BEHAVIOR OF GRAVITY TYPE QUAY WALL DURING EARTHQUAKE REGARDING DYNAMIC INTERACTION BETWEEN CAISSON AND BACKFILL DURING LIQUEFACTION

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SUMMARY

The aim of this study is to clarify the mechanism of the damage to quay walls during earthquakes. The dynamic behavior of quay wall during earthquake was observed in model shaking table tests, with the effect of liquefaction on the damage. The transient nature of the dynamic earth pressure on caisson was observed and analyzed with a mass-spring-dashpot model.

From some series of model shaking table tests on the caisson type model quay walls with different vibration properties, it was clarified that the liquefaction of backfill soil enhances the damage to quay walls. In the case of no-liquefaction of backfill the fluctuating earth pressure acts in the opposite direction to the inertia force on quay wall. On the other hand, in the case of liquefaction of backfill the direction of fluctuating earth pressure is same as that of inertia force; resultant thrusting force becomes much larger than that in the case of no-liquefaction. This transient behavior of earth pressure in the process of liquefaction is explained regarding the interaction between caisson and backfill ground; the stiffness of backfill ground reduces due to the generation of pore water induced by the liquefaction. A simple mass-spring-dashpot model was derived taking account of the vibration properties of caisson-backfill ground system.

INTRODUCTION

Many structures in port and harbor area have been damaged frequently during large earthquakes that recently occurred in Japan, and especially the significant damages to quay walls were caused by liquefaction of reclaimed lands. In these cases the function of quay walls was lost because of subsidence and displacement of caisson toward sea, and also foundations of structures and underground structures were damaged due to the laterally flow deformation of the ground, which was triggered by the displacement of the quay walls [e.g., Inagaki et al., 1996; Kamon et al., 1996].

From the close examinations on the damages to quay walls in Hokkaido Island, Japan, during three recent big earthquakes, i.e., 1993 Kushiro-oki, 1993 Hokkaido Nansei-oki and 1994 Hokkaido Toho-oki earthquakes, it was, however, clarified that the degrees of the damage are strongly related to the occurrence of the liquefaction in the backfill ground [Japanese Geotechnical Society, 1994; Hokkaido Development Bureau, 1996]. The damages were large at the quay walls where liquefaction occurred in the backfill ground. However, the quay walls whose backfill was improved against liquefaction by a combination of sand the compaction pile and gravel drain methods survived with slight damage.

The causes of the damage to quay walls are classified into

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- Inertia force of the caisson
- Earth pressure acting on the back of the quay wall
- Reduction of the bearing capacity of the foundation ground

Inertia force is a body force induced by the vibration through the foundation. Both static and fluctuating components of the earth pressure acts on the back wall of the caisson. The fluctuation of the earth pressure occurs from the interactive behavior between the vibrations of the caisson and the backfill. In the present study, the behavior of the quay wall during vibration is investigated through the shaking table tests and the analysis by simplified mass-spring-dashpot model, regarding the interactive behavior between the inertia force and earth pressure from the backfill. In these investigations the vibration property of quay wall are important for understanding for the dynamic earth pressure during earthquakes, and the vibration properties such as the natural frequencies of caisson and backfill were examined within the model shaking table tests. The vibration properties of the prototype quay walls were observed and assessed through the measurement of microtremor of gravity type quay walls in several port and harbor areas [Miura, et al, 1999].

The liquefaction of foundation ground has remarkable effect on the damage to quay walls, as seen in the damages to several types of quay walls during the 1995 Hyogo-ken Nambu earthquake. Many types of quay walls were severely damaged by the reduction of bearing capacity induced by liquefaction of the sand foundations [Inagaki, et al., 1966; and Kamon, et al., 1996], but is not discussed in this paper.

