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THE APPLIANCE OF WAVELET TRANSFORM IN THE ANALYSIS OF SEISMIC STRUCTURE DYNAMIC RELIABILITY

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SUMMARY

With good time-frequency discrimination ability and flexible time-frequency windows, wavelet transform is now widely used to analysis various signals in time and frequency domain simultaneously. Because of the simple relation between wavelet spectra and local (evolutionary) power spectra of a signal, a new method based on wavelet transform to estimate the local power spectra of earthquake ground motions is put forward. With the local power spectra (LPS) acquired, the LPS of structural response are calculated by numerical integral and the dynamic reliability of the structure based on the first passage criterion is estimated. It is proved that the local power spectra estimated by wavelet transform are accurate to reflect the time-frequency characteristics of earthquake ground motions. Wavelet transform may improve the traditional spectrum analysis to time-frequency analysis.

INTRODUCTION

In earthquake engineering, the strong non-stationary characteristics both in time domain and frequency domain, i.e. amplitude and frequency, of earthquake ground motion are well known and difficult to tackle in applying the ground motion as input to acquire structure responses. Usually, the non-stationary amplitude is dealt with by uniformly modulated stationary process [Caughey and Stumpf 1961; Lin 1963; Hammond 1968; Corotis and Marshall 1977; Gasparini and DebChaudhury 1980]. And the frequency non-stationary characteristic is not or seldom considered explicitly. Both the former and the latter will influence the dynamic reliability of structure when the structure subjected to the ground motion excitations [Yeh and Wen 1990; Papadimitriou and Beck 1992; Conte 1992; Li et al. 1998]. But the existing random vibration theory can not consider them accurately and simultaneously, so the stationary stochastic excitation transfer method has been adopted in reliability analysis in spite of the non-stationary characterization.

As a new method with obvious advantage for time-frequency analysis, wavelet transform is now applied in many fields of study. Howbeit, it is not so popular in civil engineering research. As we can see, seldom papers on this question are issued until now. In this paper, the earthquake ground motion is taken for a non-stationary process, and coped with by wavelet transform to acquire its local power spectra (LPS). Because of the good time-frequency discrimination ability and flexible time-frequency windows of wavelet

transform, the LPS estimated by wavelet transform reflect precisely the unstable characteristics both in time domain and frequency domain of the non-stationary process. With the aid of evolutionary process theory [Priestly 1967], the LPS of elastic unsteady structural response are calculated by numerical integral and so the influence of non-stability in time and frequency of the non-stationary process is considered adequately. The seismic structure is modeled as a MDOF (multi-degree-of-freedom) system with concentrated mass and its dynamic reliability based on the first passage criterion is estimated with the LPS acquired.

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WAVELET TRANSFORM

Wavelet transform is a very new mathematical tool that cuts up signals into different frequency components, and then studies each component with a resolution matched to its scale [Daubechies 1992]. The so called "scale" means the frequency band of the wavelet. The wavelet series used to analysis signals are expanded or shrunk by the mother wavelet automatically. The small scale, which can be regarded as a narrow time window, means high frequency band corresponding to shrunk wavelet and vice versa. Because of the characteristics of automatic changing windows according to the frequency being analyzed, wavelet transform has very good time-frequency discrimination ability. There are many mother wavelets (basis) to be chosen, however, and so it is with the transform method. One must choose an appropriate mother wavelet and transform method on basis of characters of signals to be analyzed. Through comparisons and studies [Cao 1999], the Littlewood-Paley (L-P) basis and the direct wavelet transform [Chen 1998] are thought as relatively adapt to analyzing the earthquake waves, for the tight support in frequency domain of the L-P basis and the same length of components gotten by the direct wavelet transform. The direct wavelet transform means that to get the components by direct convolution of wavelets and signal, and so it is time-consuming and with lower precision comparing to indirect wavelet transform. It is referenced in [Chen 1998]. To overcome the disadvantage, some changes are carried out. The first is implementing convolutions by FFT (Fast Fourier Transform). The second and more important is to utilize the formulation in frequency domain of L-P basis. The main merits of the changes are as follows: FFT runs much faster than the direct convolution; FFT with the formulation in frequency domain is much more precise than that in time domain; characters of tight support in frequency domain of L-P basis can help to prevent leakage, so can promote the precision all the more. Details can be found in [Cao 1999]. So the dyadic discrete formulation of L-P basis in time and frequency domain are expressed as

