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COMPARISON OF EQUIVALENT LINEAR ANALYSIS AND NONLINEAR ANALYSIS FOR A LIQUEFACTION PROBLEM

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SUMMARY

Dynamic response analysis of horizontal soil layers to earthquake considering liquefaction has received high attention in the recent seismic engineering. Dynamic effective stress analysis is the theoretically best method for this purpose that can solve fully integrated problem such as generation and dissipation of the excess pore pressure based on micro-mechanism. However, there is difficulty to use it in engineering practice. Influence of initial shear stress, cyclic mobility and multi-directional behaviour of stress are not well established. Therefore a new simple method to analyse liquefaction problem for horizontal soil layers is proposed. It is a cycle-wise equivalent linear analysis (CELA). The method is extension of the equivalent linear analysis (ELA). Therefore, the validity of the ordinary equivalent linear analysis is studied for the heavily nonlinear soil layers (without liquefaction) at first in this report. ELA provided good agreement with the directly integrated nonlinear analysis with Ramberg-Osgood's stress strain relationship. CELA provided better results than ELA. Then liquefaction mechanism is implemented to CELA and applied to the soil layer problem where liquefaction is anticipated. Additional data of this method is only the liquefaction strength curve so that no new soil investigation system is required. The method provides very good agreement with the results obtained by the dynamic effective stress analysis and the observed data.

INTRODUCTION

Dynamic response analysis of horizontal soil layers to earthquake is the most basic and important part in the seismic engineering. It is necessary for estimation of surface acceleration to consider degradation of material properties and/or liquefaction of the soil layer. In this aspect a procedure SHAKE [Schnabel et. al. 1972] which based on the equivalent linear analysis (ELA) is the mostly used procedure. However, there is no well-established procedure that can calculate the response for the liquefied soil layer. SHAKE can be used to calculated the safety factor of liquefaction of soil layer but not for earthquake response with liquefaction.

This paper investigates the possibility of the extension of ELA to liquefaction problem and a new procedure as simple as SHAKE is proposed. It only requires an additional data set of liquefaction strength curve. Shear stiffness is basically a function of the confining pressure. Liquefaction lowers the effective stress of the soil layer so that liquefaction reduces more scant stiffness than that reduced by shear strain in ELA. Very small effective stress is expected for liquefied soil layer. On the other hand it is said that ELA is only valid for the stiffness reduction, which is larger than 40%. So validity of ELA is studied for the large input motion at first.

Since SHAKE is linear analysis with a reduced scant shear stiffness, it theoretically gives error at the large input motion, especially if there is large impact motion during earthquake excitation. Martin et al [Martin et. al. 1982] reported that SHAKE provided good results for the problem subjected to the input motion 0.26g and calculated the double acceleration for the input motion 0.55g. Tanaka et al [Tanaka et al, 1983] showed that maximum acceleration at the surface calculated by the direct integration method in time domain for nonlinear analysis (NLA) is half of the SHAKE's acceleration in some case but it is depend on the shear modulus against shear

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strain curve (G-γ curve). They used the G-γ curve, which fits the site investigation for ELA and used Ramberg-Osgood model and Heardin-Drnevich model for NLA. Results are depending on the fitted curve. Both studies used the G-γ curve, which is not corresponding to the stress strain curve.

In the above investigation, it should be noticed that they used the inconsistent stress strain relationship with the secant shear modulus and shear strain relationship (G-γ curve). ELA showed little difference from NLA if the G-γ curve, which is obtained from the skeleton curve of stress-strain relationship, is used. Change of the G-γ curve makes much larger difference on the earthquake response. In this context their results are not exact comparison of the numerical methods.

A new procedure is proposed based on the concept of ELA. In this procedure, the equivalent linear method applied to cycle-wise in stead of whole time length of the earthquake. This cycle-wise equivalent linear analysis (CELA) should theoretically have better accuracy. Comparisons are made with nonlinear dynamic effective stress analysis (DESA). This method only requires the liquefaction strength curve in addition to the data for the ordinary ELA.