SHAKING TABLE TESTS

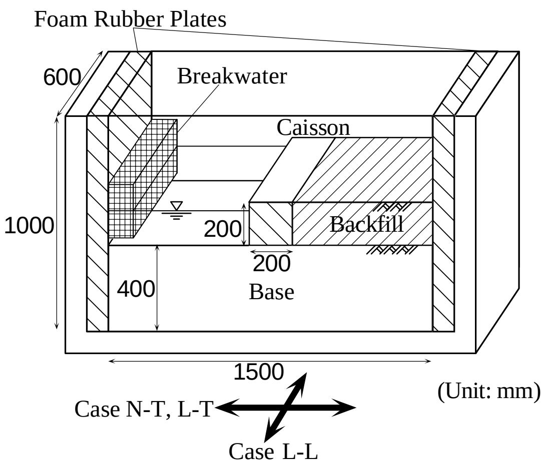


Figure 1(a): Model quay wall

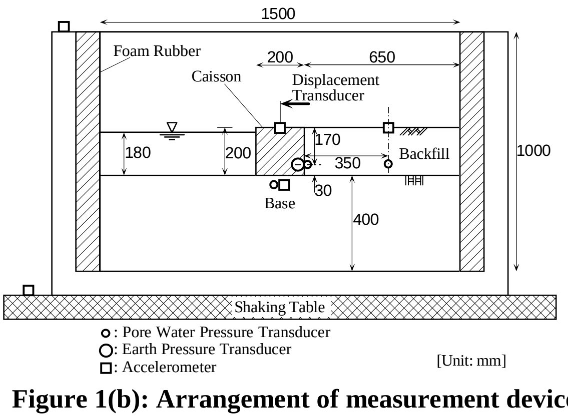


Figure 1(b): Arrangement of measurement devices

Test Method

The model quay wall and the locations of the measuring devices used in the shaking table tests is shown in Figure 1. Four types of caisson with different mass and aspect ratios of the section are shown in Figure 2 and their mechanical properties in Table 1. The caisson is a hollow box made of steel plates, in which lead plates were stacked to control the density and gravity center. Both the base ground underlying the caissons and the backfill ground were made with the same siliceous sand whose physical properties are listed in Table 2. The sand ground was deposited under water so as to obtain sufficient high degree of saturation [Kiku, 1993]. When preparing foundation grounds, the container was vibrated intermittently in order to possess a high relative density more than 90%. The density of the backfill was controlled to be either loose with Dr = 30-40% or dense with Dr = 70-80 %. Displacement and earth pressure on the caisson and pore water pressure in the backfill were measured in order to monitor the behavior of quay walls. Pore water pressure, acceleration, displacement and earth

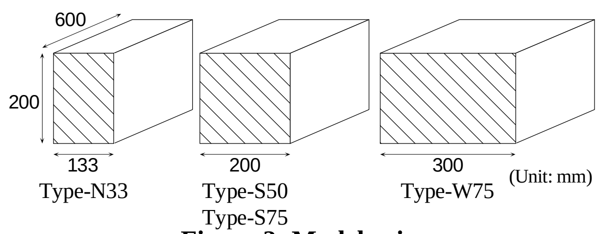


Figure 2: Model caissons

Table. 1: Properties of model caissons

Caisson Type	Breadth (m)	Mass (kg)	Height (m)	Average Density (kg/m^3)	Gravity Center from Bottom (%)	Safety Factor (Sliding)		
						Ordinary	Earthquake	Liquefaction
Type-N33	0.133	33	0.200	2.17×10^3	40	1.93	0.58	0.31
Type-S50	0.200	50		2.17×10^3	43	2.84	0.70	0.41
Type-S75		75		3.25×10^3	35	5.39	1.04	0.68
Type-W75	0.300			2.17×10^3	39	4.21	0.81	0.53

Relative Density, D_r = 75% Internal Friction Angle, ϕ = 36.7° Horizontal Seismic Intensity, k_h = 0.2

Table 2: Mechanical property of siliceous sand

Grain density ρ_s (g/cm^3)	Mean diameter D_{50} (mm)	Uniformity coefficient, U_c	Maximum density $\rho_{d\,max}$ (g/cm^3)	Minimum density $\rho_{d\,min}$ (g/cm^3)
2.717	0.18	1.82	1.610	1.255

pressure on the caisson were measured with strain gauge type devices whose locations are shown in Figure 1(b). Pore water pressure transducers were installed in the base and the backfill; the values measured by these transducers are referred to as u_b and u_g, respectively. The devices placed under the ground were suspended with threads in order to keep their original positions even under liquefaction. The accelerometers were supported in the backfill with thin percolative plate and threads in order to maintain their original positions and attitudes. An earth pressure transducer was installed on the caisson back wall, and its value is referred to as p_e. The horizontal displacement of the caisson x_c was monitored with a wire type displacement transducer. Details of the testing method are described in the reference [Kohama, et al. 1998].

Table 3: Density of backfill

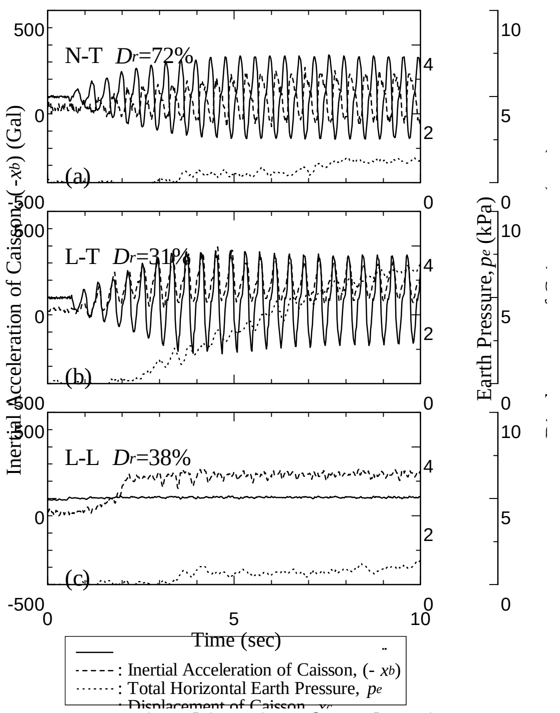
Density of Backfill	Relative Density, D_r
Loose	30 40 %
Medium	50 60 %
Dense	70 80 %