$$\psi_{m,b}(t) = 2^{m/2}\,\frac{\sin 2\pi\,\dfrac{t-b}{2^m} - \sin\pi\,\dfrac{t-b}{2^m}}{\pi(t-b)} \tag{1}$$

$$\hat{\psi}_{m,b}(\omega) = \sqrt{\frac{2^m}{2\pi}}\,e^{-i\omega b} \qquad \frac{\pi}{2^m} \le |\omega| \le \frac{\pi}{2^{m-1}} \tag{2}$$

The wavelet spectra $W_m(b)$ of a signal Y(t) are as

$$W_m(b) = \int_R \hat{Y}(\omega)\hat{\psi}_m(\omega)e^{i\omega b}d\omega \tag{3}$$

And Y(t) can be reconstructed by $W_m(b)$ as

$$Y(t) = \sum_m \frac{1}{2^m}\int_R \hat{W}_m(\omega)\hat{\psi}_m(\omega)e^{i\omega t}d\omega \tag{4}$$

$\hat{Y}(\omega)$ is the Fourier transform of Y(t), m means scale and b means time in the above equations. The scheme of changed direct wavelet transform is that: transform the signal Y(t) form time domain into frequency domain to get $\hat{Y}(\omega)$ with FFT; get the product of $\hat{Y}(\omega)$ and the $\hat{\psi}(\omega)$;then transform the product into time domain with IFFT to get $W_m(b)$.

LOCAL POWER SPECTRA OF GROUND MOTIONS

The integral of the product of Y(t) and G(t) can be obtained by their wavelet spectra as follows [Daubechies 1992]

$$\int_R G(t)Y(t)dt = \frac{1}{C_\psi}\iint_{R^2} W_Y(a,b)W_G(a,b)\frac{da}{a^2}db \tag{5}$$

if $G(t)=Y(t)$, then

$$\int_R Y^2(t)dt = \frac{1}{C_\psi}\iint_{R^2} [W_Y(a,b)]^2\frac{da}{a^2}db \tag{6}$$

by using Parseval's identity, one can get

$$\int_R \left|Y(\omega)\right|^2 d\omega = \frac{1}{C_\psi} \iint_{R^2} [W_Y(a,b)]^2 \frac{da}{a^2} db \tag{7}$$

The above equations are expressed for one signal, or say, one earthquake wave. On the viewpoint of random vibration theory, an earthquake wave is just one realization of earthquakes, i.e. a sample of a nonstationary random process. So to characterize the process, the average over an ensemble should be made. Thus the expected total energy of the process is

$$\int_R E[\left|Y(\omega)\right|^2] d\omega = \frac{1}{C_\psi} \iint_{R^2} E[W_Y(a,b)]^2 \frac{da}{a^2} db \tag{8}$$

In equations (5) to (8), $W_Y(a,b)$ are the wavelet spectra of Y(t) gotten by continuous wavelet transform. They should be discretized to implement numerical calculation. Let $a_m=2^m$, and $b_k=(k-1)$ b, then the dyadic discrete version of Eq. (8) is

$$\int_R E[\left|Y(\omega)\right|^2] d\omega = \sum_m \frac{1}{2^m} \sum_k \Delta b E[W_m(b_k)]^2 \tag{9}$$

If the summation over m in Eq. (9) is taken out, the energy corresponding to the mth frequency band is obtained as

$$\int_R E[\left|Y^m(\omega)\right|^2] d\omega = \frac{1}{2^m} \sum_k \Delta b E[W_m(b_k)]^2 \tag{10}$$

