in Fig 2.

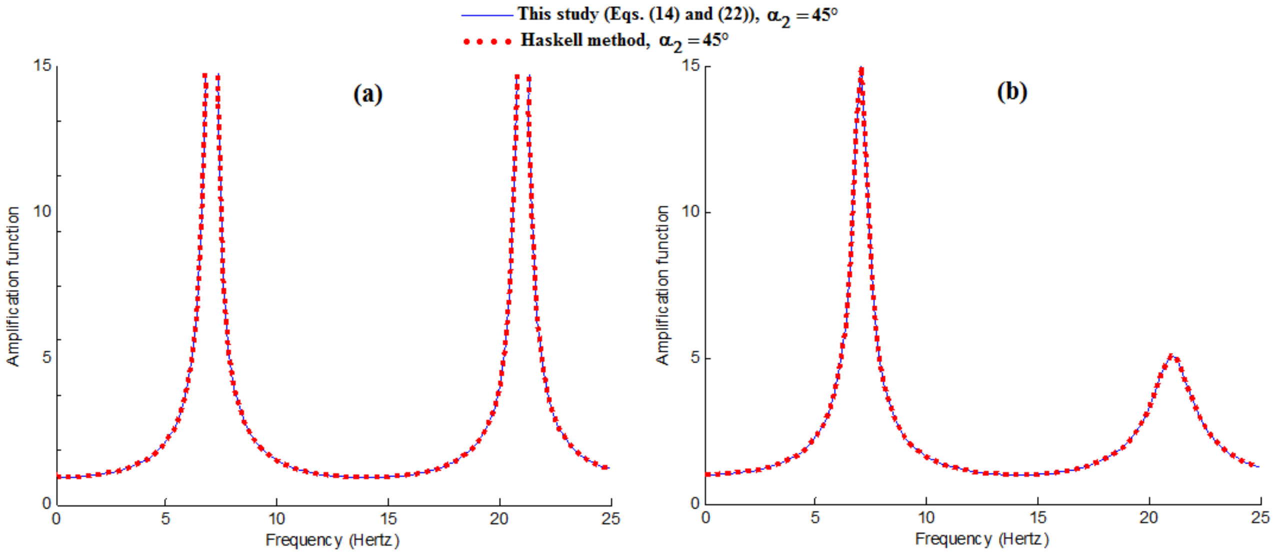

Fig. 2: Transfer functions of El – Asnam Cultural Center with respect to soft bedrock for wave incidence angle $\alpha_2 = \pi/4$: (a) elastic soil and (b) viscoelastic soil models ($\tau_e = \tau_e(\alpha_2)$; Q = 12).

Figure 2a shows that the transfer function $\left|H_{\mathrm{B},\alpha_2}(f)\right|$ for the elastic soil computed by using Eq. (14) for $\alpha_2 = \pi/4$, becomes infinite at the two resonant frequencies $f_k = kV_{e1}/4H_1 = k/4\tau_e$ with k = 1 and 3 for the case at hand. Figure 2b shows that the amplitudes of the transfer function $\left|H^{Q}_{\mathrm{B},\alpha_2}(f)\right|$ of the viscoelastic soil model computed by using Eq. (23) for $\alpha_2 = \pi/4$ have now finite values at the resonant frequencies but due to damping in the soil, the peak amplitudes now decrease with increasing frequency. It can be shown that $\left|H^{Q}_{\mathrm{B},\alpha_2}(f)\right|$ has its peak amplitudes at the same frequencies as $\left|H_{\mathrm{B},\alpha_2}(f)\right|$.

Also for accuracy checking purposes, superimposed in dotted lines over Fig. 2a and Fig. 2b, are the corresponding transfer functions calculated by using the classical Thomson-Haskell technique (e. g. Aki and Richards, 2002) for $\alpha_2 = \pi/4$. It is clearly seen that the results are in perfect agreement.