At last, liquefaction analyses are compared with the observed data and results obtained by the other program.

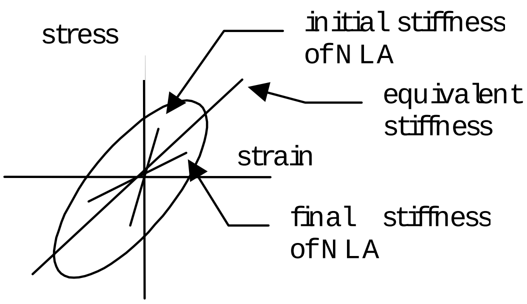
NOLINEARITY OF SHEAR BEHAVIOUR

Nonlinearity of soil starts from very small strain. Therefore, consideration of that is inevitable. To take into account that, ELA is well accepted and the program SHAKE [Schnabel et. al., 1972] is most popular for ELA. SHAKE is well established because the procedure to obtain the material properties for the program is also well-establish. The authors intend to extend the concept of ELA to the liquefaction analysis so that method of ELA and its range of the validly are studied first.

Then, we introduced the cycle-wise equivalent linear analysis for non-liquefaction and liquefaction problems. The method is verified in comparison with NLA. At first, theoretical summary of each method is presented.

Equivalent linear analysis

Basic idea of ELA is to solve the non-linear problem using linear analysis with a reduced scant stiffness, so that ELA can simulate the response at the cycle of the largest amplitude. To obtain the proper stiffness the linear analysis is repeatedly solved until the stiffness and the maximum shear strain response at each layer satisfy the material relationship between stiffness and the shear strain (namely, G-γ curve). Therefore ELA well simulates the response for the maximum magnitude cycle. This can be seen in Figure 2. Nevertheless, the response of the other cycle has error to the response of the NLA since stiffness used is the stiffness at the largest amplitude.



1. set the possible stiffness K_i
2. do dynamic linear analysis and obtain the maximum shear strain
3. evaluate the stiffness K_{i+1} for the maximum shear strain from G-γ curve
4. if the $K_{i+1} = K_i$, exit. otherwise repeat from step 2 with new stiffness K_{i+1} .

Figure 1: Concept of the ELA

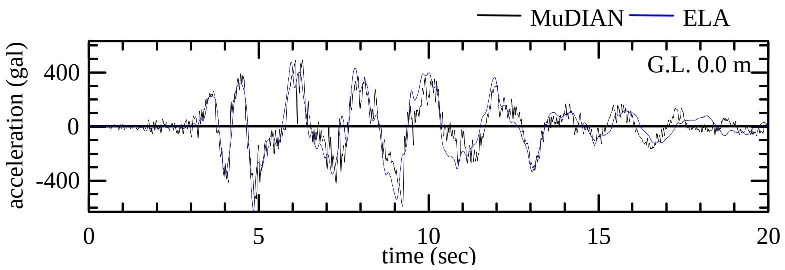


Figure 2: Time history of Equivalent Linear Analysis

Dynamic Nonlinear Analysis

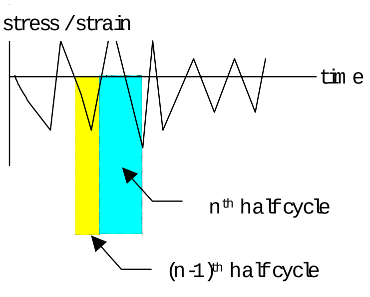
For nonlinear analysis, the following numerical method is used. Newmark-β method with the coefficient 0.25 and 0.5 for time integration, Ramberg-Osgood model and Hardin-Drnevich model for stress strain relationship, Masing's rule for cyclic behaviour, 4 node element for finite element modelling, Fixed condition for the bottom boundary. There are many errors could occurs such as due to time-stepping algorithms (numerical damping, time interval), stress strain relationship, finite element modelling, etc. They should also be taken into account but not discussed here. One thing should be noticed is the stress strain curve chosen here is not perfect to describe the reality but we pay less attention here to concentrate to study difference due to numerical procedures of ELA, CELA and NLA. But the consistency of the data for each numerical procedure kept same. It is always difficult to model the macro behaviour by the micro-mechanism model. Determination of the parameters is also not easy for this case. For example, shear failure is very much affected by initial condition of soil. $K_o = 0.5$ is most possible value but is not clearly known [Shiomi and Shiogeno 1993a]. This is one of the reason that ELA is preferably used since it does not account the initial stress. ELA can be used with less material parameters. However, in case that failure mechanism is requested to predict, NLA is only the method to use.

