Analytical Investigation of Vertical Waving Phenomenon Focused on Earthquake Damage of a Horizontally Long Building with Pile Foundation

M. Motosaka(1) and T. Abe(2)

(1) Professor Emeritus, Tohoku University, motosaka@irides.tohokuac.jp
(2) Graduate Student, School of Engineering, Tohoku University, ffcc.ciaran.football@gmail.com

Abstract

The earthquake damage related to guided wave propagation is important to be investigated. Regarding the earthquake damage such as a suspended ceiling board drop during the 2005/08/16 Miyagi-ken Oki earthquake (M7.2) and also horizontally long pile foundation building damage during the 2011 Tohoku earthquake (M9.0), the author suggested that the earthquake damages are due to the vertical waving phenomenon caused by the guided wave propagation. The vertical waving phenomenon is interpreted that dominant frequency of earthquake-induced force is in the higher frequency range than cut-off frequency, which can be moved to lower frequency due to stiffness reduction of non-linearity of supporting springs.

In this paper, vertical and horizontal coupling behavior with focus on vibration energy transmission bounded by cut-off frequency is investigated for the horizontally long building with pile foundation. To explain the vertical waving phenomenon of the building constructed in an inclined supporting layer, a simple vertical and horizontal coupling equation of motion is considered as partial differential equation with matrix form coefficients by modelling building as shear rod in the longitudinal direction supported by springs representing vertical and horizontal coupling stiffness of pile foundation. The corresponding characteristic wave equation is solved as an eigenvalue problem of the coupling equation of motion. Phase velocity and group velocity of characteristic waves are discussed focused on different wave propagation characteristics in the two propagation zone bounded by two cut-off frequencies. Then, FEM model of the building considering the coupling wave equation of motion has been prepared and wave propagation characteristics are investigated for vertical and/or horizontal harmonic wave excitation, impulse response, and incident earthquake ground motions. The non-linearity of supporting spring is considered in this study, not only linear cases. Based on the investigation analyses, the author concluded that the vertical waving phenomenon which caused the structural damage can be explained by vertical guided wave propagation.

Keywords: earthquake damage, waving phenomenon, guided wave propagation, cut-off frequency, horizontally long building

1. Introduction

One of the interesting wave propagation phenomena is guided wave propagation which is bounded by cut-off frequency, which is border of propagation zone and attenuation zone. During the 2005/08/16 Miyagi-ken Oki earthquake (M7.2), a suspended ceiling board of swimming pool facility was dropped showing waving phenomenon and caused human damage. The author suggested that the earthquake damage of suspended ceiling board drop is related to the guided wave propagation, which is explained by a simple model of a beam/ rod supported by springs (Motosaka, 2006)[1]. As other example, the author reported that a horizontally long 5-story RC building (one span in the transverse direction and 16 spans in the longitudinal direction) with pile foundation was damaged during the 2011 Tohoku earthquake (M9.0), showing vertically waving phenomenon (AIJ, 2011)[2]. In the damaged building site, supporting soil layer is strongly inclined in the transverse direction. Structurally important beams and non-structural walls were damaged. Foundation at the center in the longitudinal direction is also severely damaged. These damages are possibly due to the vertical waving phenomenon.

The characteristic building damage during the 2011 Tohoku earthquake at a 5-story RC pile-foundation building with horizontally long plan (single span in the transverse direction and 16 spans in the longitudinal direction, refer to Photo 1(a)) has been reported. The building is located at a sharply inclined supporting soil. The shearing crack of base mat at the difference in level (Photo 1(b), crack of beam (Photo 1(c)), and shearing crack of non-structural wall seemed to be caused by the induced vertical waving phenomena during the earthquake. It has been suggested that the vertical waving phenomena can be explained by transfer of vibration energy in a shearing and/or bending element (beam or plate) supported by springs [3]. The vertical wave propagation problem has been investigated focused on the vibration energy transmission in the axial (horizontal) direction bounded by the cut-off frequency determined from the weight and the spring, and indicated that the vibration energy transmission occurs in the higher frequency range than the cut-off frequency but not in the lower frequency range (Ohka and Motosaka, 2015[4], Cao and Motosaka, 2015[5], Motosaka, 2017[6]). The vertical waving phenomenon is interpreted that dominant frequency of earthquake-induced force is in the higher frequency range than cut-off frequency, which can be moved to lower frequency due to stiffness reduction of non-linearity of supporting springs. It is noted that the waving phenomenon leads to energy transmission horizontally.