3.Sensitivity analysis of site amplification with respect to bedrock subjected to*obliquely* incident SH waves: Effects of angle of incidence on transfer functions

In thepreceding section, we have considered the discrete time wave propagation formulation for site amplification of vertically and *obliquely*incident SH waves in both elastic and viscoelasticsoilsoverlying bedrock. In this section,a sensitivity analysis of seismic site amplification in viscoelastic soils overlying bedrock subjected toSH waves with non-vertical incidence is carried out. We will analyze the effects of angles of incidence, first on site amplification functions and spectral peak ratiosfor both soft and rigid bedrock cases (Subsections 3.1 and 3.2 respectively). Effects on earthquake ground accelerations time historieswill then be discussed (Section 4).

Using the analytical model of the site of El – Asnam Cultural Center, the actual and equivalent properties of which are listed in Table 1, it is possible to investigate the sensitivity of seismic site amplification in viscoelastic soils overlying bedrock, to various angles of SH wave incidence.

Table 1: Actual and equivalent properties of the El – Asnam Cultural Center soil profile model (H_1=11) overlying actual soft bedrock (V_2 = 900, ρ_2 = 2.4) [H (m), V (m/s), ρ (g/cm^3) and f (Hz)]

Actual properties		**Equivalent properties**				
			α_2			
			0°	**45°**	**60°**	**76° (α_{cr})**
V_1	300	V_{1e}	300	308.51	313.05	316.67
ρ_1	1.650	ρ_{1e}	1.650	1.560	1.515	1.480
f_1	6.81	f_{1e}	6.81	7.01	7.11	7.20
f_2	20.43	f_{2e}	20.43	21.03	21.33	21.60

Four angles of SH-wave incidence are examined: α_2 = 0, α_2 = 45°, α_2 = 60°, and $\alpha_2 = \alpha_{cr}$ = 76° . The angle value α_{cr} = 76° is defined as the critical angle beyond which the amplitude of the dominant (i. e. first resonant) peak of the transfer function is less than one. It follows that no amplification occurs for values of α_2 greater than α_{cr}. Moreover, two values of the rock-soil impedance ratio corresponding to soft bedrock and rigid bedrock conditions will be considered.

Figure3 shows the site amplification functions of the Cultural Center soil profile evaluated for the four angles of SH wave incidence considered in the present study.

3.1 Soft bedrock case (V_2 = 900 m/s, ρ_2 = 2.4 g/cm^3)

The properties of the considered soil profile are such that$V_1/V_2 = 0.33$ and hence, as indicated in Table 1, $\rho_e < \rho_1$ i. e. the equivalent soil profile becomes lighter and the amplification peaks for non-vertical incidence will thus evidence a shift towards higher frequencies as previously mentioned in subsection 2.2 and clearly illustrated in Fig. 3. This shifting is also highlighted in Table 1 by the increasing values of f_1 and f_2 (the first and second resonant frequencies) with increasing changes of the angle of incidence α_2. It is also of interest to note that the frequency shifts associated with the second resonant peak are larger than those for the first resonant peak.

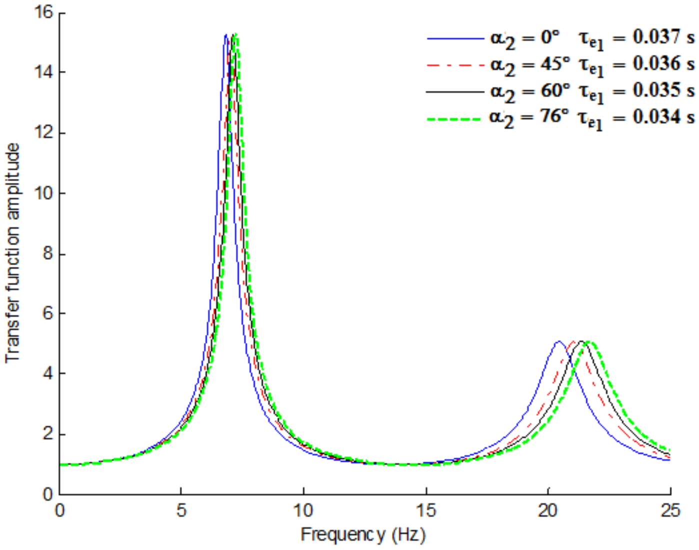

Fig.3: Amplifications functions of El-Asnam Cultural Center site assuming viscoelastic soil layer models($\tau_e = \tau_e(\alpha_2)$; Q = 12) ;for various angles of SH-wave incidence with respect to soft bedrock, (V_2 = 900 m/s).

