# GENETIC ALGORITHM STRATEGY FOR EVALUATING FUNDAMENTAL PERIOD OF VERTICAL IRREGULAR FRAMES

S. Marasco[(1)], G.P. Cimellaro[(2)], R. Greco[(3)]

[(1)] *Postdoctoral research associate, Department of Structural, Geotechnical and Building Engineering, Politecnico di Torino, Italy, email: sebastiano.marasco@polito.it*

[(2)] *Associate professor, Department of Structural, Geotechnical and Building Engineering, Politecnico di Torino, Italy, email: gianpaolo.cimellaro@polito.it*

[(3)] *Associate professor, Department of Civil Engineering and Architecture, Technical University of Bari, Italy email : r.greco@poliba.it*

## Abstract

The assessment of the natural period of vibration of a Reinforced Concrete (RC) framed structure is a key issue in earthquake design. The main seismic design codes proposed simplified formulations to evaluate the fundamental period of regular structures based on the total height. Indeed, the natural period of vibration depends on several parameters directly connected to the mass and stiffness of the structure and on the irregularities on the frame system. This paper proposes an accurate mathematical formulation to evaluate the fundamental period of vibration of RC frames which have various vertical geometry irregularities and for different mechanical and geometrical design parameters. Different RC Moment Resisting Frame (MRF) configurations are analyzed to study the effect of wide range of building's parameters and the presence of infilled walls and weaker first story in the fundamental period estimation. Furthermore, vertical irregularities are accounted through several randomly generated configurations. Evolutionary Polynomial Regression (EPR) technique is employed to find the best fitted polynomial expressions of natural period of vibration from the huge search space identified by the numerical results of the computational procedure. This technique merges the evolutionary approach which is inspired by Darwin's theory of evolution with a multiple robust regression technique. A Matlab computer program is developed to search a symbolic form of fundamental period of vibration by assuming a wide set of mechanical and geometrical variables. Furthermore, the computer program is capable of identifying the absolute importance of each selected structure's parameters and providing the optimal solution of polynomial model structures through a Single-Objective Genetic Algorithm (SOGA). The entire computational flow is performed through a multiprocessing process run on a Rack Server consisting in two processors Intel Xeon E5-2698 v4 with a number of 20 cores and 40 threads each.

*Keywords: Fundamental period; vertical irregularity; Evolutionary Polynomial Regression; Sensitivity analysis*

## 1. Introduction

The evaluation of the fundamental period of vibration of a Reinforced Concrete (RC) framed buildings is an essential requirement in earthquake design and assessment. Current seismic design codes propose simplified mathematical expression to assess the natural period of vibration based on total height of the frame. These equations have been obtained derived from regression analysis on statistical dataset composed by the period of vibration effectively measured during past earthquake. The most of the empirical formulations assume the form of Eq. (1).

$$T_1 = \alpha \cdot H^{\beta} \tag{1}$$

where α is depending on the structural system, while is specified by the seismic code. Firstly, ATC3-06 [1] proposed this semi-empirical formulae with $\beta$ equal to 0.75, and $\alpha$ was calibrated as 0.06 for $H$ expressed in meters and RC Moment Resisting Frames (MRFs). Analogously, European seismic design regulation [2] adopted a value of 0.075 for $\alpha$ and 0.75 for $\beta$ with H expressed in meters and RC-MRFs. Goel and Chopra [3] demonstrated that the semi-empirical code formulae tended to provide shorter periods than measured ones. Therefore, they proposed another empirical formulation capable of improving the correlation with the measured data collected from eight Californian earthquakes, starting with the 1971 San Fernando earthquake and ending with the 1994 Northridge event. The same mathematical expression of Eq. (1) was adopted by Goel and Chopra [3], while providing an upper and lower limits of estimated period. [4] guidelines recommended an alternative formulae for both RC and steel MRF buildings obtained by multiplying the number of stories by 0.1. This mathematical expression can be used for buildings with maximum number of stories equal to 12. Although design code formulae provide a simplified approach, it is still an important issue to enhance its accuracy while taking into account further parameters. In fact, the natural period of a MRF is dependent on the mass, strength and stiffness of the structure. Many factors such as structural regularity (in plan and elevation), number of story and bays, section properties, presence of infilled panel or other constructive elements, need to be considered. Verderame, Iervolino [5] assessed the longitudinal and transversal elastic period of two groups of existing RC-MRF buildings having similar structural configuration concluding that the only height is not sufficient to accurately describe the period variability. Therefore, the general code formulae was further modified to account for the plan dimensions of buildings as given in Eq. (2).