First, three test cases listed in Table 3 are examined. The first characters “L” and “N” in the names indicate the occurrence of the liquefaction in the backfill ground: “L” for liquefy and “N” for not liquefy. The second characters “T” and “L” indicate the direction of the vibration: “T” for transverse and “L” for longitudinal to the direction of caisson. Type-N50 caisson was used in these test cases. The input sinusoidal wave acceleration at the shaking table is common for all these test cases, and is 220 Gals in amplitude and 2.5 Hz in frequency.

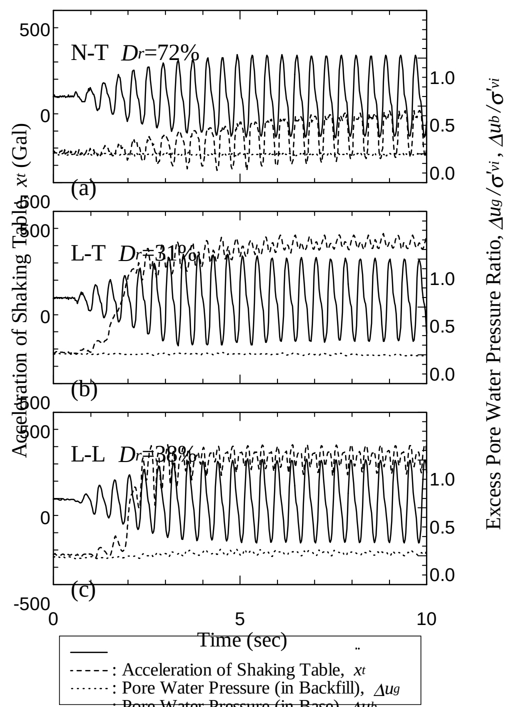
Test Results and Discussion

Dynamic behavior of quay wall in liquefaction and no-liquefaction cases

Shown in Figure 3 are time histories of the total horizontal earth pressure, acceleration and displacement of the caisson. Figure 4 shows the time histories of acceleration at shaking table and excess pore water pressure ratio in base and backfill ground. Acceleration was not detected in case L-L in Figure 3 because direction of vibration is perpendicular to the direction of measurement. The earth pressure also did not fluctuate, although static change was observed from initial to liquefied state due to the reduction in effective stress as shown in Figure 5. Time histories of the acceleration at top of the caisson, at the base beneath the caisson and on the shaking table are shown in Figure 6, where only predominant component (frequency of 2.50Hz) was extracted by using Fourier’s Analysis. All the waveforms in different cases and at different locations are almost the same to each other; phase difference between cases N-T and L-T was intentionally set only for the demonstration. This suggests that the base is sufficiently rigid and resonance between the base and the caisson did not occur in these



Figures 3(a-c): Time histories of acceleration, earth pressure and displacement measured in; (a) Case N-T, (b)Case L-T, (c) Case L-L.



Figures 4(a-c): Time histories of acceleration and pore water pressure in backfill measured in the test. (a) Case N-T, (b)Case L-T (c) Case L-L.

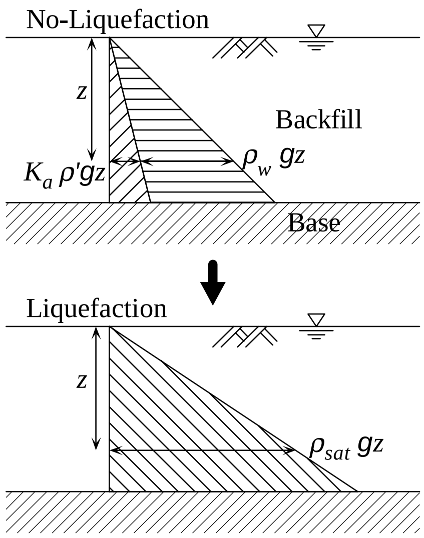
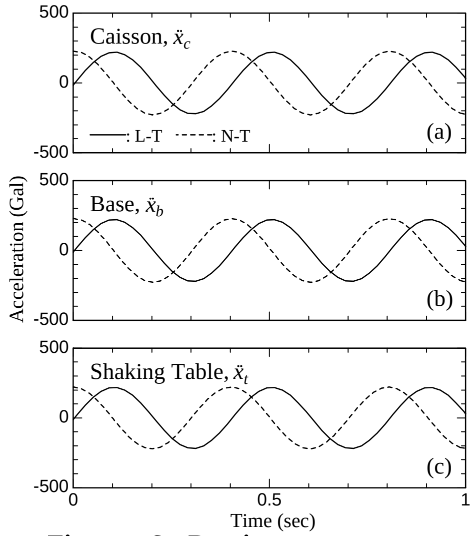