Cycle-wise Equivalent Linear Analysis (CELA)

As mentioned above, ELA can not change the stiffness and damping in time and can not simulate the change of material (elasto-plastic behaviour) in time. To overcome this, cycle-wise equivalent linear analysis (CELA) is introduced. CELA is based on the direct time integration method and it updates stiffness and damping of the dynamic system at each half cycle. The half cycle is defined as period between a zero-crossing time and the next zero-crossing time. This method does not need complex stress strain curve and cyclic rule. It uses scant stiffness as well as ELA. The difference of the procedure is summarised in Figure 3. There are some efficient and deficient aspects. The efficient aspect is that it can simulate big change of material strength such as ground liquefaction. ELA can not analyse two extremely different conditions, i.e. condition before liquefaction and after liquefaction but CELA can. The deficient aspect is that it can not use stiffness of complex number since it is integrated in time domain. This can be overcome by using the general form of normal damping matrix.

$$\mathbf{C} = \mathbf{M}\sum_n a_n \left[\mathbf{M}^{-1}\mathbf{K}\right]^n \tag{1}$$

Stiffness and damping are changed at each cycle. They are calculated as a function of the maximum shear strain of the previous cycle. G = G(coefficient * γ_{max} of the n-1 th half-cycle). In stead of the previous cycle, the previously calculated results can be also used. The damping ratio is also obtained by the same manner.



	ELA	CELA	NLA
Integration	in frequency	in time	in time
Change of K and C	constant for whole period	constant for each cycle	changes at each step
Stiffness K	scant	scant	tangent
Damping	complex number	propotional to M and K	propotional to M and K

Figure 3: Concept of Cycle-wise Equivalent Linear Method

Shear modulus can be expressed as a function of the mean effective stress σ_m' and the normalised shear strain γ/γ_{50}. γ_{50} is the shear strain where the shear stuffiness is half of the stiffness at small strain ($\gamma<10^{-5}$).

$$G = G_0\left(\frac{\sigma_m'}{\sigma_{m_{ref}}'}\right)\Gamma\left(\frac{\gamma}{\gamma_{50}}\right) \tag{2}$$

σ_m' and γ is the value at (n-1)th half cycle. G_o( ) is a function of normalised confining pressure. Γ( ) is a function represented by the reduction curve of the scant stiffness against the normalised shear strain γ/γ_{50}. This can be obtained by a laboratory test (cyclic triaxial test).

Comparison of the numerical procedures

The comparison was made for the maximum acceleration for ELA (SHAKE), CELA and NLA (MuDIAN). The model for the comparison is the soil layer at the Port Island in Kobe City, Japan. Material properties, such as unit weight γ, shear modula G_o at small strain and at the depth of each layer, are shown in Table 1. G-γ curves listed in Table 1 are shown in Figure 5 for sand (S-1, S-2) and clay (C-1, C-2, C-5). KPU, KPM and KPL in the table are the liquefaction strength curve shown in Figure 4, and will be used in the later section. Surface layers G.L. –2.3 m to –18.6 m is possible to liquefy. In the first study of non-liquefied soil layer, this data are ignored. These data for Port Island are based on many reports published such as by Hatanaka et al [Hatanaka et al, 1997].

To compare the two numerical procedures, the G-γ curve is modelled from the stress strain curves given in Figure 5 into the Ramberg-Osgood stress strain relationship. For cyclic behaviour of the nonlinear analysis, double amplitude of the skeleton curve for the cyclic process proposed by Masing is used.