In this paper, considering the damaged pile foundation building constructed in sharply inclined supporting soil, dynamic behaviors focused on vertical waving phenomena are investigated based on the extended version of horizontal and vertical coupling wave propagation problem of shearing rod supported by non-linear springs. In the following Section 2, modelling of the horizontal and vertical coupling wave propagation analysis is described. In Section 3, analysis of vertical wave propagation using shearing rod supported by non-linear springs is addressed for the horizontal and vertical independent model and in Section 4 for the he horizontal and vertical coupling model. Concluding remarks are described in Section 5.

(a) Overview

(b) Damage of base mat at the level difference

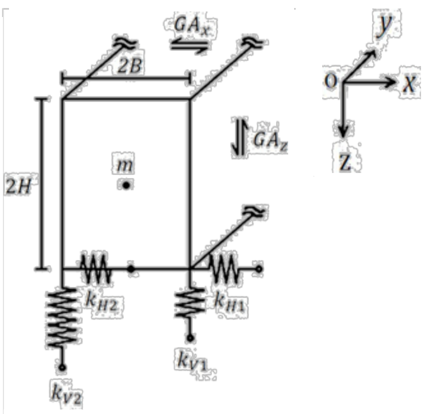
(c) Damage of a structural beam

Photo 1－Damaged 5-story RC pile-foundation building in Sendai and damage feature[2]

2. Modelling of the Horizontal and Vertical Coupling Wave Propagation Analysis

2.1 Basic theory

A shearing rod supported by horizontal and vertical springs is considered as is shown in Fig.1.



m (=ρA) : mass of structure [kg/m]

k_{H1}, k_{H2} : horizontal springs [N/m^2]

k_{V1}, k_{V2} : vertical springs [N/m^2]

Fig. 1－A shearing rod model supported by horizontal and vertical springs

Equation of motion for the horizontal and vertical coupling wave propagation is described as follows.

$$\begin{bmatrix} m & \\ & m \end{bmatrix}\frac{\partial^2}{\partial t^2}\begin{Bmatrix} u_x \\ u_z \end{Bmatrix} - \begin{bmatrix} GA_x & \\ & GA_z \end{bmatrix}\frac{\partial^2}{\partial y^2}\begin{Bmatrix} u_x \\ u_z \end{Bmatrix} + \begin{bmatrix} \widetilde{k_H} & \widetilde{k_{HV}} \\ \widetilde{k_{HV}} & \widetilde{k_V} \end{bmatrix}\begin{Bmatrix} u_x \\ u_z \end{Bmatrix} = \begin{Bmatrix} f_x \\ f_z \end{Bmatrix} \tag{1}$$

where,

$$\alpha = \frac{k_{V1}-k_{V2}}{k_{V1}+k_{V2}},\ e_x = \frac{B}{i},\ e_z = \frac{H}{i}\ (i: rotation\ radius)$$

$$\widetilde{k_H} = k_H - (k_H e_z{}^2 + k_V e_x{}^2)^{-1} k_H{}^2 e_z{}^2$$
$$\widetilde{k_{HV}} = \widetilde{k_{VH}} = -(k_H e_z{}^2 + k_V e_x{}^2)^{-1}\alpha k_H k_V e_x e_z$$
$$\widetilde{k_V} = k_V - (k_H e_z{}^2 + k_V e_x{}^2)^{-1}\alpha k_V{}^2 e_x{}^2$$

As eigenvalue problem of Eq. (1), characteristic wave solution can be obtained in the following form.

$$\lambda^2 = \frac{1}{2}\left\{\left[\frac{(\widetilde{\omega_h}^2-\omega^2)}{\nabla_x{}^2} + \frac{(\widetilde{\omega_v}^2-\omega^2)}{\nabla_z{}^2}\right] \pm \sqrt{\left[\frac{(\widetilde{\omega_h}^2-\omega^2)}{\nabla_x{}^2} - \frac{(\widetilde{\omega_v}^2-\omega^2)}{\nabla_z{}^2}\right]^2 + \frac{4\widetilde{\omega_{hv}}^2\cdot\widetilde{\omega_{vh}}^2}{\nabla_x{}^2\nabla_z{}^2}}\right\} \tag{2}$$

where, $\widetilde{\omega_h} = \sqrt{\frac{\widetilde{k_H}}{\rho A}}$, $\widetilde{\omega_v} = \sqrt{\frac{\widetilde{k_V}}{\rho A}}$, $\widetilde{\omega_{hv}} = \sqrt{\frac{\widetilde{k_{HV}}}{\rho A}}$, $\widetilde{\omega_{vh}} = \sqrt{\frac{\widetilde{k_{VH}}}{\rho A}}$