Figure 5: Static increase in earth pressure due to reduction in effective stress by liquefactiion



Figures 6: Dominant components of acceleration

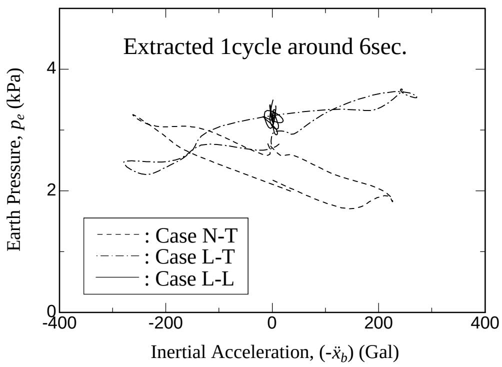
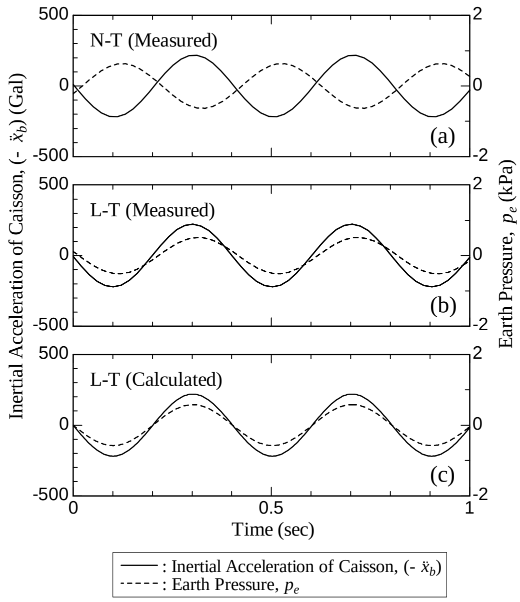


Figure 7: Relationship between earth pressure and inertial acceleration



Figures 8: Dominant components of inertia acceleration and earth pressure analyzed from time histories;

(a) measured in case N-T,

(b) measured in case L-T,

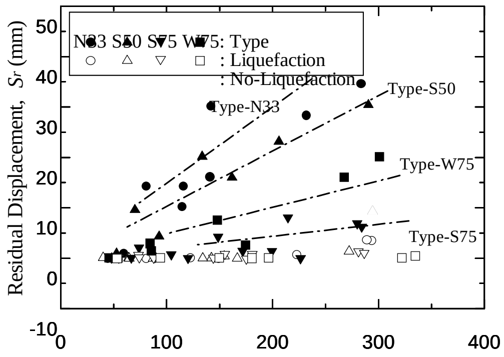
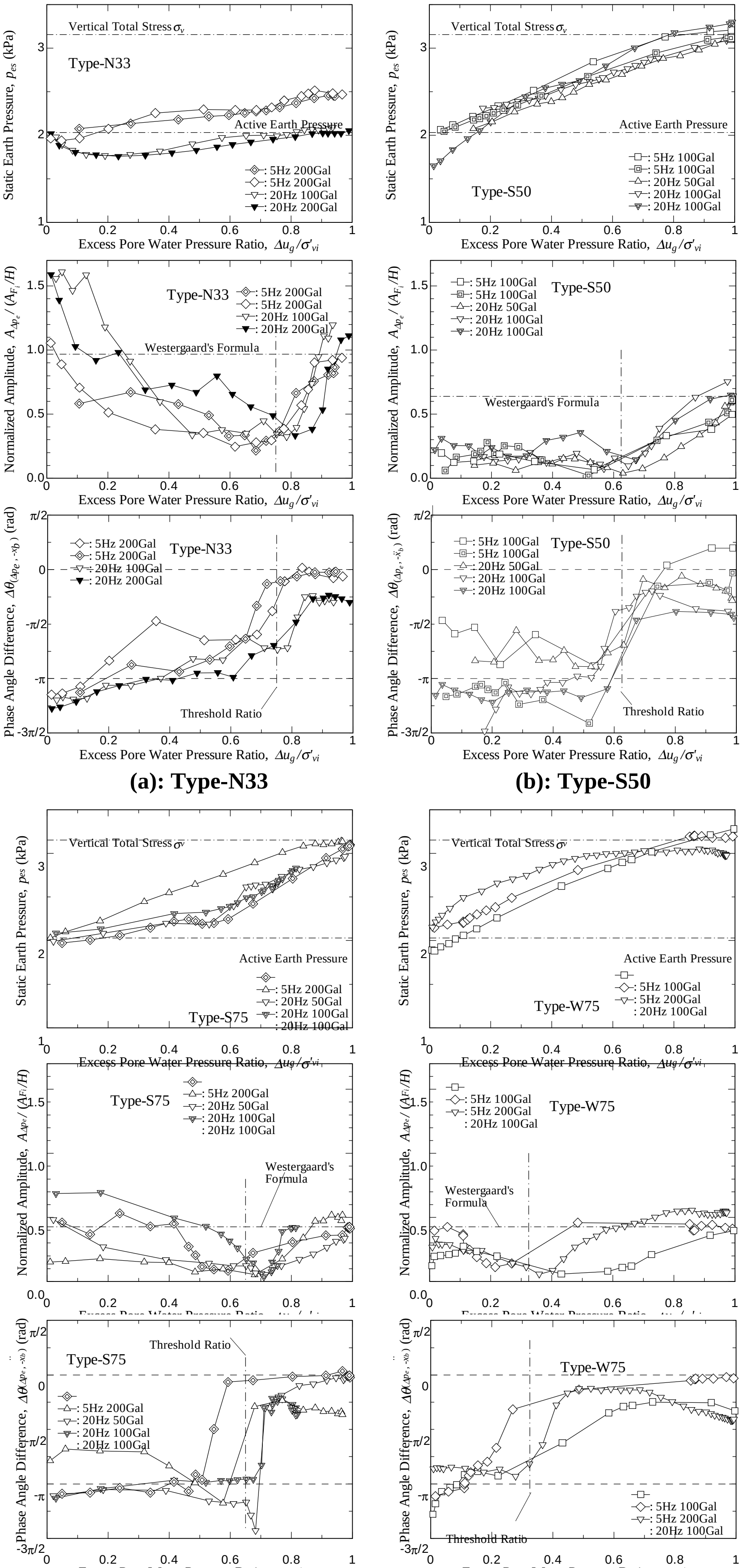