Table 1: Material properties of soil layer

Layer	Depth (m)	V_s (m/sec)	γ (kN/m^3)	Shear modulus G_0 (kPa)	$G\sim\gamma$ curve	Liquefaction strength curve
sand and gravel	-2.3	170	19.60	57800	S-1	
	-10	210	19.60	88200	S-1	KPU
	-14.7	210	19.60	88200	S-2	KPM
	-18.6	210	19.60	88200	S-2	KPL
alluvial clay	-23	180	16.07	53136	C-1	
	-27	180	16.17	53460	C-2	
sand	-32.8	245	17.15	105044	S-2	
sand and gravel	-50	305	17.93	170236	S-2	
sand	-61	350	18.23	227850	S-2	
dilluvial clay	-67	303	16.66	156075	C-5	
	-79	303	16.66	156075	C-5	
sand and gravel	-83.8	320	19.60	204800		

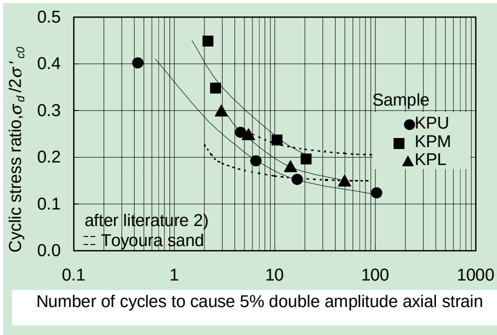


Figure 4: Liquefaction strength

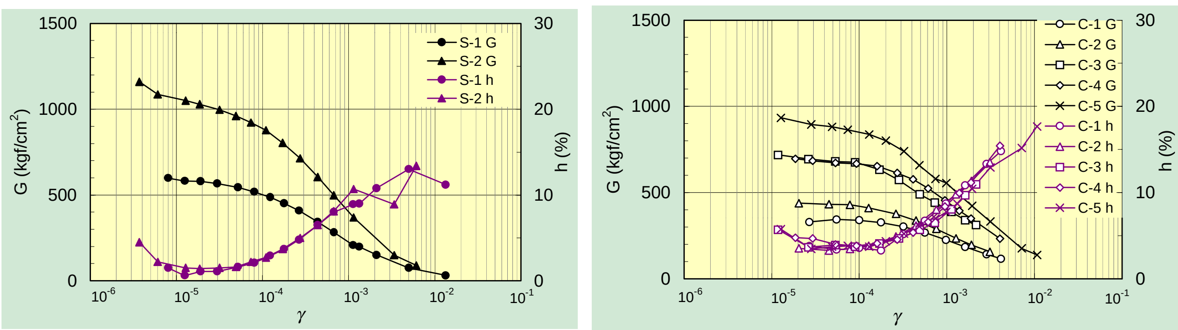


Figure 5: Scant shear modula for soil layers

Earthquake data used in this study are three sets of earthquakes recorded at Port Island during 1995 Hyokogen-Nanbu earthquake [Iwasaki and Tai, 1996]. The record at G.L. -83.8m is used as the input motion at the fixed bottom boundary.

The maximum response accelerations at the surface by ELA, CELA and NLA are shown in Figure 6. All numerical procedures show the same tendency to the magnitude of input motion. In this example, the larger the input motion is, the smaller the amplification factor is. Amplification factor for the maximum acceleration at the surface for the input motion 50 gal is 1.4, 1.0 for 200 gal, and 0.8 for 723 gal as seen in the profile for depth in Figure 6. In this sense, even for the input motion 50 gal, soil layers may be in the range of quiet large nonlinearity.