From the eigenvalues in equation (2), the characteristic wave velocity in the y-direction Γ is obtained from the relation $\lambda = -i\omega/\Gamma$. The relation of wave propagation characteristics and eigenvalue is as follows.

$$\text{Propagation characteristics}\begin{cases} \text{propagation:} & \Gamma\ (\text{real}),\ \lambda^2<0 \\ \text{non-propagation:} & \Gamma\ (\text{complex}),\ \lambda^2>0 \end{cases} \tag{3}$$

The necessary and sufficient condition for the propagation condition $\lambda^2 < 0$ is expressed as follows from Eq. (2).

$$\omega^2 \geq \omega_c{}^2 \tag{4}$$

Where, ω_c denotes the cut-off circular frequency described as

$$\omega_c{}^2 = \frac{1}{2}\left\{ (\widetilde{\omega_h}^2 + \widetilde{\omega_v}^2) \pm \sqrt{(\widetilde{\omega_h}^2 - \widetilde{\omega_v}^2)^2 + 4\widetilde{\omega_{hv}}^4} \right\} \tag{5}$$

Considering $\alpha < 0$ and $4\widetilde{\omega_{hv}}^4 \approx 0$, the cut-off circular frequency (Eq. (5)) is approximately expressed as follows.

$$\omega_{c1} \approx \widetilde{\omega_h} \quad , \quad \omega_{c2} \approx \widetilde{\omega_v} \tag{6}$$

It is noted that the cut-off frequency, which becomes important boundary conditions to induce waving phenomena in the building, is determined from the mass of the rod (building weight) and stiffness of piles.

2.2 Analytical model by FEM

The relation of the discretized nodes of the continuous shearing rod for the horizontal and vertical coupling wave propagation analysis is shown in Fig.2. FEM model is constructed as shown in Fig.3 by discretizing Eq. (1). The superstructure's shearing resistance between nodes is considered as element stiffness, and the pile foundation is modelled as horizontal and vertical soil springs.

The modelled superstructure has 103m in the longitudinal direction comprising 32 elements, and 9m in the transverse direction comprising 1 element. The horizontal shear wave velocity (Vx) is determined as 71m/s, and the vertical one is 28.9m/s based on the shearing stiffness and weight of the building[4],[5]. Regarding soil springs, horizontal and vertical spring stiffness is described in Section 3 for the horizontal and vertical independent model and in Section 4 for the he horizontal and vertical coupling model.

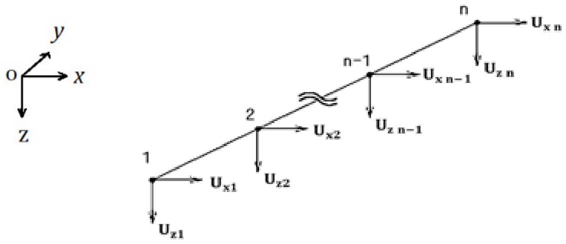


Fig. 2 — Relation between discritized nodes of shearing rod

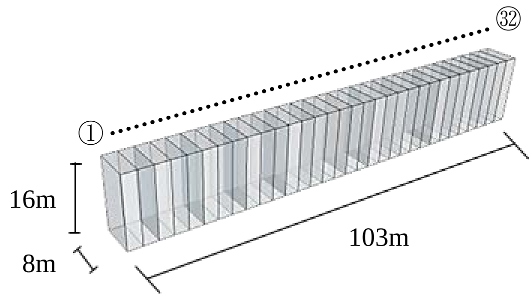


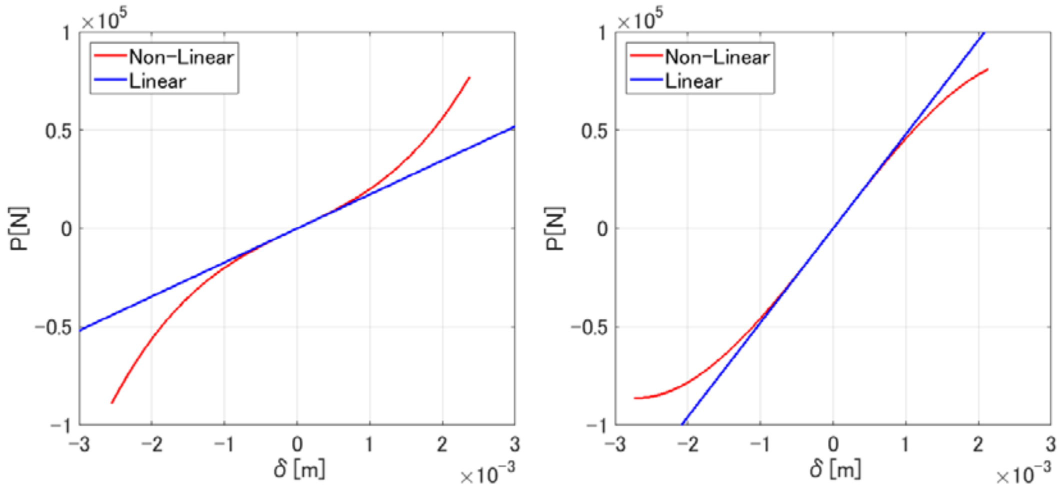
Fig. 3 — Imaged figure of FEM model of the holizontally long building