Figure 9: Residual horizontal movement

test cases. As shown in Figure 4, actually accumulation of the excess pore water pressure was negligibly small in the base ground.

As shown in Figure 4, the backfill ground liquefied in both cases L-T and L-L; the excess pore water pressure increased resulting in the perfect loss of the effective stress within 3 seconds from the start of the shaking. On the other hand, in case N-T where backfill did not liquefy, pore water pressure fluctuated but only a half of the initial effective stress was lost at maximum in the backfill. The caisson moved significantly only in case L-T, but the movement or slide ceased at the time of the termination of shaking.

The stability of the caisson was evaluated by assuming the friction angle between the caisson and the base to be of 35 degrees. The safety factor F_s against sliding was 2.65 under ordinary condition under which both earth pressure and water pressure at rest works. It was 0.70 under the earthquake condition under which inertia force and earth pressure under horizontal seismic intensity of k_h = 220/980 were considered, where 220 and 980 are



Figures 10: Transient change in earth pressure during liquefaction

maximum horizontal acceleration and acceleration of gravity, respectively. The earth pressure increase more when liquefaction occurs and effective stress is lost in the backfill ground as shown in Figure 5, in which case F_s is 0.62. The caisson could slide during the earthquake even in case N-T (F_s = 0.70) based on the stability analysis with seismic coefficient method. The displacement of the caisson was, however, small to be only 3 mm or 2% of the height of the caisson. This indicates that both inertia force and liquefaction of backfill are needed to make the caisson instable. Comparison of the behaviors between cases L-T and L-L suggests that the inertia force and fluctuation component of the earth pressure of liquefied backfill play an important role for the instability of the caisson. These experimental results agree with the trend of the actual damage to quay walls during earthquakes; damage was significant only when the liquefaction in backfill ground occurred.

The relationships between the inertial acceleration and earth pressures are plotted in Figure 7 for 1 cycle after 6 seconds from the beginning of the vibration. Since the input vibration wave is a little apart from the perfect sinusoidal wave and the deformation properties of backfill and foundation grounds are not linear, the loops shown in the figure are not complete elliptic shape. In case L-L, neither inertial acceleration or earth pressure fluctuated, only a rise in earth pressure caused by the occurrence of liquefaction was recognized (see Figures 2(c) and 5). The same amount of the rise in excess pore water pressure is also found in case L-T. On the other hand, in cases N-T and L-T, both inertial acceleration and earth pressure oscillated, but the directions of the loops are different from each other. The predominant components of the inertial acceleration and the fluctuating earth pressure are shown in Figures 8(a, b). The directions of the inertial acceleration and fluctuating component of the earth pressure are almost opposite in case N-T, whereas almost the same in case L-T. The cause why instability of the caisson occurred only in case L-T is explained from this contrast in the direction of inertial acceleration relative to earth pressure. Figure 8(c) shows the fluctuating earth pressure calculated by Westergaard's formula. This formula is derived to evaluate the dynamic water pressure on dams by Westergaard [1933]. The density of fluid is set to be 1.96x10^3 kg/m^3 in this calculation. Both the measured and calculated behaviors in Figures 8(b) and (c) show that fluctuating component is the same in phase angle with the inertial acceleration, and measured fluctuating earth pressure is in good agreement with the calculated one.