As well as, the energy of the process in time $t=b_k$, i.e. the instantaneous mean-square value of the process, is gotten from equation (9) as

$$E[Y^2(t)]\Big|_{t=b_k} = \sum_m \frac{1}{2^m} E[W_m(b_k)]^2 \tag{11}$$

Eq. (10) can be discretized with time b_k like Eq. (11) to obtain the expected energy of the process corresponding to the mth frequency band in time $t=b_k$ as

$$\frac{1}{\Delta b} \int_R E[\left|Y_k^m(\omega)\right|^2] d\omega = \frac{1}{2^m} E[W_m(b_k)]^2 \tag{12}$$

The formulation in the left of Eq. (12) is usually called as the instantaneous power spectra of the process. Nevertheless, the true instantaneous energy distribution of the process or a sample can not be acquired by any time-frequency analysis method for the reason of the spectral uncertainty principle or Heisenberg 's uncertainty principle [Priestly 1967]. In fact, it represents the local energy distribution over frequency of the process. Hence on the viewpoint of the author, it is more exact to call it as the local power spectra (LPS). As we can say, a simple relation between the LPS and the wavelet spectra of the process exists as shown in Eq. (12). So the LPS of the process can be easily estimated by wavelet transform.

It is well known that the earthquake ground motion is a nonstationary random process as mentioned previously. To characterize a nonstationary random process, infinite samples are needed in mathematics sense. In practice, it is impossible. A volume of samples, e.g. 30, is usually used. Thirty waves are adopted here to construct a sample base, which describes an earthquake ground motion process in some extent. These samples are simulated with the method suggested by [Cao 1999] using the LPS of a famous actual wave, EL centro NS record. In addition, the normalized acceleration responses of these artificial waves are all resembled in order to correctly represent the process in random vibration theory sense. Firstly each sample is decomposed into wavelet spectra by wavelet transform method aforementioned and then all the wavelet spectra are square-averaged over the ensemble, i.e. the sample base. According to Eq. (12), the LPS of the process can be obtained by the square-averaged wavelet spectra of the samples. Thus, the LPS estimated by wavelet transform are used to characterize the nonstationary process.

STRUCTURAL RESPONSES TO NONSTATIONARY EXCITATIONS

As introduced previously, structural responses to nonstationary excitations are difficult to be acquired with the existing theory. So an uniformly modulated process or a stationary process is often used as the earthquake excitation. But now, the LPS of a nonstationary process can be applied to obtain structural responses with the aid of evolutionary process theory.

With the use of the evolutionary process theory [Priestly 1967], a nonstationary process Y(t) can be expressed as the Fourier-Stieltjes integral of a stationary process X(t) and a deterministic function $A(t,\omega)$ as

$$Y(t)=\int_{-\infty}^{\infty} A(t,\omega)e^{i\omega t}dZ(\omega) \tag{13}$$

in which $dZ(\omega)$ is a zero-mean orthogonal-increment relevant with $X(t)$ having property

$$E[dZ(\omega_1)dZ^*(\omega_2)]=S_{XX}(\omega_1)\delta(\omega_2-\omega_1)d\omega_1 d\omega_2 \tag{14}$$

where the superscript * denotes the complex conjugate operator and $S_{XX}(\omega)$ is the power spectral density function (PSDF) of $X(t)$. Then the time-dependent (evolutionary) PSDF of $Y(t)$ can be denoted by

$$S_{YY}(t,\omega)=|A(t,\omega)|^2 S_{XX}(\omega) \tag{15}$$

The ith modal displacement response $R_i(t)$ of a MDOF structure to the excitation Y(t) is available with the Duhamel integral as

$$R_i(t)=\int_0^t h_i(t-\tau)\int_{-\infty}^{\infty} A(\tau,\omega)e^{i\omega\tau}dZ(\omega)d\tau=\int_{-\infty}^{\infty} m_i(\omega,t)e^{i\omega t}dZ(\omega) \tag{16}$$

where $h_i(t)$ is the unit impulse response function of the ith mode and $m_i(\omega,t)$ is the time-frequency modulated function of the ith modal displacement response, $m_i(\omega,t)$ is expressed as

$$m_i(\omega,t)=e^{-i\omega t}\int_0^t h_i(t-\tau)A(\tau,\omega)e^{i\omega\tau}d\tau \tag{17}$$