3. Analysis of Vertical Wave Propagation using Shearing Rod Supported by Non-linear Springs

3.1 Input motion and non-linear modelling of pile stiffness

Ricker wavelet with center frequency of 4Hz is used as input motion with phase lag. The phase velocity moving from Node 1 to Node 33 is assumed to be 50 m/s.

In this study, two non-linear spring models are considered for pile stiffness. One is hardening type non-linear elastic spring of which initial stiffness corresponding to cut-off frequency of 3Hz. The other is softening type non-linear elastic spring of which initial stiffness corresponding to cut-off frequency of 5Hz. The analyses for the two non-linear models are compared with the corresponding initial stiffness linear models. Therefore, vertical waving propagation characteristics using 4 models are investigated. Fig.4 shows force-displacement relation of supporting spring models for pile stiffness.



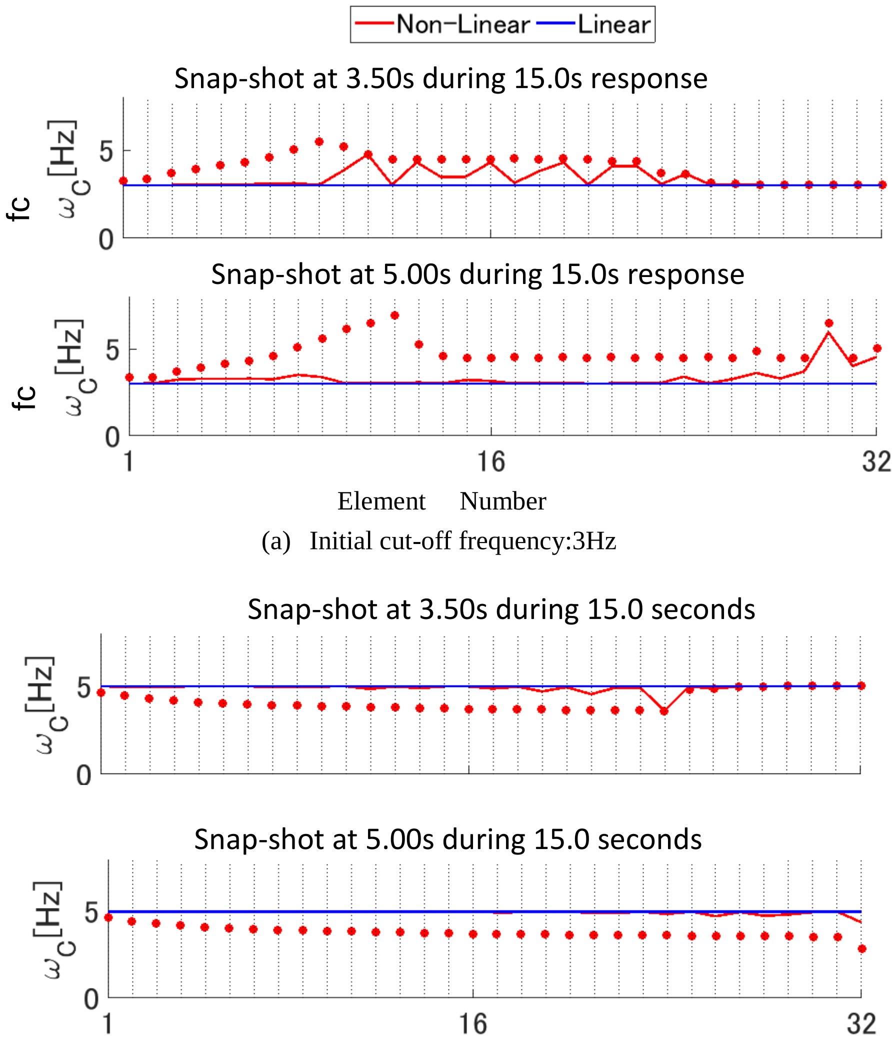
(a) Cut-off frequency:3Hz (b) Cut-off frequency:5Hz

Fig. 4 — Force-displacement relation of supporting spring models for pile stiffness