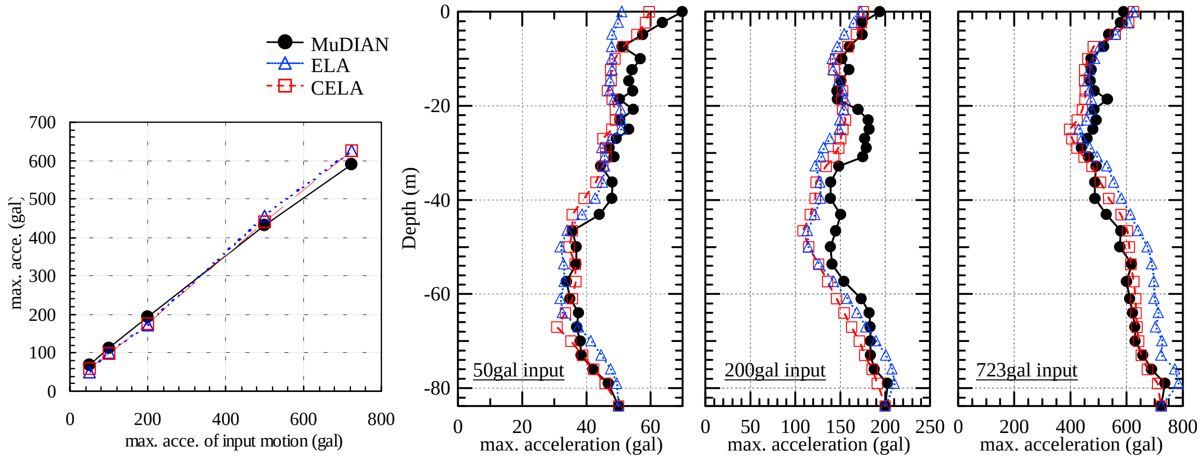


Figure 6: Comparisons of numerical methods for the magnitude of earthquake

NUMERICAL PROCEDRUE FOR LIQUEFACTION PROBOLEM

Dynamic effective stress analysis (DESA) has been received significant attention for recent years and is used for real engineering [Zienkiewicz et. al. 1998]. The analysis is based on the two-phase dynamic equation and the constitutive equation, which can simulate dilatancy behaviour of soil material. DESA can take into account the dissipation of pore pressure as well as its generation. This analysis is not as popular as ELA. One of the reason is that setting parameters of the constitutive equation is not clearly proposed with parameters / indices which can be obtained by ordinary procedure of the site investigation. This is not easy because the most constitutive equations have more unknowns than the properties that soil investigation can provide. Therefore, a numerical procedure is proposed, aiming the simple determination of the parameters for the analysis. Indeed, only liquefaction strength curve is necessary in addition to the ordinary set of the material properties of ELA.

Cycle-wise Equivalent Linear Liquefaction Analysis

Cycle-wise equivalent linear liquefaction analysis (CELLA) [Shiomi et al, 1999] is for an analysis procedure of dynamic response of horizontal soil layer taking into account liquefaction. In this method, excess pore pressure ratio is calculated from the accumulated damage ratio. Liquefaction is the phenomenon which reduces the mean effective stress that causes the shear strength and stiffness decrease. The reduction is calculated by equations 4 to 6. The left-hand side of equation includes not only the confining pressure due to upper load but also the change due to liquefaction. γ_{50} also changes as the confining pressure changes due to liquefaction.

$$G_0 = G_0\left(\frac{\sigma'_m}{\sigma_{m_{ref}}}\right)^n \quad \text{and} \quad \gamma_{50} = \gamma_{50\,ref}\left(\frac{\sigma'_m}{\sigma_{m_{ref}}}\right)^n \tag{4}$$

The above mean effective stress is calculated from the liquefaction ratio r_u as follows.

$$\sigma'_m = \sigma'_{mo}(1 - r_u) \tag{5}$$

The liquefaction ratio r_u can be obtained from empirical function of the accumulated damage ratio D. Seed et al [Seed et. al., 1976] proposed a function as follows.

$$r_u = \frac{1}{2} + \frac{1}{\pi}\sin^{-1}\left(2D^{\frac{1}{\alpha}} - 1\right) \tag{6}$$

The relationship is varies wide range [Tatsuoka et. al. 1980] so that it can simply be proportional, i.e. $r_u = D$.

The damage ratio is calculated using the liquefaction strength curve. The curve shows the stress ratio against the number of cycle applied at a triaxial test or a hollow cylinder test. The damage ratio is calculated by accumulating the incremental damage ratio ΔD for each half cycle. ΔD is obtained by 1/(2N) where N is number of cycle corresponding to the maximum shear stress ratio of the previous half cycle. The following figure illustrates whole procedure of CELLA.