Transient dynamic behavior of quay wall in the course of liquefaction in backfill ground

The transient behavior of quay walls in the course of liquefaction was investigated; the caissons of different 4 types were observed in model shaking tests. In all these test cases the frequency of sinusoidal base input motion was 5 or 20 Hz, and the amplitude was linearly increased up to a prescribed value in first 8-10 seconds and held constant in the following 15 seconds. This loading specification was employed for the observation of behavior of the quay walls especially in the process of liquefaction. First, the amount of residual horizontal movement of caisson S_{res} is plotted against the amplitude of base acceleration $\ddot{x}_b$ in Figure 9. In this figure, excess pore water pressure ratio reached unity only in liquefaction cases. Only in liquefaction cases which include all test cases on loose backfill ground and part of test cases on medium dense backfill ground, the caissons were displaced depending on the intensity of base acceleration but not depending on frequency. The occurrence of liquefaction is a main factor controlling instable behavior of caisson. The residual amount of displacement is also dependent on the type of caisson. The amount of displacement can be correlated with safety factor shown in Table 1; Type N33 was displaced most severely.

Relationship between the earth pressure and the excess pore water pressure ratio $\Delta u_g /\sigma'_{vi}$ were examined in Figures 10(a-d), where top, middle and bottom figures show the static component p_{es}, the amplitude of dynamic components of earth pressure Δp_e and phase angle difference between dynamic earth pressure and inertial acceleration $-\ddot{x}_b$, respectively. General feature observed in all the liquefaction cases can be explained as follows. Static component p_{es} increases monotonically and linearly with an increase in excess pore water pressure ratio $\Delta u_g /\sigma'_{vi}$ as can be explained in Figure 5 where the ratio $\Delta u_g /\sigma'_{vi}$ is assumed constant through depth of backfill. Normalized amplitude $A_{\Delta p_e}/\left(A_{F_i}/H\right)$ first reduces to a fairly small value, then start to increase suddenly up to the values estimated from Westergaard's solution at liquefaction state with excess pore water pressure ratio $\Delta u_g /\sigma'_{vi}$ of unity. Phase angle difference $\Delta\theta\left(\Delta p_e, -\ddot{x}_b\right)$ is almost $-\pi$ with some scatters, and leaps to 0 together with the sudden increase in $A_{\Delta p_e}/\left(A_{F_i}/H\right)$ as seen in Figure 10. This indicates that the ground begins to behave like a liquid when effective stress reduces to a certain amount; the excess pore water pressure ratio $\Delta u_g /\sigma'_{vi}$ at this state is named as critical excess pore water pressure ratio $(\Delta u_g /\sigma'_{vi})_c$. It should be noted that $(\Delta u_g /\sigma'_{vi})_c$ is different with the type of caisson; the lowest value was obtained for Type W75 and the largest value for Type N33.

As will be discussed in the next section, this transient feature of dynamic earth pressure in liquefaction process is related to the vibration properties of caisson and backfill. Therefore, their natural frequencies were examined with amplification properties as shown in Figure 11; amplification factor was evaluated from the vibration test with various frequencies of minimal acceleration of 50Gal and the impact test, where amplitude of acceleration was compared. As shown in this figure, the natural frequencies of the caissons were lower than that of backfill, and the natural frequency was highest for Type W75 and lowest for Type N33. For prototype quay walls free vibration behavior was observed in several port and harbor areas by the authors [Miura, et al., 1999]. As a result of the measurement and analysis of micro tremor of caisson and backfill ground, it was found that the natural frequency of caisson was in negative correlation with the ratio of height to width as shown in Figure 11 for model caissons. And it was conformed also for prototype quay walls that without exceptions the natural frequency of caisson is smaller than that of backfill.