Moreover, it is seen, from Fig. 3 and Table 2, that the dominant amplification peaks $\left|H_{\mathrm{B}}^{Q}\right|_{max,\alpha}^{(1)}$ and $\left|H_{\mathrm{B}}^{Q}\right|_{max,\alpha}^{(2)}$ of the transfer functions with respect to bedrock, corresponding to f_1 and f_2 respectively, essentially do not change with respect to increasing changes of the angle of incidence but merely shift.

Table 2: Spectral peak values $\left|H_{\mathrm{B}}^{Q}\right|_{max}$ of site of El – Asnam Cultural Center versus wave incident angle α_2(V_2 = 900 m/s)

α_2	0°	45°	60°	76°
$\left\|H_{\mathrm{B}}^{Q}\right\|_{max,\alpha}^{(1)}$	15.28	15.28	15.27	15.27
$\left\|H_{\mathrm{B}}^{Q}\right\|_{max,\alpha}^{(2)}$	5.06	5.06	5.06	5.06

3.2 Rigid bedrock case (V_2 = 3000 m/s, ρ_2 = 2.7 g/cm^3)

When we theoretically increase the rock/soilimpedance ratio$\rho_2 V_2$ / $\rho_1 V_1$ between the rock and the soil deposit, we obtain the new equivalent propertieslisted in Table 3. Notice that the value of the critical angle becomes now equal to α_{cr} = 85°.

Table 3: Actual and equivalent properties of the El – Asnam Cultural Center soil profile model (H_1=11m) overlying a hypothetical rigid bedrock (V_2 = 3000, ρ_2 = 2.7)
[V (m/s), ρ (g/cm^3) and f (Hz)]

Actual properties		**Equivalent properties**				
			α_2			
			0°	**45°**	**60°**	**85° (α_{cr})**
V_1	300	$V_{1\mathrm{e}}$	300	300.75	301.13	301.50
ρ_1	1.650	$\rho_{1\mathrm{e}}$	1.650	1.641	1.637	1.633
f_1	6.81	$f_{1\mathrm{e}}$	6.81	6.83	6.84	6.85
f_2	20.43	$f_{2\mathrm{e}}$	20.43	20.49	20.52	20.55

It is seen from Table 3 that the equivalent shear wave velocity $V_{\mathrm{e}1}$ = V_1 for all practical purposes and hence no shift is observed as clearly illustrated in Fig. 4. As expected, the equivalent mass density $\rho_{\mathrm{e}1}$ = ρ_1 and the equivalent wave travel times $\tau_{\mathrm{e}1}$ are constant.