$$T_1 = \alpha \cdot H^{\beta} \cdot S^{\gamma} \tag{2}$$

where $S$ is the footprint area of RC-MRF building. For the two building groups, the mass and stiffness matrix were evaluated and the associated periods of vibration were calculated. Finally, the least squares regression were adopted to estimate both transversal and longitudinal elastic periods based on Eq (3). The available code, experimental and numerical expressions, tend to return different results of periods of vibration for the same structural configuration. Hong and Hwang [6] monitored more than 30 buildings in Taiwan to identify the fundamental vibration periods. As results of the collected data regression analysis, an empirical formulae was proposed to estimate the fundamental period of RC-MRF buildings and to compare the influence of certain structural parameters on the period definition. Hong and Hwang [6] found that the height of a buildings is more important than building’s width and length in the prediction of fundamental period, while the monitored buildings tends to be stiffer than those in U.S. This discrepancy clearly shows the code-to-code variability in period definition. On the other hand, the discrepancy increases when comparing results obtained from code formulae with those returned by numerical analyses. Kose [7] investigated the effects of some structural parameters (i.e. building height, number of bays, shear walls area ratio, infilled panels ratio and type of frame) on the fundamental period of RC buildings. A typical building structural configuration was modeled in SAP2000 [8] and an iterative linear modal analysis was carried out. The influence of each parameter was determined through sensitivity analysis, while an Artificial Neural Network procedure was employed to assess the relationship between the period and the considered parameters for 189 different computational models.



Varadharajan, Sehgal [9] dealt with the influence of building vertical geometrical irregularities in its period estimation. A single parameter ($\lambda$) was adopted to quantify mass, stiffness and strength irregularity in terms of both magnitude and location. Moreover, different structural configuration of irregular buildings were investigated and subjected to 27 ground motions. Regression analysis was conducted to estimate the fundamental period as given in Eq. (3).

$$T_1 = \lambda \cdot 0.075 \cdot H^{0.75} \tag{3}$$

Asteris, Repapis [10] investigated on the assessment of fundamental period of vertically irregular RC frame buildings with infilled walls. Three different building groups were defined based on the type of vertical irregularity. For each group, 8, 12, 16, 20, and 24 storeys configurations were analyzed, while a predefined dataset of strength and mechanical characteristics were assumed. Infilled panels were modeled using the equivalent strut nonlinear cyclic model proposed by Crisafulli [11]. Analyses results showed that the fundamental period of irregular buildings are consistently smaller than those regular.

$$\lambda = \frac{1}{N^{0.1}} \tag{4}$$

According to Eq. (3), Asteris, Repapis [10] proposed a reduction factor for fundamental period of irregular RC frame building based on the number of storeys (Eq (4)).

The main objectives of this work are: (*i*) investigate the influence of a wide set of RC buildings parameters on the fundamental period estimation; (*ii*) develop a new comprehensive formula capable of providing an accurate estimate for any kind of building configurations and vertical irregular plane frame. With this aim, a novel computational procedures is herein proposed. First, a MATLAB [12] code is developed to iteratively perform modal analysis of given structural building configuration while automatically modify its inherent geometrical, mechanical, and strength parameters. Furthermore, a wide range of vertical irregularities, the presence of infilled walls and first weaker story are investigated. As an attempt to achieve period estimates that are not only accurate but also practical, Evolutionary Polynomial Regression (EPR) technique is employed herein. EPR merges a Genetic Algorithm (GA) paradigm for finding the best mathematical structure and the Least-Squares Method (LSM) for the identification of the multi-regression parameter. This effective combination produces a nonlinear mapping of numerical data obtained by the modal analyses with few constants, avoiding well-known over-fitting issues and improving the generalization of the final mathematical model.

The paper starts with detailed description of building modeling and definition of the input dataset. Section three tackles the core of the computational procedure that is represented by the application of the EPR technique used both as sensitivity analysis tool and to identify the optimal formula of the fundamental period of RC-MRF buildings. Results and discussions are given in the fourth section of the manuscript. This work provides an innovative computational technique to accurately estimate the fundamental period of RC-MRF buildings based on their structural configuration and main parameters.

## 2. Building dataset

The influence of building's parameters and on the fundamental period of RC-MRF plane frame is herein investigated.