3.2 Analytical results and discussions

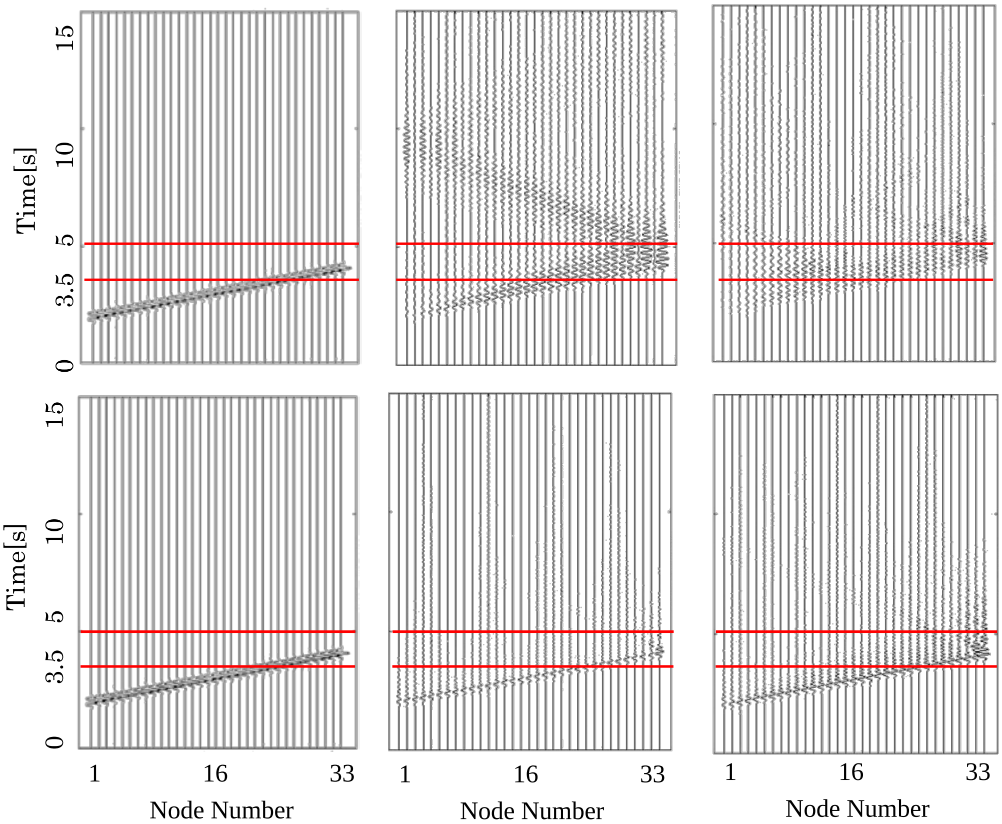
Fig.5 shows cut-off frequencies corresponding to maximum displacement at each element for each nonlinear model compared to the initial linear model. Fig.6 shows paste up curves for each linear and non-linear model together with input wave. In this figure, the snap-shot time is shown by red line. Fig.7 shows the maximum displacement for each model.

It is found from Fig.5 that cut-off frequency is different depending on displacement at each node. In the case of hardening non-linearity, the maximum stiffness changes by 5.33 times compared to initial stiffness. In the case of softening non-linearity, the minimum stiffness becomes 0.32 times. It is noted that the cut-off frequency is the same as initial one where incident wave does not arrive yet in the larger node number area at the snap-shot at the time of 3.50s as is recognized from Fig.6 (a).



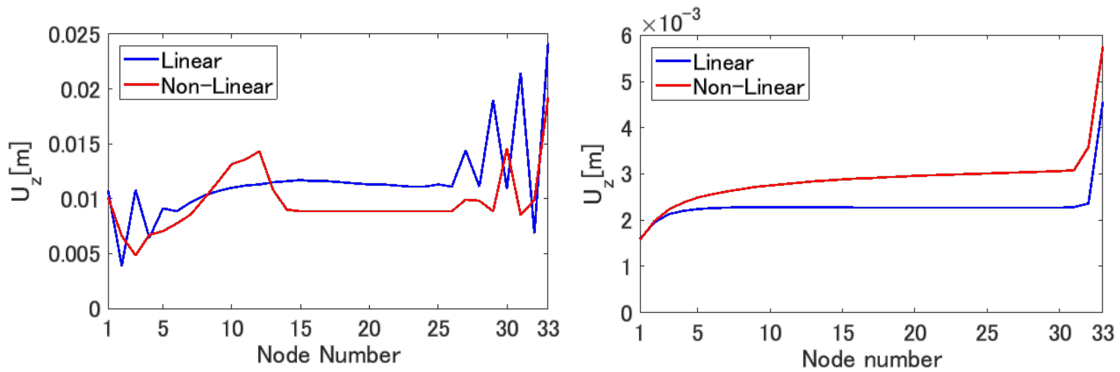
(b) Initial cut-off frequency:5Hz

Fig.5 ― Cut-off frequencies corresponding to maximum displacement at each element for each nonlinear model