So the cross-correlation function of R_i and R_j can be expressed as

$$\phi_{R_iR_j}(t,\tau)=E[R_i(t)^* R_j(t+\tau)]=\int_{-\infty}^{\infty} m_i^*(\omega,t)S_{XX}(\omega)m_j(\omega,t+\tau)e^{i\omega\tau}d\omega \tag{18}$$

If τ=0, Eq. (18) reduces to

$$\phi_{R_iR_j}(t,0)=\int_{-\infty}^{\infty} m_i^*(\omega,t)S_{XX}(\omega)m_j(\omega,t)d\omega=\int_{-\infty}^{\infty} S_{XX}(\omega,t)d\omega \tag{19}$$

where

$$S_{XX}(\omega,t)=m_i(\omega,t)^* S_{XX}(\omega)m_j(\omega,t) \tag{20}$$

is the evolutionary cross PSDF of the ith and jth modal response. It is obvious to see if $m_i(\omega,t)$ and $m_j(\omega,t)$ can be found, $S_{XX}(\omega,t)$ is available. But the integral in Eq. (17) is often difficult to implement, mainly because of the adaptive formulation of $A(t,\omega)$ is hard to get. Nevertheless, if $S_{XX}(\omega)$ is regarded as 1 forever, $A(t,\omega)$ can be expressed as follows according to Eq. (15)

$$A(t,\omega)\quad (S_{YY}(t,\omega))^{1/2} \tag{21}$$

Though the formulation of $S_{YY}(\omega,t)$ is usually more difficult to find than that of $A(t,\omega)$, the $S_{YY}(\omega,t)$ can be obtained by wavelet transform as the LPS, and with numerical integral method the integral in Eq. (17) can be carried out without problem. The LPS of the nth structural layer displacement response hence can be obtained as

$$S_n(t,\omega)=\sum_i\sum_j \phi_n^i\phi_n^j\alpha_j\alpha_i m_i^*(\omega,t)m_j(\omega,t) \tag{22}$$

in which ϕ^i_n and ϕ^j_n is the nth component, α_i and α_j is the participant coefficient of the ith and jth mode respectively. And, it is easy to deduct the LPS of the nth inter-layer displacement response as

$$S_n(t,\omega) = \sum_i \sum_j (\phi_n^i - \phi_{n-1}^i)(\phi_n^j - \phi_{n-1}^j)\alpha_j \alpha_i m_i^*(\omega,t) m_j(\omega,t) \tag{23}$$

Therefore, the elastic nonstationary structural responses to the nonstationary process excitation are characterized by the LPS of the response process.

STRUCTURAL DYNAMIC RELIABILITY

The structural dynamic reliability considered here is based on the first passage criterion and has been studied by many researchers for a long time. One of the reliability formulae adopted in this paper is put forward by [Corotis et al. 1972]

$$P_s(t,b) = \exp\left\{ \int_0^t -2v_b(\tau) \frac{1-\exp[-v_b^e(\tau)/2v_b(\tau)]}{1-(\frac{v_b(\tau)}{v_0(\tau)})} d\tau \right\} \tag{24}$$

and the other is suggested by [Wu 1989]

$$P_s(t,b) = \exp\left\{ -\int_0^t v_b^e(\tau) d\tau \right\} \tag{25}$$

where $v_0(t)$, $v_b(t)$ and $v_b^e(t)$ is the instantaneous expected up-crossing rate for the random process to cross 0, to cross level b, and the instantaneous expected up-crossing rate for the envelope of the random process to cross level b respectively, and expressed as follows

$$v_0(t) = \frac{1}{2\pi}\sqrt{\frac{a_2(t)}{a_0(t)}} \tag{26}$$

$$v_b(t) = \frac{1}{2\pi}\sqrt{\frac{a_2(t)}{a_0(t)}} \exp(-\frac{b^2}{2a_0(t)}) \tag{27}$$

$$v_b^e(t) = \frac{b}{\sqrt{2\pi}a_0(t)}\sqrt{(a_0(t)a_2(t) - a_1^2(t))/a_0(t)} \exp(-\frac{b^2}{2a_0(t)}) \tag{28}$$

in addition, another formula of $v_b^e(t)$ can be used in Eq. (25), that is $v_b^{e'}(t)$ [Wu 1989]

$$v_b^{e'}(t) = \frac{2b}{\pi\sqrt{2\pi}a_0(t)}\sqrt{(a_0(t)a_4(t) - a_2^2(t))/a_2(t)} \exp(-\frac{b^2}{2a_0(t)}) \tag{29}$$