### 2.1 Building configurations

Regular plane frame configurations are herein adopted to investigate the influence of certain structural parameters (Fig. **1**.a). Constant values of story height ($h$), span length ($l$) number of spans ($n_s$), and uniform distributed load ($q$) have been assumed. For all types of RC-MRF a number of five spans is assumed. Furthermore, the variation in building's natural period caused by the presence of infilled walls and weaker first story is investigated. To do that the building configuration depicted in Fig. 1.b is considered, where the first floor height ($h_I$) is greater than $h$, and the infilled panels are regularly distributed along the building height except for the first level. These kind of building configuration are commonly employed in urban environment where the first story is designated for commercial purposes while the high levels have a residential occupancy.

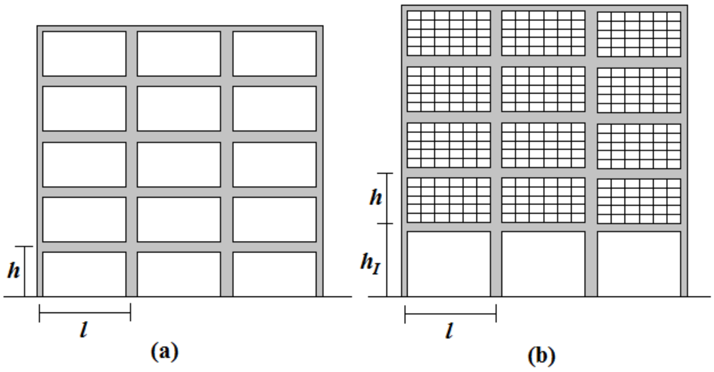



Fig. 1 – Types of building: regular (a), and with presence of infilled panels and weaker first story (b)

Beside the regular RC building frames, different vertical setback irregularities have been investigated. In this study the vertical irregularities are intended as gradual variation of setbacks along the building height. To investigate on vertical irregularities, the parameter proposed by Karavasilis, Bazeos [13] is herein adopted for quantifying the setback irregularity (Eq. (5)).

$$\varphi_I = \frac{1}{n_s - 1} \cdot \sum_{i=1}^{n_s - 1} \frac{L_i}{L_{i+1}} \tag{5}$$

where, $n_s$ is the number of story, while $L_i$ represents the width of the $i^{th}$ story (Fig. 2).

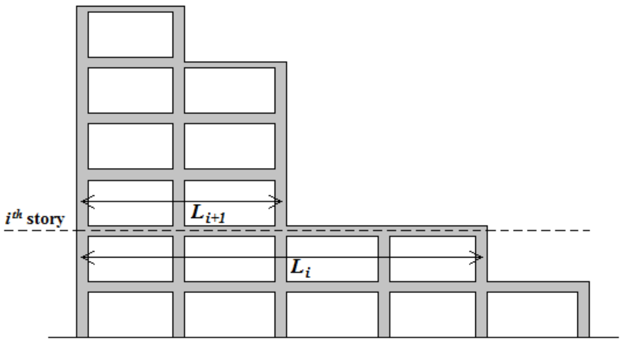



Fig. 2 – MRF geometry to define irregularity indices proposed by Karavasilis, Bazeos [13]

A wide range of randomly assigned irregularities are applied to each building's type for investigating the effect of Karavasilis, Bazeos [13] parameter on the fundamental period estimation.

## 2.2 Building design

The data quality is a key issue in data-driven technique; therefore the adopted building's information have been designed to satisfy accuracy (parameters values are representative of the observed quantity), consistency (data are in agreement with desired property of their behavior), and completeness (data contains all the desired information). The frames are designed for being representative of different year of construction periods and therefore referring to different structural design regulations. For this purpose, four different categories of design procedures have been taken into account for being representative of gravity design only buildings and structures designed to withstand horizontal actions with 0.05 g, 0.15 g , and 0.25 g design acceleration. The frames structural members have been uniformly designed along the story level and tapered in elevation for each three levels. The mechanical parameters such as characteristic compression strength of the concrete ($f_{ck}$), elastic modulus of concrete ($E_c$) have been set to 25 GPa and 31 GPa, respectively. A characteristic steel tensile strength of 450 GPa have been assumed for reinforced bars. Infilled panels are modeled using the equivalent strut model proposed by Al-Chaar [14]. The panel is represented by two parallel struts that carry only compression axial load as shown in Fig. 3.