(a) Input wave (b) Linear response (c) Non-linear response

Fig.6 — Paste up curves for each model
(Upper: Initial cut-off frequency is 3Hz, Lower: Initial cut-off frequency is 5Hz)



(a) Initial cut-off frequency:3Hz (b) Initial cut-off frequency:5Hz

Fig.7 — Maximum displacement for each model

It is confirmed from Fig.6 that quantity of vibration energy transmission of hardening model is decreased compared to the linear model corresponding to the initial stiffness, but that of softening model is increased compared to the corresponding linear model.

It is also found from Fig.7 that displacement response values of the hardening model decrease generally and the peak value appeared at Node 13, and that those of the softening model increase generally while the maximum response is appeared at the end Node 33, which is the opposite side of incident wave input side.

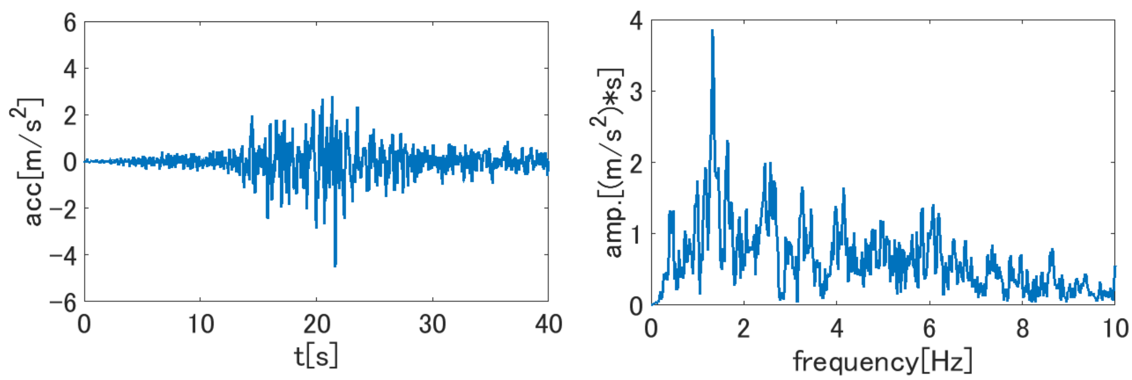
As was described in the previous research, incident wave in propagation zone is reflected in the non-propagation zone. The interference seems to cause the larger displacement values of the hardening model recognized at around Node 13. While in the case of softening model, input wave from Node 1 is reflected at the free boundary, Node 33 and then Node 1. Therefore, interference is caused at the both free sides and displacement response values become large at both ends, especially response value become the largest at the opposite end Node 33 to input from Node 1 because of damping effect of traveling wave.

4. Wave Propagation Analysis Using Horizontal and Vertical Coupling Model with Non-linear springs

4.1 Input motion and non-linear modelling of pile stiffness

As input ground motion, horizontal ground motion at the building site is used. The ground motion is estimated by using 250mx250m mesh in Sendai city and earthquake observation network data [7]. Fig.8 shows acceleration waveform and its Fourier amplitude spectrum. The phase-lag input motions with the 4 moving velocities, 50m/s, 100m/s, 150m/s, and 200m/s, are considered.

Regarding modelling of pile stiffness as soil spring, vertical stiffness is determined so as that the cut-off frequency becomes 6 Hz considering the eccentric ratio α =0.2 (20%) in equation (1). And horizontal stiffness is determined so that the cut-off frequency becomes 2Hz for the initial stiffness of softening type nonlinear model as shown in Fig.9. Fig.10 shows characteristic waves of the model. In this figure, the number in parentheses indicates cut-off frequency. It is confirmed from this figure that the horizontal phase velocity curve shows notching at the crossing point with the vertical phase velocity curve due to effect of horizontal and vertical coupling.