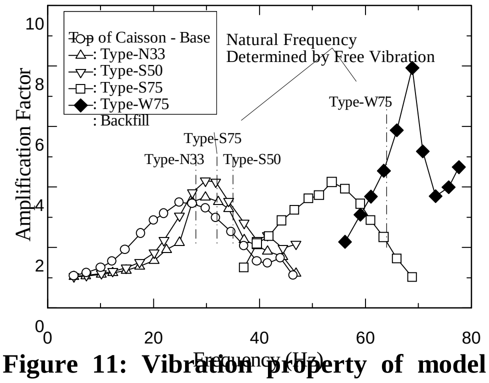


Figure 11: Vibration property of model quay wall

SIMULATION OF THE BEHAVIOR OF QUAY WALL DURING EARTHQUAKE

From examination of the vibration properties of caissons in both model quay walls and prototype quay walls, it is generally the case that natural frequency of backfill ground is higher than that of caisson. This general feature suggests that the natural frequency of caisson is relatively larger than that of backfill ground at ordinary condition, and becomes smaller in the course of liquefaction due to the generation of pore water pressure in backfill ground. That is, the stiffness and natural frequency of backfill ground are higher than those of caisson at the beginning of liquefaction process, and then decrease with increasing excess pore water pressure. At the state near liquefaction, the stiffness and natural frequency of backfill become lower than those of caisson. This change in the relative stiffness between backfill and caisson plays an important role; accordingly, the following simulation is carried out for the interaction between caisson and backfill, and dynamic earth pressure on the basis on this feature.

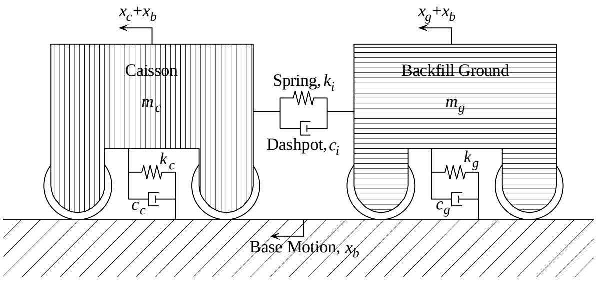


Figure 12: Mass-spring-dashpot model

Mass-Spring-Dashpot Model

A mass-spring-dashpot model is devised in order to explain the transient behavior of the caisson-backfill ground system in the process to liquefaction. The caisson and backfill ground were modeled as lumped masses whose mass are m_c and m_g. They are connected by a spring with spring constant of k_i and a dashpot with viscous coefficient of c_i. The base input motion x_b (=X_b exp(i$\omega_b t$)) is propagated to the caisson and the backfill through the springs (k_c, k_g) and dashpots (c_c, c_g) as shown in Figure 12. The natural angular frequency ω, critical damping ratio h and complex spring coefficient K are defined as follows:

$$\omega_c = \sqrt{k_c / m_c}\,,\quad h_c = c_c / \sqrt{4 m_c k_c}\,,\quad K_c = k_c + i\omega_b c_c \tag{1}$$

$$\omega_g = \sqrt{k_g / m_g}\,,\quad h_g = c_g / \sqrt{4 m_g k_g}\,,\quad K_g = k_g + i\omega_b c_g \tag{2}$$

$$\omega_i = \sqrt{k_i / m_c}\,,\quad h_i = c_i / \sqrt{4 m_c k_i}\,,\quad K_i = k_i + i\omega_b c_i \tag{3}$$

Table 4: mechanical parameters for simulation

ω_g / ω_c	0.0 3.0
m_g / m_c	4.0
ω_i / ω_c	0.25 2.0
ω_b / ω_c	0.22 (= 10 / 45)
h (= h_c= h_g= h_i)	0.2

where suffixes 'b', 'c', 'g' and 'i' correspond to the parameters for base, caisson, backfill ground and the interface between caisson and backfill ground, respectively, and ω_b is an angular frequency of the base motion. Simultaneous momentum equation for the model is given by

$$\begin{bmatrix} -\omega_b^2 m_c + K_c + K_i & -K_i \\ -K_i & -\omega_b^2 m_g + K_g + K_i \end{bmatrix} \begin{Bmatrix} X_c \\ X_g \end{Bmatrix} = \begin{Bmatrix} \omega_b^2 m_c X_b \\ \omega_b^2 m_g X_b \end{Bmatrix} \tag{4}$$

where x_c (=X_c exp(i$\omega_b t$)) and x_g (=X_g exp(i$\omega_b t$)) are relative displacements of the caisson and backfill ground, respectively. The inertia force F_i to which the caisson is subjected and the force generated at the interface ΔF_e, which corresponds to the earth pressure, are calculated by

$$F_i = m_c \omega_b^2 X_b \exp(i\omega_b t)\,,\quad \Delta F_e = -K_i (X_c - X_g)\exp(i\omega_b t) \tag{5}$$

The values of the mechanical parameters used in the simulation are listed in Table 4. The frequency of the sinusoidal base motion was 10 Hz. The same value of h of 0.2 is used for all the spring-dashpot systems. Spring constants at the interface k_i were parametrically varied in the simulation.