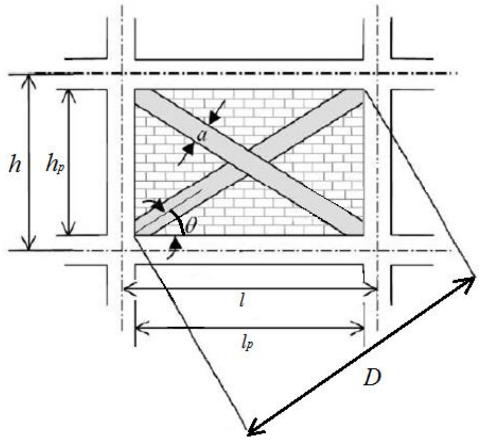



Fig. 3 – Equivalent strut model proposed by Al-Chaar [14]

The height of the panel is $h_p$, while its length is represented by $l_p$. The equivalent panel thickness ($a$) is calculated using the Mainstone [15] formula (Eq. (6)).

$$a = 0.175 \cdot D \cdot \lambda_1^{-0.4} \tag{6}$$

where $D$ is the diagonal length of the panel and the coefficient $\lambda_1$ is given by Eq. (7) [16].

$$\lambda_1 = h \cdot \sqrt[4]{\frac{E_p \cdot t_p \cdot \sin(2\theta)}{4 \cdot E_c \cdot I_c \cdot h_p}} \tag{7}$$

where $E_p$ and $t_p$ are the compression elastic modulus and thickness of the panel, respectively. $\theta$ represents the slope angle of the diagonal panel line, while $I_c$ is the moment of inertia of the columns. To take into consideration the reduction of panel strength due to the openings, Al-Chaar [14] proposed to multiply the parameter $a$ by a reduction factor $R_1$ given in Eq. (8).

$$R_1 = 0.6 \cdot \left(\frac{A_o}{A_p}\right)^2 - 1.6 \cdot \left(\frac{A_o}{A_p}\right) + 1 \tag{8}$$

where the area of openings and panels are represented by $A_o$ and $A_p$, respectively. In this study, the panel thickness has been fixed to 0.2 m.

## 2.3 Building modeling

A large number of geometrical, mechanical and material-based parameters have been taken into account to study their effects on the fundamental period estimation. In particular, the total building height ($H$), the index of joint rotation ($\rho$, Blume [17]), the maximum axial load rate ($v$), the total building-s length ($L$), the ratio between the first story height and the story height of the other level ($h_1/h$), Stafford Smith and Carter [16] parameter ($\lambda_1$), uniform distributed panel load ($q_p$), and opening area on the panel ($A_{open}$) have been considered. To reduce the number of explanatory variables, the coefficient $\beta_p$ given in Eq. (9) is introduced.



$$\beta_p = \frac{A_o}{A_p} \cdot \frac{q}{q_p} \tag{9}$$

This parameters accounts for both panel's mass and stiffness characteristics and openings. Table 1 lists the observed building's parameter and their relative range and step variability.

Table 1 – Observed building's parameters and relative range values and step variability

| **Parameter** | H [m] | $\rho$ [-] | $\upsilon$ [-] | L [m] | $h_I/h$ [-] | $\beta_p$ [-] | $\varphi_I$ [-] |
|---|---|---|---|---|---|---|---|
| **Variability** | 6 - 30 | 0.10 - 3.00 | 0.07 - 0.60 | 15-30 | 1 - 3 | 0.25 – 4.5 | 1.05 – 3.00 |
| **Step variability** | 3 | f(design) | f($A_c$, q) | f(design) | f($h_I$) | f(q, $A_0/A_p$) | f(irr.) |

Some steps variability depends on single characteristic, such as span length, cross section area of base columns ($A_c$), distributed load, first story height, elastic modulus of the panel, volumetric weight of the panel ($\gamma_p$), and irregularity setback. Table 2 lists the maximum and minimum values and the step variability associated with each building's characteristic.

Table 2 – Maximum and minimum values of building's characteristic and their step variability

| **Parameter** | l [m] | $A_c$ [m$^2$] | q [kN/m] | $h_I$ [m] | $E_p$ [GPa] | $\gamma_p$ [kN/m$^3$] | $A_o/A_p$ [-] |
|---|---|---|---|---|---|---|---|
| **Min value** | 3.50 | 0.06 | 15.00 | 3.00 | 0.5 | 15 | 0.2 |
| **Max value** | 6.50 | 0.30 | 60.00 | 6.00 | 4.50 | 25 | 0.8 |
| **Step variability** | 1 | f(design) | 15.00 | 1 | 2.00 | 5 | 0.2 |

The step variability of the joint rotation index is dependent on the procedure used to design the frame elements. A MATLAB [12] code is developed to iteratively perform modal analysis of given structural building configuration while automatically modify the selected parameters within the ranges listed in Table 1. Numerical analyses are accelerated implementing multiprocessing analysis.