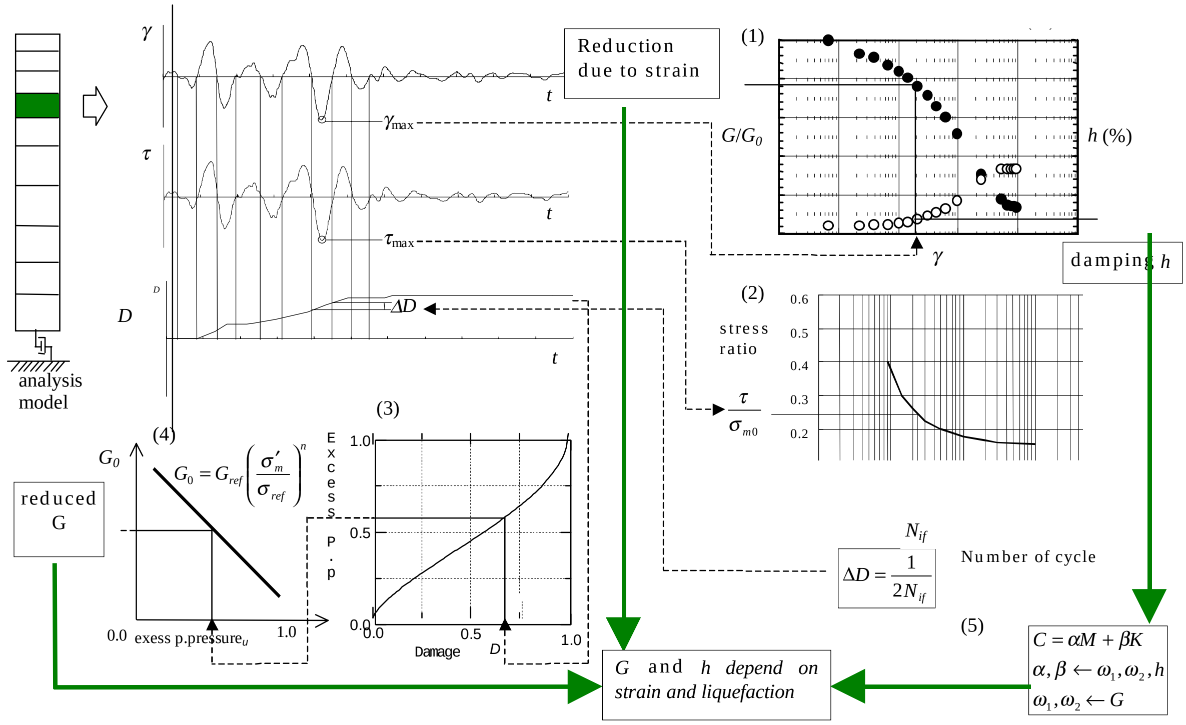


Figure 7: Concept of cycle-wise equivalent linear analysis for liquefaction

Verification of Cycle-wise Equivalent Linear Liquefaction Analysis (CELLA)

CELLA method is verified by comparing with DESA. The program MuDIAN [Shiomi et al, 1993b] and DIANA-SWANDYNE-II [Zienkiewicz et al, 1998] is used for DESA. In that, constitutive equation is Ramberg-Osgood model for shear behaviour and the densification model for the dilatancy behaviour for MuDIAN and Pastor-Zienkiewicz model is for SWANDYNE-II. Numerical Verification for CELLA is also studied by Nukui et al [Nukui et al, 1999].

Figure 8 shows the maximum acceleration at the surface against the magnitude of the input motion. The larger the input motion is, the larger the response acceleration (Figure left-hand side). The maximum acceleration at the surface seems to reach a limit, as the input motion becomes so large. The profile of the maximum acceleration against the depth is different between the non-liquefaction (Figure-6) and liquefaction (Figure 9, right-hand side) problems. The study for liquefaction shows less amplification ratio. The maximum acceleration at the surface is 1.1 - 1.3 times the input motion for 50 gal, 0.8 - 1.0 for 200 gal, 0.6 - 0.7 for 723 gal, depend on the numerical procedures. The maximum accelerations of any numerical methods for the input motion 723gal agree to the observation.