Calculation Results and Discussion

The results of the parametric calculation are shown in Figure 13; the amplitude of the earth pressure normalized by the inertia force $A_{\Delta F_e} / A_{F_i}$ and phase angle difference $\Delta\theta_{(\Delta F_e, F_i)}$ are plotted against the ratio of the natural angular frequency of backfill ground ω_g to that of caisson ω_c. The reduction of the natural frequency ratio ω_g/ω_c corresponds to the reduction of the stiffness of backfill ground due to the generation of pore water pressure in the process to liquefaction. As shown in these figures the amplitude of the earth pressure first descends and vanishes when the natural frequencies of the caisson and backfill coincide, i.e., ω_g/ω_c =1. Then it starts to ascend. The phase angle difference leaps from around $-\pi$ to around 0 at ω_g/ω_c =1 or the amplitude $A_{\Delta F_e}$ equals 0. Comparison between Figures 10 and 13 shows that the transient feature of the earth pressure in the process to liquefaction is simulated qualitatively by the analysis.

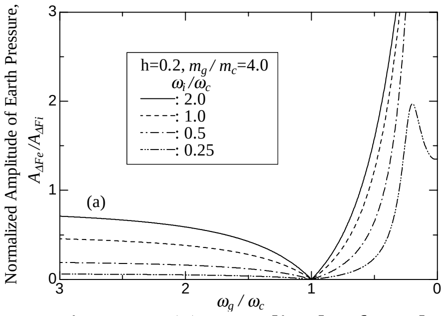


Figure 13 (a): Amplitude of earth

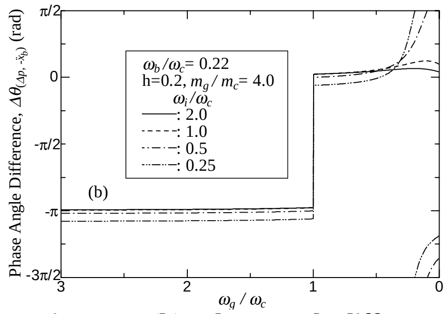


Figure 13 (b): Phase angle difference

CONCLUDING REMARKS

Dynamic behavior of gravity type quay walls during earthquake was investigated experimentally and analytically, and the mechanism of the damage to the quay walls due to the liquefaction of backfill was clarified. The following conclusions were obtained from the examination of the results of model shaking table tests and simplified model analysis.

- The caisson becomes unstable when the backfill ground is liquefied, and the amount of displacement of caisson is a function of liquefaction as well as shaking intensity and vibration properties of caisson. The fluctuation earth pressure and inertia force cooperate together to slide the caisson only in the case of liquefaction. This trend was confirmed in model shaking table tests on the several caissons with different vibration properties.
- When a caisson-backfill ground system is subjected to the earthquake loading, the fluctuation of the earth pressure on the caisson first decreases as excess pore water pressure generates. It reaches a negligibly small value when natural frequency of the backfill ground becomes equals to that of the caisson at critical excess pore water pressure ratio state. After that, according to the softening of backfill ground, the amplitude increases rapidly and the phase difference to inertia force leaps to 0.
- The transient dynamic behavior of quay wall can be explained taking account of the vibration characteristics of caisson-backfill system. Based on the observation in the model shaking table tests, critical excess pore water pressure ratio is a function of the vibration properties of caisson; the caisson with higher natural frequency possesses lower critical value.
- A simple mass-spring-dashpot system was employed to explain the mechanism of transient dynamic behavior of quay wall.

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REFERENCES

Hokkaido Development Bureau (1996): *Report on the Improvement o the Seismic Resistance of Port Facilities, Cold Region Port and Harbor Research Center* (in Japanese).

Inagaki, H., Iai, S., Sugano, T., Yamazaki, H. and Inatomi, T. (1996): "Performance of Caisson Type Quay Walls at Kobe Port," *Special Issue of Soils and Foundations*, pp.119-136.

Japanese Geotechnical Society (1994): *Report of the Damages during 1993 Kushiro-oki and Notohanto-oki Earthquakes*, pp.175-182 (in Japanese).

Kamon, M., Wako, T., Isemura, K., Sawa, K., Mimura, M., Tateyama, K. and Kobayashi, S. (1996): "Geotechnical Disasters on the Waterfront," *Special Issue of Soils and Foundations*, pp.137-147.

Kiku, H. (1993): *A Study on the Permanent Displacement of Liquefied Ground*, Thesis for the Partial Fulfillment for the Degree of Doctor of Engineering, *Kitakyushu Institute of Technology* (in Japanese).

Kohama, E., Miura, K., Yoshida, N., Ohtsuka N., and Kurita, S. (1998): "Instability of Gravity Type Quay Wall Induced by Liquefaction of Backfill during Earthquake," *Soils and Foundations*, Vol.38, No.4, pp.71-84.

Miura, K., Kohama, E., Inoue, K., Ohtsuka, N., Sasajima, T., Hayashi, T., and Yoshida, N. (1999): "Instability of Gravity Type Quay Wall during Earthquake with Regard to Dynamic Interaction with Backfill Ground," *Proceedings of ISOPE-99*, Vol. 4, pp.512-519.

Westergaard, H. M. (1933): "Water Pressures on Dams during Earthquakes," *Trans. ASCE*, Vol.98, pp.418-432.