## 3. EPR-based procedure

The huge amount of output data generated from the iterative modal analyses need for techniques capable of extracting useful information and knowledge. In this study, EPR are employed to search among the possible space of mathematical models. To cope with the main objective of this manuscript, the search space consists in the possible mathematical expressions aimed at evaluating the fundamental period of RC-MRF buildings. The analyzed buildings parameters identify the input variables of the expressions, while the output of the numerical analyses represent the known observations. A polynomial symbolic structure is adopted, where a set of exponents are assigned at each input variable. These exponents are selected from a user-defined set of candidate values and modified and combined through Genetic Algorithm (GA) strategy. The polynomial terms are then multiplied each other defining the so called transformed variable. At this stage, multiple regression is performed to find the best fit between the observations and the transformed input variables. The mathematical expression is then found and it is adopted to provide an estimate of the fundamental period $T$ (Eq (10)).



$$T = a_0 + \sum_{j=1}^{m} a_j \cdot X_1^{ES(j,1)} \cdot \quad \cdots X_k^{ES(j,k)} \tag{10}$$

where $m$ is the number of model coefficient ($a_0, \ldots, a_m$) to be estimated throug multiple regression; $k$ is the number of input variables ($X_1, \ldots, X_k$), while $ES(j,z)$ (with $z = 1, \ldots, k$) is the exponent of the $z^{th}$ input within the $j^{th}$ term. The last step of the procedure consists in checking the goodness of the mathematical model through a specific objective function based on the maximization of the model accuracy or minimization of the model's complexity (Fiore et al., 2014). EPR combines GAs to search the optimal polynomial form of the expressions and regression techniques (e.g. Ordinary Least Square Method, OLSM) to estimate coefficient of the mathematical expression.

### 3.1 GA technique

GA were introduced by Holland [18] as model that uses selection and recombination operators to generate new sample points in a search space. GA begins with a population of chromosome randomly assigned that represent the candidate solutions of the problem. Initialization consist in multiple assignment of input variable's exponents creating a generation of candidate functional forms (individuals). In each generation, the "goodness" of every individual needs to be evaluated through a fitness function that gives a measure of how close a given individual is to the target solution. Based on the fitness score, the best fitted individuals are selected to breed a new generation. The selected individuals are considered as parent solutions for a new generation of offspring solutions. The key idea is to simulate the mixing of genetic material that can occur when organisms reproduce. The reproduction of the parent's individuals is performed through a combination of genetic operators called crossover and mutation. After reproduction phase, the new generation is replaced to the previous one. Therefore, the new created set of individuals will represent the next parent generation and the aforementioned steps are repeated. The algorithm terminates when either a maximum number of generations has been produced, or a satisfactory fitness level has been reached for the population.

### 3.2 Multiple regression technique

Observations may have large residuals (outliers) that do not match the general trend of the rest of the data. In other cases, some observations may have extreme values of the independent variable that are named as leverage points. In these cases, robust regression approach is required to estimate the parameters of the regression model. The idea of robust regression is to weigh the observations differently based on how well behaved these observations are (Eq. (11)). Therefore, the size of the weight indicates the precision of the information contained in the associated observation.

$$\sum_{i=1}^{n} w_i \cdot \left[ T_i - \left( X^{ES} \right)_i \cdot a \right] \cdot \left( X^{ES} \right)_i = 0 \tag{11}$$

where $T_i$ refers to the $i^{th}$ observed fundamental period, while $w_i$ is the weight associated with the $i^{th}$ observation and it is inversely proportional to the standard error of the observation. Being capable of measuring the precision of the observation, an iterative procedure is required. At each iteration, a set of weigth are selected and a least square is performed to estimate the regression parameters. The goodness of the results are mesured and if it does not match the fixed requirement, the process continues until the values of the coefficient estimates converge within a specified tolerance. The approach herein used is the M-estimation method [19] which aims to minimize an objective function dependent on the residuals.