(a) Acceleration waveform of input motion (b) Fourier amplitude spectrum

Fig.8－Waveform of input motion and its Fourier amplitude spectrum

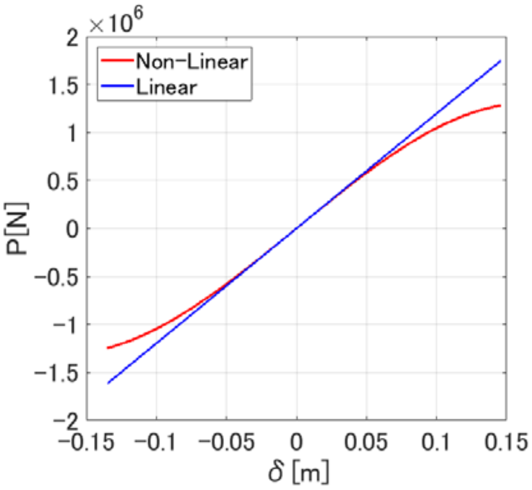


Fig.9－Force-displacement relation of supporting spring model motion

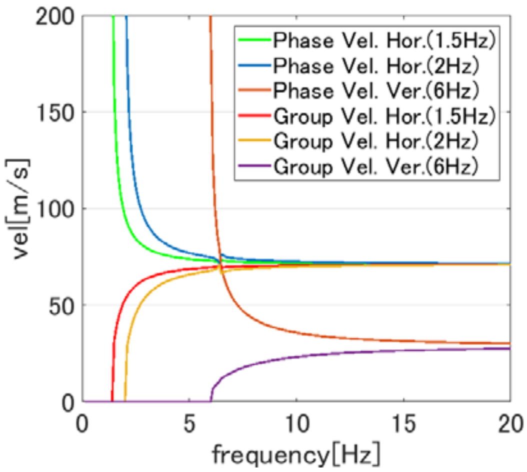


Fig.10－Characteristic velocity curves of the coupling system

4.2 Analytical results and discussions

In Fig.11, the vertical displacement response at Node 33 of each model is shown, including the linear and the softening type-nonlinear cases of supporting springs, due to the input motion with moving velocity of 200m/s. It is found from this figure that the second wave packet appears at around 27s to 34s in the linear model, but 25s to 30s in the in the non-linear model, which means earlier arrival due to higher group velocity for the dominant frequency of input wave. Similar phenomenon is recognized for the third wave packet. It is confirmed from Fig.10 that the group velocities become faster with decrease of cut-off frequencies. Considering the non-linear model as softening type in this analysis, cut-off frequency of the non-linear model becomes lower compared with that of the initial stiffness for some time periods, which leads to the larger group velocity in the non-linear model compared to the linear model. The above-described response characteristics are caused by vibration energy transmission and the difference of group velocity lead to the time difference of the wave packet response. Therefore, it is found that the vertical response characteristics are affected by the horizontal wave propagation conditions in the coupling model.

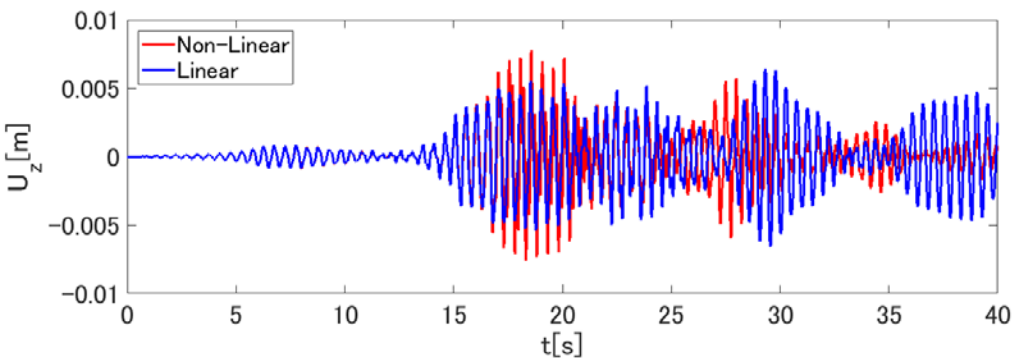


Fig. 11－Vertical displacement responses waveforms at Node 33 for the linear and the softening type-nonlinear models in the case of moving velocity 200m/s

Fig. 12 shows Fourier amplitude spectra of vertical responses at Node 33 of the non-linear model for different moving velocities, 50m/s, 100m/s, 150m/s, and 200m/s. It is found from this figure that the response value is smaller in the case of 50m/s compared to other higher velocities, and that the frequency response values in the cases except 50m/s are shifting to lower frequency range. Considering that the input motion with moving velocity and observer's situation at Node 33, the difference of spectral amplitudes due to the moving velocity seems to be Doppler effects. Considering the horizontal phase velocity is about 71m/s as is recognized from Fig.10, the case of moving velocity 50m/s is only the case of with moving velocity is smaller than the phase velocity. Moving velocities of other three cases are faster than the phase velocity. That is why only the response values in the case of 50m/s are small in relation to moving velocity. It is noted that Doppler effect is expressed in the following Eq. (7). When moving velocity is faster than 100m/s, denominator of Eq.(7) becomes larger with increasing the moving velocity and the responses move to lower frequency range, which is consistent with the analytical results. Therefore, it is confirmed that Doppler effect is caused in relation to the phase velocity of the system compared to the moving velocity, and it is found that horizontal response affects vertical response as coupling effect.