$a_0(t)$, $a_1(t)$, $a_2(t)$ and $a_4(t)$ in above equations are the various order moment of the evolutionary PSDF, which found to be

$$a_k(t) = \int_{-\infty}^{\infty} S_{XX}(\omega,t)\omega^k d\omega \tag{30}$$

Let the evolutionary PSDF in Eq. (30) be the LPS of structural responses obtained previously, the not passage of level b reliability of each layer's inter-layer displacement is available by above equations.

Eq. (24) and (25) are expressed for nonstationary process, i.e. the nonstationary structural response process. For comparison, formulae for stationary process are also employed here, which given by [Davenport 1964]

$$P_s(T,b) = \exp[-v_0 T \exp(-b^2/2a_0)] \tag{31}$$

and by [Corotis et al. 1972]

$$P_s(T,b) = \exp\left\{ -v_0 T \exp(-\frac{b^2}{2a_0}) \frac{1-\exp[-\sqrt{\pi/2}\sqrt{(a_0 a_2 - a_1^2)/a_0}\, b/a_0]}{1-\exp(-\frac{b^2}{2a_0})} \right\} \quad (32)$$

in which a_0, a_1 and a_2 are various order moment of PSDF of the stationary process, having the same formulation of Eq. (30), except for they are now time-independent, so the $S_{xx}(\omega,t)$ should be replaced by $S_{xx}(\omega)$. The structural response process is actually nonstationary, however, it is the aim of this paper to compare the results of Eq. (24), (25), (31) and (32). By the way, the time-independent PSDF $S_{xx}(\omega)$ of the structural response process can be acquired by traditional Fourier transform and modal analysis method. Things have been discussed are just for the reliability of structure of one-degree-of-freedom. About a MDOF structure with N layer, the formula suggested by [Ou 1984] can be used

$$P_s(T) = \prod_{i=1}^{N} P_{si}(T) \quad (33)$$

where P_{si} is reliability of the ith layer which can be obtained by the above equations. Eq. (33) means that the whole reliability of the structure is the product of reliability of each layer.

NUMERICAL EXAMPLE

The structure considered here is modeled as a concentrated mass MDOF shear-beam type fixed-base, five-story building, with critical damping ratio being 5%. The mass and story stiffness and height of each layer is 847, 268.4, 108.1, 130.6 ,224.5 kN and 4.5, 1.64, 3.5, 4.4, 2.07×10^4 kN/m, and 3.5, 3.5, 3.2, 3.0, 4.6 m, from the lower to the upper layers respectively. The natural frequencies of this building are computed to be 3.55, 9.96, 18.38, 27.27 and 32.27 rad/s.

Under the excitation of the nonstationary process characterized by its LPS aforementioned, the LPS of each structural layer inter-layer displacement response are obtained by Eq. (23). The LPS of the excitation and the first layer displacement response is shown in Fig.1 and Fig.2 respectively. The mean-square value history of the first displacement response is also acquired with the LPS and Eq. (30), and shown in Fig.3. Contemporarily, the mean-square value history is calculated using the time-history analysis method (step by step integral method) with the 30 samples described earlier as the inputs, and square-averaging the results over the ensemble, shown in Fig.3 as well. Let *b* be the H/450 (H is the height of each layer), the dynamic reliability can be gotten by Eq. (24), (25), (31), (32) and (33), with the acquired LPS of each structural layer inter-layer displacement response. The last results are shown in Fig.4, in which, Stationary 1 means the result by Eq. (31), Stationary 2 by Eq. (32), Nonstationary 1 by Eq. (24), Nonstationary 2 by Eq. (25) with $v_b^e(t)$, Nonstationary 3 by Eq. (25) with $v_b^{e'}(t)$. Eq. (33) is used for all things to get the whole reliability.