### 3.3 Optimal solution

The goodness of each symbolic expression is measured through a Single Objective (SO) function or by employing different objective (Multi Objective, MO) functions. Generally, the objective functions are based on some criteria such as maximization of the model accuracy and minimization of the model complexity [20]. A SO-based approach is adopted in this study, where global accuracy of a symbolic expression is determined through the squared R ($R^2$). The best fitted model structures are then selected and used for recombination through the crossover and mutation genetic operators. The optimal symbolic expression of the fundamental period is obtained when a satisfactory fitness level is reached that corresponds to the maximum achievable $R^2$. Fig. 4 resumes the whole workflow describing the SO EPR–based procedure adopted in this study.

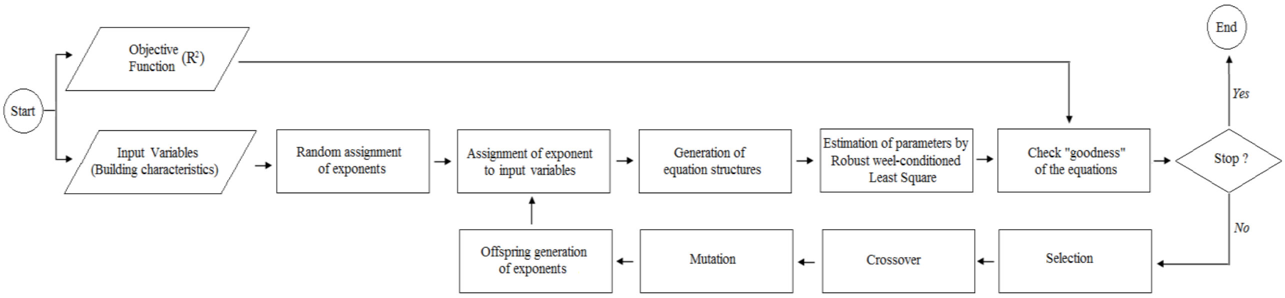



Fig. 4 – Workflow of the adopted SO EPR–based procedure

## 3.4 Analysis settings

The EPR-based procedure has been conducted for the three selected building's types in order to examine the influence of the observed parameters (Table 1) on the fundamental period estimation. A MATLAB [12] code is implemented to perform sensitivity analysis, while the entire EPR approach is based on the regression parameters and genetic operators values listed inTable 3 and Table 4, respectively.

Table 3 – Setting of the regression parameters

| **Regression Parameters** | | **Exponents** | | |
|---|---|---|---|---|
| Min | Max | Min | Max | Step |
| 2 | 6 | -1 | 1 | 1/4 |

Table 4 – Setting of the genetic operators

| **Population size (P) [-]** | **Selection Rate (SR) [%]** | **Crossover Rate (CR) [%]** | **Mutation Rate (MR) [%]** |
|---|---|---|---|
| 500 | 30 | 40 | 10 |

The number of parameters used in the regression model may be selected based on the principle of parsimony stating that for a set of equivalent mathematical models the simplest one must be chosen to explain a set of data [21]. Beside the concept of simplicity, the model needs to show a certain quality in fitting the observed data. A trade-off between model complexity and accuracy needs to be measured. In this study, the maximum number of regression parameters has been fixed to 6 in order to examine a wide range of mathematical model and, at the same time limit the complexity of the model itself. Instead, the selection of the exponent upper and lower bounds have been set accordingly to the previous studies, where the main parameters affecting the fundamental period do not exceed the unit value (i.e. Crowley and Pinho [22], Goel and Chopra [3], Verderame, Iervolino [5], Asteris, Repapis [10]).

## 3.5 Sensitivity analysis

EPR technique is herein adopted to identify how the fundamental period varies in response to variations of a certain building parameter and then to explore its optimal mathematical structure. A importance measure of each symbolic model can be provided by the Akaike weight [23] which can be interpreted as the probability that a given model is the best among the all possible models. To deal with the relative importance of the input variables, the Akaike weight are determined in each iteration (Eq (12)).

$$w_{A,l} = \frac{\exp\left(-0.5 \cdot \Delta_{AIC,l}\right)}{\sum_{p=1}^{N_m} \exp\left(-0.5 \cdot \Delta_{AIC,p}\right)} \tag{12}$$