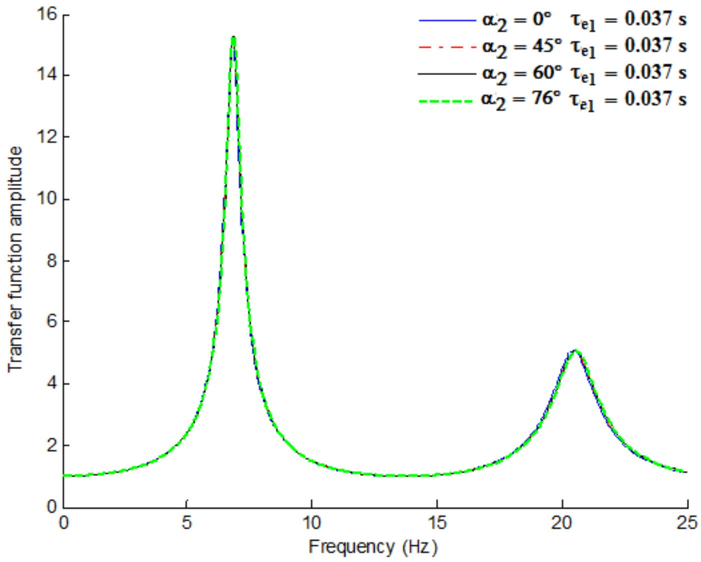

Fig. 4: Amplifications functions of site of El-Asnam Cultural Center assuming viscoelastic soil layer model, for various angles of SH-wave incidence with respect to rigid bedrock (V_2=3000 m/s, $\tau_e = \tau_e(\alpha_2)$and Q = 12).

The absence of frequency shift is also highlighted by the practically constant value $f_1 \cong 6.81$ Hz for increasing angles of incidence α_2 (i. e. the clustering effect is enhanced). Similar trends are observed for the second resonant frequency $f_2 \cong 20.43$ Hz

Furthermore, it is found that the peak spectral amplitudes of the transfer functions do not change for increasing changes of the angle of incidence (i. e. the same values previously reported in Table 2 are obtained again).

Thus, it may be concluded that for large rock/soil impedance ratios, the effect of angle of incidence on the amplification function with respect to bedrock is negligible, both in frequency shift and spectral peaks of the site amplification functions.

4. Sensitivity analysis of site amplification with respect to bedrock subjected to*obliquely* incident SH waves: Effects of angle of incidence on surface motions

For this purpose, the surface acceleration time history $u_{1A,x}(t)$for given site characteristics and angle of incidence can be assessed by linking the impulse response function $f^Q_{B,\alpha}(t,x)$ (i. e. the inverse Fourier transform of $F^Q_{B,\alpha}(f,x) = H^Q_{B,\alpha}(f)e^{-i\theta(x)}$ where $\theta(x) = \frac{2\pi f\, sin(\alpha_1)}{V_1}x$) to the bedrock motion $u_{2B}(t)$ through the convolution product (designated by the symbol '*') as follows:

$$u_{1A,x}(t) = f^Q_{B,\alpha}(t,x) * u_{2B}(t) \tag{24}$$

It is to be noticed that the present procedure does not require an envelope function to simulate the transient characteristics of ground motions,contrarily to the widely used simulation techniques based onspectral density functions(e. g. Clough and Penzien, 2003)).

To illustrate the method,the El – Asnam Cultural Center vicoelastic soil profile model is again considered and acceleration time historiesare computed at the soil surface level by considering different values of the angle of incidenceα_2.

In order to better enhance the effects of the angle of incidence on the sensitivity of acceleration time histories, we again consider a large rock/soil impedance ratio. As before, the same values oftheangle of incidence $0° \leq \alpha \leq \alpha_{cr} = 85°$ are considered (see Table 3).

Since no surface accelerogram has been recorded at the studied site, the acceleration time history at the bedrock level (Fig. 5a) was obtained by the deconvolution of the free surface N-S component recorded at the nearby Sogedia site during the strongest aftershock (M_S = 5.3) of the non recorded October 10, 1980 El – Asnam Earthquake in the El - Chellif region (North - Western Algeria)(Milutinovic and Petrovski, 1981).

The predicted acceleration time histories of soil surface motions with respect to bedrock at the El – Asnam Cultural Center site, computed for different values of the angle of incidence$\alpha 2$ are plotted in Fig. 5b. It is of interest to observe fromthe figure, that the PGA values are practically constant withcorresponding times of occurrence increasingconsistently with the increase intime it takes for incident wavesto travel from bedrock to the soil surface, when the angle of incidence increases from $\alpha_2 = 0°$ to $\alpha_2 = \alpha_{cr} = 85°$. Furthermore, it is seen, in the present case, that due to site amplification, the soil site motions have increased up to 2 times the bedrock excitation regardless of the angle of wave incidence.