CONCLUSIONS

A method to estimate the LPS of a nonstationary process with a fast and accurate wavelet transform devised by the author is introduced in this paper and applied to acquire the LPS of a structural elastic unstable response under the excitation of the process. With the LPS acquired, the reliability of the structure is determined based on the first passage criterion.

In Fig.3, the mean-square value history of layer displacement gotten by the LPS transfer is well compatible with those by the time-history analysis. It is proved that the LPS of the process estimated by wavelet transform describe its nonstationary characteristics accurately and the LPS transfer has enough precision. So it can be said that wavelet transform is a good tool adaptive to time-frequency analysis in earthquake engineering. It is shown that in Fig.4, except for Nonstationary 3, the results obtained by nonstationary methods are bigger than those by stationary methods. That is, the latter is more conservative than the former. It is consistent with the foregone knowledge. And it is accordant with the conclusions of [Wu 1989] for the abnormality of Nonstationary 3. Anyhow, it is possible that with good time-frequency discrimination ability, wavelet transform can improve the studies of earthquake engineering from conventional frequency spectrum analysis to more accurate time-frequency analysis.

REFERENCES

Cao Hui (1999) *Studies on Wavelet Transform and Random Vibration Theory in Earthquake Engineering*, Ph.D Thesis,Chongqing Jianzhu University

Caughey, T.K. and Stumpf, H.J. (1961) "Transient Response of a Dynamic System under Random Excitation", *J. Appl. Mech. ASME* 28(E), 563-566

Chen, Zhi-qui (1998) *Studies on the Wavelet Bases of Engineering Signals and the Wavelet Transform Analysis Instrument System*, Ph.D Thesis, Chongqing University

Conte, J. P. (1992) "Effects of Earthquake Frequency Nonstationary On Inelastic Structural Response", *Proc. of the 10th WCEE*, Vol.7

Corotis, R.B. , Vanmarcke, E.H. and Cornell, C.A.(1972) "First Passage of Nonstationary Random Process", *J. Eng. Mech. ASCE*, Vol.98, No.2, 289-302

Corotis, R. B. and Marshall, T. A. (1977) "Oscillator Response to Modulated Random Excitation", *J. Eng. Mech. ASCE*, Vol.103, EM4, 501-513

Daubechies, I. (1992) *Ten Lectures on Wavelets, Society for Industrial & Applied Mathematics*, Philadelphia, PA

Davenport, A. G. (1964) "Note on the Distribution of the Largest Value of a Random Function with Application to Gust Loading", *Proc. ICE*, Vol.28, 187-196

Gasparini, D. A. and DebChaudhury, A. (1980) "Dynamic Response to Nonstationary Nonwhite Excitation", *J. Eng. Mech. ASCE*, Vol.106, EM6, 1233-1248

Hammond, J. K. (1968) "On the Response of Single and Multidegree of Freedom Systems to Nonstationary Random Excitations", *J. Sound and Vibration*, Vol.7, No.3, 417-430

Li, Ying-min, et al. (1998) "Prominence Analysis of the Earthquake Ground Motion Nonstationary Character in Frequency Domain Influence to Elastic Response Spectra", *Proc. of the 5th National Conference on Earthquake Engineering*, Beijing, 482-488

Lin, Y.K. (1963) "Application of Nonstationary Shot Noise in the Study of System Response to a Class of Nonstationary Excitations", *J. Appl. Mech. ASME*, 555-558

Ou, Jin-ping and Qu, Wei-lian (1984) "The Dynamic Reliability Analysis of the Non-linear MDOF System Under the Stationary Random Excitation", *Earthquake Eng. and Eng. Vib.*, Vol.4, No.1, 20-35

Papadimitriou, K. & Beck, J. L. (1992) "Stochastic Characterization of Ground Motion and Applications to Structural Response", *Proc. of the 10th WCEE*, Vol.2, 835-838

Priestly, M. B. (1967) "Power Spectral Analysis of Non-stationary Random Process", *J. Sound and Vibration*, Vol.6, 86-97

Wu, Wen-Fang (1989) "Envelope Crossing Rate and First Passage Probability", *Proc. of the 5th International Conference on Structural Safety and Reliability*, 1271-1278