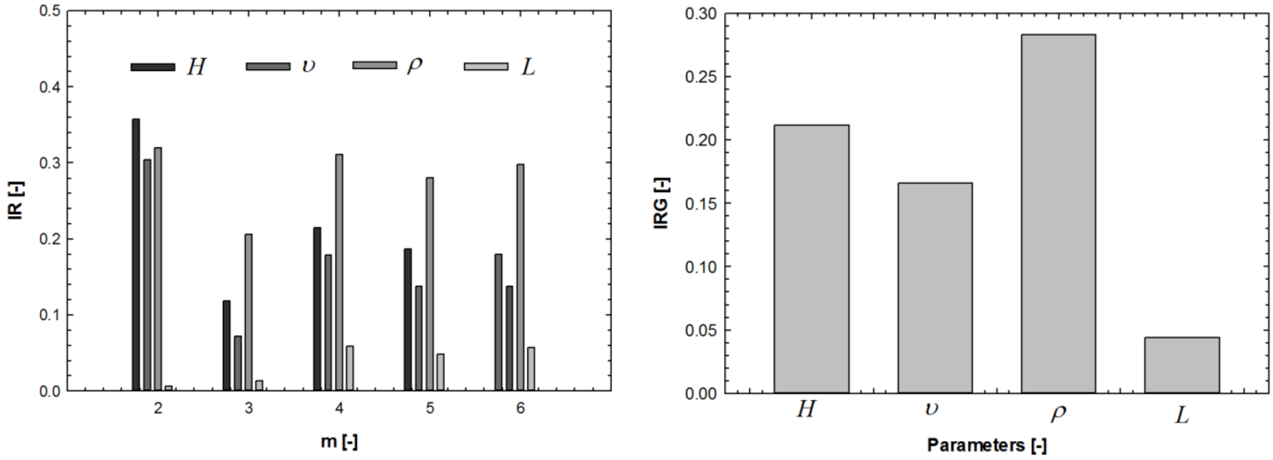
where $\Delta_{AIC,l}$ is the difference between the Akaike Information Criterion (AIC) value [24] and the AIC of the optimal solution for the $l^{th}$ model. The weight are then normalized with respect to the weight of the model which considers the complete input dataset ($w_{A,complete}$). The relative importance of the $l^{th}$ input variable ($IR(X_l)$) is given in Eq. (13).

$$IR\left(X_l\right) = \left(\frac{w_{A,l}}{w_{A,complete}}\right)^{-1} - 1 \tag{13}$$

The input variable may be classified as irrelevant if its relative importance factor is close to zero. The relative importance factor is defined for each model that consists in a given number of regression parameters. Therefore, a global indicator of relative importance ($IR_G$) of an input variable may be calculated as average of those obtained for each regression model.

## 4. Results and discussions

Sensitivity analysis has been conducted for regular building configurations aimed at measuring the relative importance of the observed parameters (Table 1) on the fundamental period estimation. Fig. 5 depicts both relative importance and global indicator of relative importance of the explanatory variables.




(a) (b)

Fig. 5 – Relative importance (a) and global indicator of relative importance (b) for Type 1building

The total building's length is found to have a relative importance lower than the others parameters. Global indicators of relative importance of 0.042 is obtained which represents the 19.85 % of the relative average indicator. Total heigth of the building assumes a global relative importance of 0.21, while its relative importance reaches the value of 0.36 for the two parameters model. An almost uniform relative importance of 0.28 is found for the joint rotation index, while an IRG of 0.17 is assessed for the axial load rate of base columns. Therefore, sensitivity analyses have revelaed the unrelevance of *L* parameter while confirming the importance of *H*, *ν,* and *ρ* parameters. Therefore these last three parameters are considered as explanatory variables to identify the mathematical model of buildin's fundamental period. EPR-procedure is then applied to find the optimal mathematical form for estimating the fundamental period based on the settings listed in Table 3 and Table 4.

The simplest model is described by two regressions parameters. For this model, a maximum $R^2$ coefficient of 0.885 is identified. The optimal model has been chosen through a trade-off between model complexity and accuracy which is represented by the Parsimony Coefficient (PC) as given in Eq. (14).

$$PC_j = \frac{R^2}{m^\alpha} \tag{14}$$

where $\alpha$ is fixed to 0.5, while $R^2$ is the maximum accuracy obtained for the $j^{th}$ model. The greater PC value is assumed by the model with two parameters, therefore it has been selected has the optimal model. The mathematical expression of the fundamental period of a RC-MRF building is given in Eq. 15.

$$T_1 = 0.147 + 0.109 \cdot \sqrt[4]{H^3 \cdot \rho \cdot \nu} \tag{15}$$

It is possible to observe how the proposed formulation is consistent with those adopted in all the current seismic standards. In fact, the total building heigth has the traditional exponent of 3/4. Instead, the axial compression rate of base column and the joint rotation index have exponent of 1/4. These parameters are representative on beam-column joint capacity and on the mass of the frame which can be useful to describe a wide range of RC-MRF designed according to different seismic codes.