$$f = \frac{V}{|V - v|} f_0 \tag{7}$$

Where, V denotes moving velocity, and v denotes propagation velocity(phase velocity)

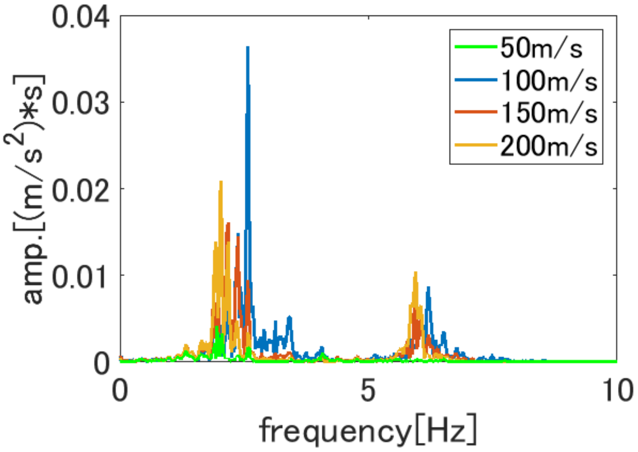


Fig. 12 －Fourier amplitude spectra of vertical responses at Node 33 of the non-linear model for different moving velocities

5. Concluding Remarks

In this paper, dynamic characteristics of horizontal and vertical coupling rod supported by nonlinear springs are analytically investigated focused on the vertical waving phenomena related to earthquake damage of a horizontally long building with pile foundation. In the analysis, difference of vibration energy transmission characteristics due to difference of non-linearity and moving velocity for input motions with phase-lag is investigated.

In the analytical cases using horizontal and vertical independent model, it is found that the vibration energy transmission of non-linear stiffness model is not simply increased or decreased compared to the linear model because of the different non-linearity level of each element. While, in the horizontal and vertical coupling model, it is recognized that wave propagation condition in the horizontal affects vertical response, and that the large/small relation of the phase velocity and moving velocity of input motion is related to the dynamic response of the building together with the absolute value difference of the two velocities.

Although dynamic response characteristics for only one directional input motion is discussed in this paper, those for the horizontal and vertical coupling input motion is expected to be investigated in the future tasks, for a more precise quantitative verification of the actual damage.

Acknowledgements

The authors thank Dr. Cao Miao, Lecturer of Tohoku Institute of Technology, and Ms. Yuka Ohka, former bachelor student of Motosaka laboratory (currently, MLIT), who contributed to the research materials in this paper. The authors also thank Doctor S. Ohno, Assoc. Prof. of IRIDeS, Tohoku University, and Mr. Ryota Tazuke, former master student (currently, Research Institute of Haseko Corporation) , for processing the research related to this paper.

References

[1] Masato Motosaka, Basic investigation on dynamic behaviour of the damaged suspended ceiling board of indoor swimming pool facility, the 25th Meeting of Japan Association of Natural Disaster Science, November, 2006.

[2] AIJ, Preliminary Reconnaissance Report of the 2011 Tohoku-Chiho Taiheiyo-Oki Earthquake, 2011.

[3] John P. Wolf, Dynamic Soil-Structure Interaction in Time Domain, (Chapter3, Sec.3.2), Prentice Hall, 1988.

[4] Yuki Ohka and Masato Motosaka, Basic Study on Earthquake Damage Accompanying Waving Phenomena Based on Horizontal-Vertical, Coupling Wave Propagation Analysis with Cut-off Frequencies, Annual meeting of Tohoku Natural Disaster Science, 2015.

[5] Miao CAO and Masato MOTOSAKA, Study on Guided Wave Propagation of Seismic Vibration, Summaries of Technical Papers of AIJ Annual Meeting, pp.39-40, 2015.

[6] Masato MOTOSAKA, Theoretical Investigation of Vertical Waving Phenomenon Focused on Earthquake Damage of a Horizontally Long Building with Pile Foundation, 2017-AIJ Annual Meeting, 2017.8

[7] Ryota Tazuke, Study on Seismic Micro-zoning of Sendai City Area Based on Earthquake Observation Data, Master Thesis of Tohoku University, 2015.