Yeh, C.-H. and Wen, Y.K.(1990) "Modeling of Nonstationary Ground Motion and Analysis of Inelastic Structural Response", *Structural Safety*, 8, 281-298

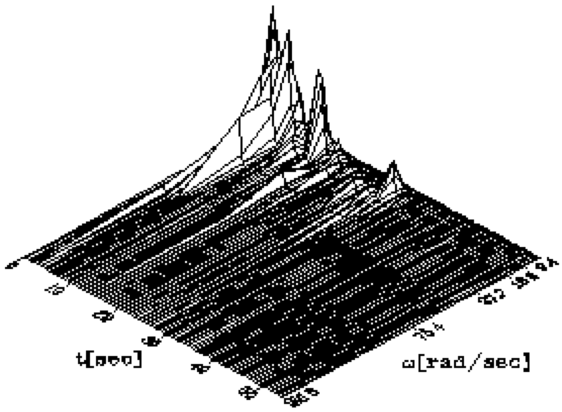

Fig.1 The Local Power Spectra of the Nonstationary Process

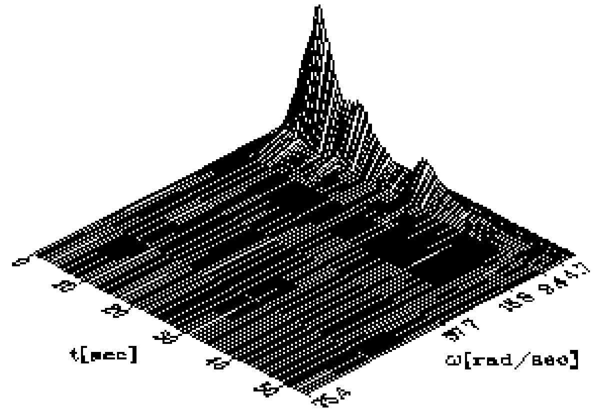

Fig.2 The Local Power Spectra of the Nonstationary Response Process of the First Layer Displacement

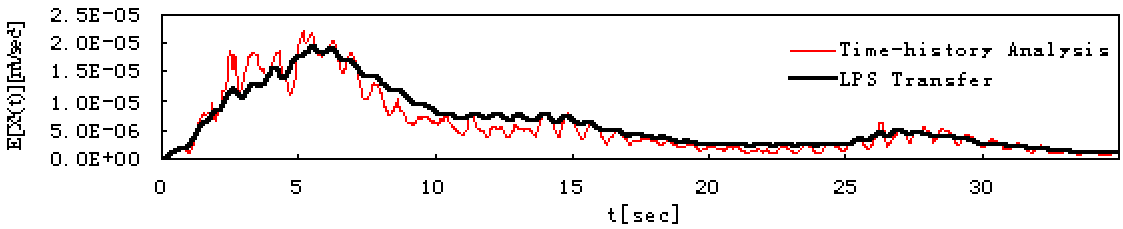

Fig.3 The Mean-Square Value Time History of the First Layer Displacement

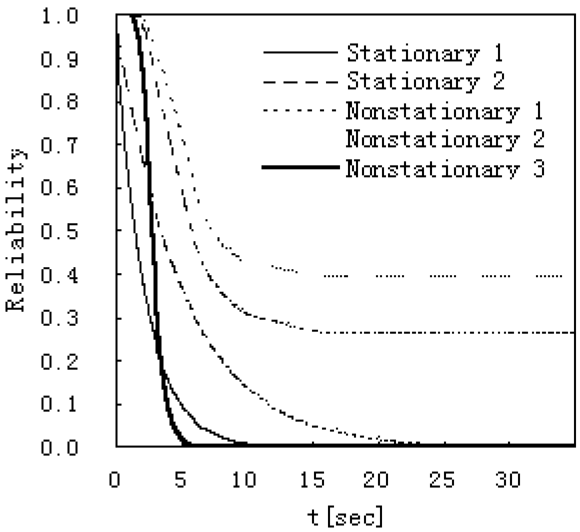

Fig.4 Structural Reliability Estimated by Various Method