EPR-procedure is further performed to find the fundamental period variation ($\Delta T_1$) caused by the presence of infilled panels and weaker first story. This variability has been intended as the ratio between the period of *Type 2* and *Type 1*. Eq. (16) gives the optimal mathematical model found for the fundamental period reduction.

$$\Delta T_1 = 0.593 \cdot \sqrt[4]{\frac{\beta_p \cdot (h_I / h)}{\lambda_1^2}} - 0.015 \tag{16}$$

Period variation increases with the ratio $h_I/h$ which leads to a more flexible behavior of the frame. Furthermore, $\lambda_1$ coefficient causes a reduction of the fundamental period due to the presence of stiffer infilled panels. Therefore, the fundamental period of a RC frame with infilled panels can be assessed by multiplying the period reported in Eq. (16) by the reduction coefiicient $\Delta T_1$.

Finally, the effect of vertical irregularities in the first period of vibration estimation is investigated. In this study the vertical irregularities are intended as gradual variation of setbacks along the building height. The parameter $\phi_I$ proposed by Karavasilis, Bazeos [13] is adopted to quantify the vertical irregularities. A huge set of irregular building configurations are explored through a random process aimed at progressively removing a certain number of spans along the building height. The random process has been implemented in Matlab, where the number of irregular configurations increases with the building's story numbers through a quadratic polynomial function. Furthermore, the random-based process attempts to analyze a broader searching space allowing a better estimation of the fundamental period estimation due to vertical irregularities. The analyzed searching space ranges from 1.05 to 3.00 of $\phi_I$ values. EPR-procedure is further performed to find the fundamental period variation caused by the vertical irregularities ($\Delta T_{1,irr}$). Eq. (17) gives the best mathematical model capable of describing the period variation due to the vertical irregularities.

$$\Delta T_{1,irr} = 0.437 + \frac{0.546}{\varphi_I} \tag{17}$$

By fixing a unitary value for $\phi_I$ which corresponds to a regular building configuration, the period variation is equal to 0.983. This result is strictly close with what expected for regular building, where the period variation must be equal to the unit value. Furthermore, the obtained mathematical formula reveals that whatever vertical irregularity implicates a reduction in the fundamental period with respect to a perfectly regular building configuration.

## 5. Conclusion

Although the fundamental period of vibration of a RC-MRF buildings is a key issue in earthquake design, the main seismic design codes proposed simplified formulations based on the total building's height which are feasible for regular structures. Indeed, the natural period of vibration depends on several parameters directly connected to the mass and stiffness of the structure and on the irregularities on the frame system. In this paper, a novel mathematical formulation is proposed to assess the fundamental period of vibration of RC frames based on their main principal characteristics, frame configurations, and vertical irregularity setback. An EPR technique is implemented to find the optimal polynomial expressions of natural period of vibration from the huge search of the numerical results. From the present study, new comprehensive formulations of fundamental period are obtained for different building configurations taking into account the presence of infilled walls, weaker first story, and vertical irregularities. First, sensitivity analyses have demonstrated the importance of some building's parameters in the period estimation that are currently neglected from standard provisions. Furthermore, the presence of infilled panels and soft-story have found to be important aspects for accurately predict the fundamental period of vibration. Finally, the effects of vertical irregularities have been accounted through a coefficient which tends to reduce the natural period with respect to a regular frame. It has also been found that this period reduction is purely affected by the setback irregularity accounted through the Karavasilis coefficient. The mathematical expressions proposed in this study accurately assess the first period of vibration of a wide variety of RC-MRF buildings taking into consideration the most relevant building's parameters. Beside the building height, the selection of dimensionless parameters avoids to use further conversion coefficients, leading to a more effective mathematical form. The mathematical forms presented herein can be employed in the assessment of the fundamental period of any plane frame. In cases of irregular and atypical frame configurations, the proposed formulae provide high fidelity results that cannot be achieved with the current simplified formulations contained in the seismic codes.

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## 7. Copyrights



## 8. References

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