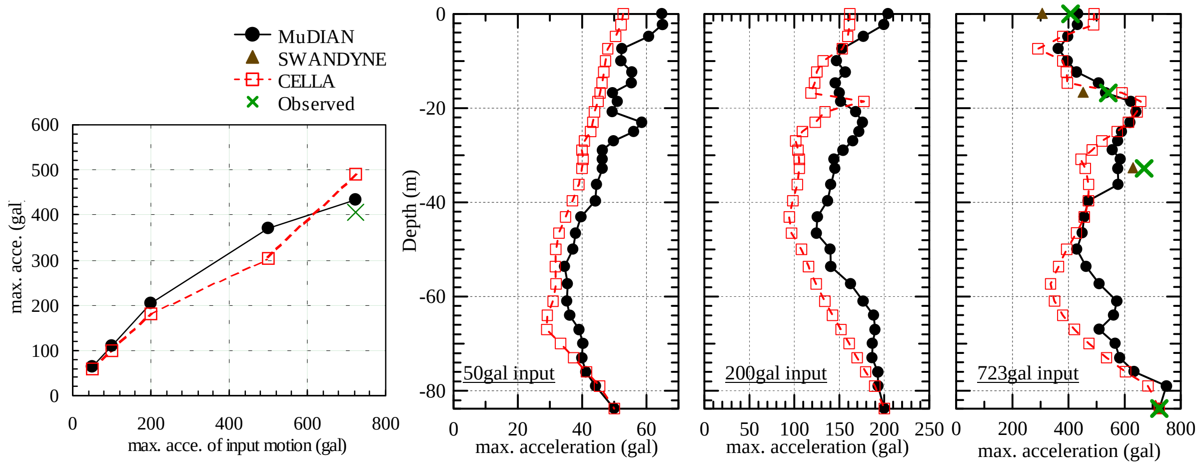


Figure 8: Comparisons of CELLA with stepwise nonlinear analysis

Time history of CELLA, MuDIAN, SWANDYNE-II is compared in Figure 9. All procedures capture the elongation of the period of response wave due to liquefaction. Liquefaction takes place almost same time of 4 seconds for all procedure in case of that the input motion is 728 gal. The transfer functions at the depth G.L. 0.0m, -16.8m and –32.8m against the input motion are compared. It is significant that characteristics of ELA and NLA that NLA (MuDIAN) shows significant magnitude of transfer function at high frequency (2 – 5Hz) as well as the observed data. CELLA does not have these components. Both CELLA (red line) and MuDIAN (black line) shows peak at about 0.2Hz. CELLA has peak at about 0.4Hz but MuDIAN does not. This may be reason of the different response at surface layers.