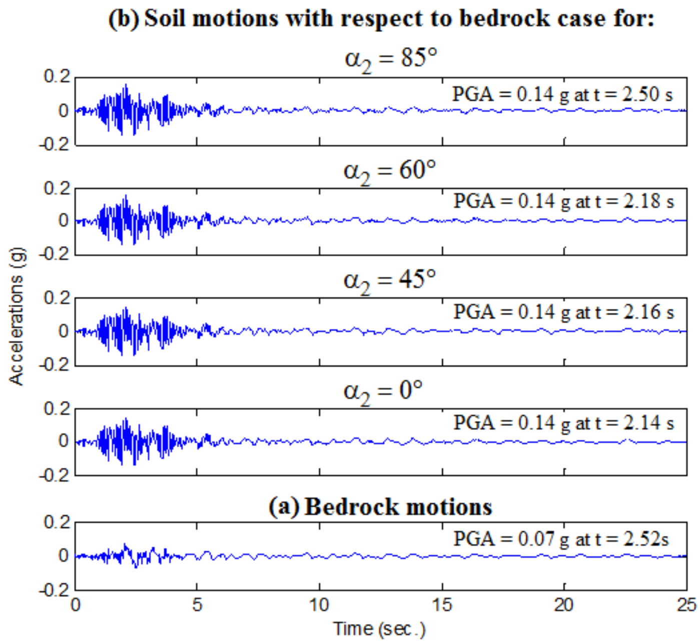

Fig. 5: Acceleration time histories of soil site motions for different values of angle of incidence with respect to rigidbedrock (Site of El-Asnam Cultural Center, $\tau_e = \tau_e(\alpha_2)$and Q = 12).

4.Summaryof main results and conclusion

In summary, a time domain recursive filter,based on discrete time wave propagation of vertically incident SH waves in elastic soil models overlying bedrock is developed in the present paper to calculate directly motions at the soil site from the bedrock motions at the soil – bedrock interface and vice versa.

Appropriate modifications are then introduced to extend the formulation to investigate soil amplification with respect to bedrock for *obliquely* incident propagating SH waves (i. e. 2-D soil amplification) in both elastic and viscoelastic models of soil deposits overlying bedrock.

Sensitivity analyses are performed for the soil profile model of the site of El – Asnam Cultural Center, in the El – Chellif region (North - Western Algeria), formulated on the basis of field tests and laboratory investigations. Numerical results are presented and discussed in terms of site amplification functions,spectral peaks ratios and surface acceleration time histories.

In addition to the analytical developments presented herein, the following main results can be drawn from the present study:

•*With regards to the effects of the angle of incidence on transfer functions with respect to bedrock*

- Amplification peaks in thesoft bedrock case are practicallyindependent of the angle of incidence but evidence a shift towards higher frequencies of the order V_{e1}/V_1 with increasing angle of incidence.

- For large rock/soil impedance ratios the effect of angle of incidence on the frequency shift and peak amplitudes of site amplification with respect to bedrock is found to be negligible.

•*With regards to the effects of angle of incidence on surface acceleration time histories*

- Due to soil amplification, PGA's of soil site motions at the studied site have increased up to the order of twice the bedrock excitation PGA regardless of angle of incidence.

- the times of occurrence ofPGA's ofof soil site acceleration time historiesincreasewith increasing angle of incidence.

• *With regards to the discrete – time wave propagation formulation*

- The discrete – time wave propagation formulation results in simple analytical models of seismic site amplification for *obliquely* propagating shear waves in both elastic and viscoelastic soil deposits overlying bedrock and requires at most two parameters (τ and Q).

- For a homogeneous soil layer over bedrock subjected to *obliquely* propagating waves, the discrete time wave formulation provides, when compared to the frequency domain approach, exact solutions and deeper physical insight.

The analytical procedures for seismic site amplification functions presented herein for *obliquely* incident bedrock motions can be advantageously used for studies of soil-foundation interaction problemsand site specific ground motion simulation without requiring the need to define envelope functions. Future works can include extension to seismic site amplification with respect to *obliquely* incident rock outcroppingmotions, and to site amplification of P-SV waves.

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