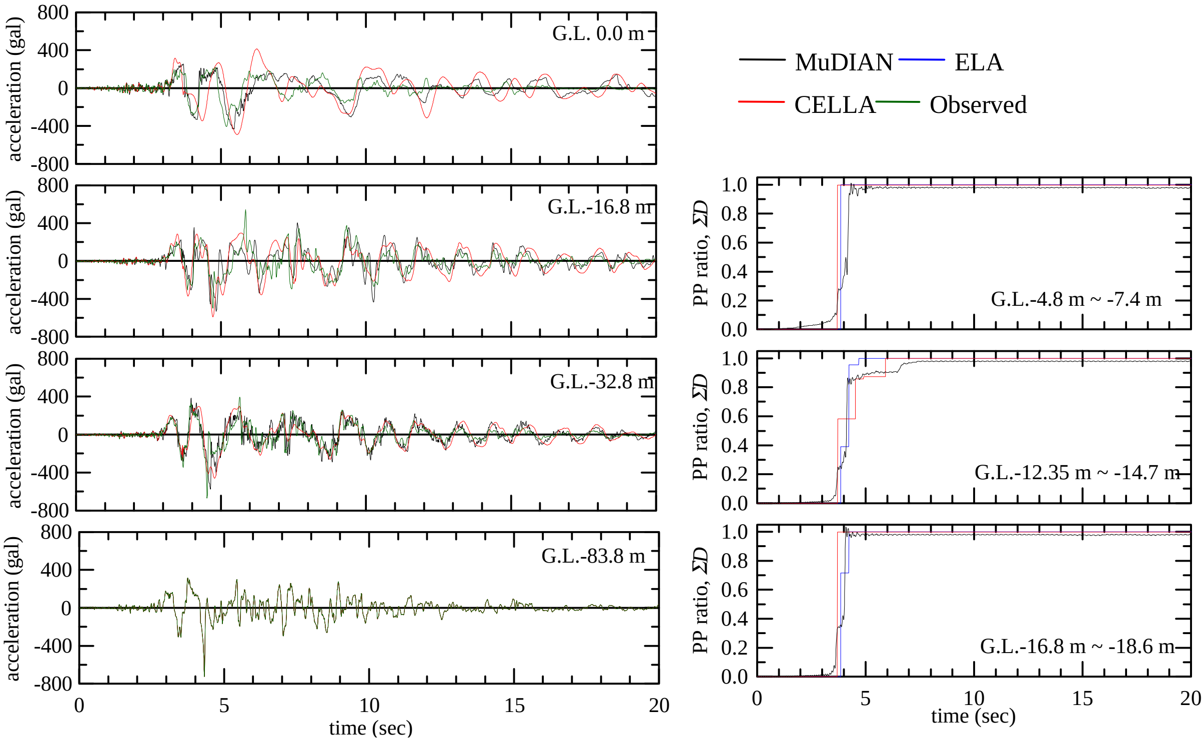


Figure 9: Liquefaction Simulation of Port Island (Time history of acceleration and excess pore pressure ratio / accumulated damage ratio)

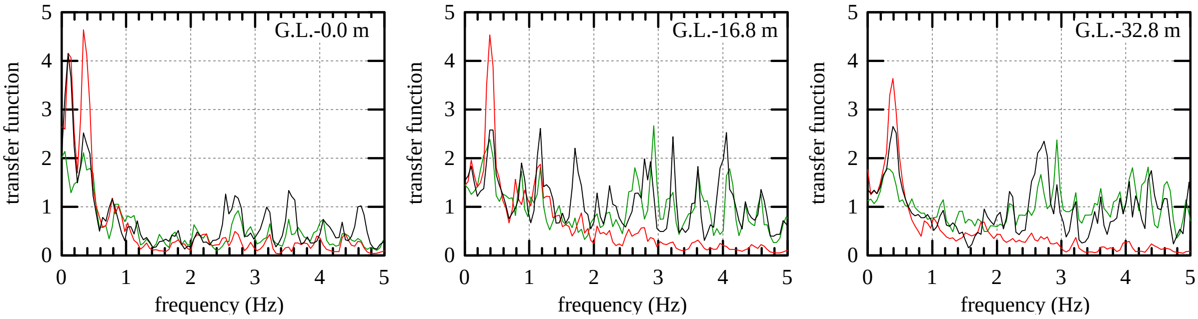


Figure 10: Transfer function to the input motion at GL 0.0m, -16.8m and –32.8m

CONCLUSIONS

A simple procedure for the liquefaction analysis of the dynamic response analysis of horizontal soil layers to earthquake motion is proposed. The method is the cycle-wise equivalent linear analysis and only uses the liquefaction strength curve as an additional data to the ordinary equivalent linear analysis such as SHAKE. The method was demonstrated in the two steps. At first the method tested without liquefaction and showed excellent agreement with the ordinary equivalent linear analysis and stepwise nonlinear analysis. At second, the method tested with liquefaction and showed very good agreement with the effective stress analysis. Theoretically cycle-wise equivalent linear analysis can not calculate the permanent deformation but it can be used as the first liquefaction analysis at least. It could be used as a one of effective stress analysis.

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