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SELF-CONSISTENT SOURCE-SCALING RELATIONS FOR INTERFACE SUBDUCTION EVENTS

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Abstract

In recent years, several scaling relations have been developed for megathrust events (e.g. [1], [2], [3], [4], [5] and [6]) that allow the estimation of the various characteristics of the fault (e.g. rupture length, width, area, average slip) as a function of seismic moment or moment magnitude. In the majority of these studies the relationship between the two variables for each equation are estimated by linear regression independently in an effort to find the best fit that would describe the specific dataset. This very common approach results in optimum results for each individual scaling relation however, the final set of equations is not self-consistent in the sense that they do not enable moment or moment magnitude, rupture length, width, area, and slip to be estimated from each other.

While this inconsistency is usually not significant for seismic hazard analysis, where all GMPE's depend on magnitude, it becomes important when the target parameter is related to displacement, as in Probabilistic Fault Displacement Hazard Analysis (PFDHA) and Probabilistic Tsunami Hazard Analysis (PSHA). Even if the integration is carried out over magnitude, the subsequent conversion to displacement to compute the hazard, or the initial condition for the tsunami is not correct when the scaling relations are not internally consistent. Furthermore, it is also currently not possible to produce PSHA and PTHA/PFDHA source models that are in agreement with each other.

The underlying cause is the fact that in the case of scaling relations there are errors in both variables (X and Y). For example, in the moment magnitude – rupture area scaling relation, there are errors in the magnitude estimation and in the derived rupture area.

In standard linear regression, the aim is to predict the Y value from the X value – so we typically account for the error only in the Y values. The technique that allows a regression that can account for errors in both X and Y values is called Generalized Orthogonal Regression ([7], [8], [9]).

We propose to use the [5] database of source parameters for megathrust events and utilize the Generalized Orthogonal Regression technique to estimate a set of scaling relations that are self-consistent and allow moment or moment magnitude, rupture length, width, area, and slip to be estimated from each other.

This would lead not only to a correct relationship between the rupture properties (area, displacement) used in a single analysis, but also allow complete consistency between PSHA, PFDHA and PTHA models.

Keywords: megathrust event, scaling relations, self-consistent

1. Introduction

In recent years, several scaling relations have been developed for megathrust events (e.g. [1], [2], [3], [4], [5] and [6] among others) that allow the estimation of the various characteristics of the fault (e.g. rupture length, width, area, average slip) as a function of seismic moment or moment magnitude. These models play an important role in the forward simulation of ground motion and tsunami effects from this type of events.

In the majority of these studies the relationship between the two variables for each equation are estimated by linear regression independently in an effort to find the best fit that would describe the specific dataset. This very common approach results in optimum results for each individual scaling relation however, the final set of equations is not usually tested for self-consistency in the sense that they do not enable moment or moment magnitude, rupture length, width, area, and slip to be estimated from each other.

While this inconsistency is usually not significant for example in seismic hazard analysis, where all GMPE's depend on magnitude, it becomes important when the target parameter is related to displacement, as in Probabilistic Fault Displacement Hazard Analysis (PFDHA) and Probabilistic Tsunami Hazard Analysis (PSHA). Even if the integration is carried out over magnitude, the subsequent conversion to displacement to compute the hazard, or the initial condition for the tsunami is not correct when the scaling relations are not internally consistent. Furthermore, it is also currently not possible to produce PSHA and PTHA/PFDHA source models that are in agreement with each other. The underlying cause is the fact that in the case of scaling relations there are errors in both variables (X and Y). For example, in the moment magnitude – rupture area scaling relation, there are errors in the magnitude estimation and in the derived rupture area.

We propose to use the databases of source parameters for megathrust events from [5], [6], [10] and [11] and test the corresponding scaling relations for self-consistency and allow users to choose the set of scaling relations that fits best their application.

2. Dataset properties

We selected 4 published datasets that have been used to derive scaling relations in the past, namely the [5], [6], [10] and [11] datasets. The processing methods for estimating the source parameters in these datasets vary and can estimate different values even for the same events, as shown in Fig 1. and Fig. 2. We believe that these differences should be considered as epistemic uncertainty of the corresponding source parameters.

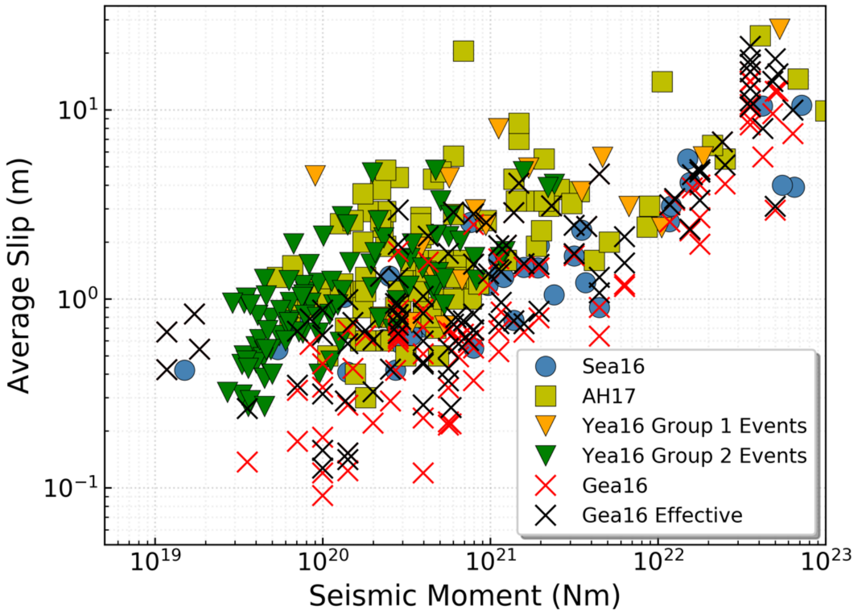

Fig. 1 – Average slip (m) as reported in the four databases that are used in the analysis plotted against seismic moment. In the figure legend: Sea16:[5], AH17:[6], Yea16:[10] and Gea16:[11]

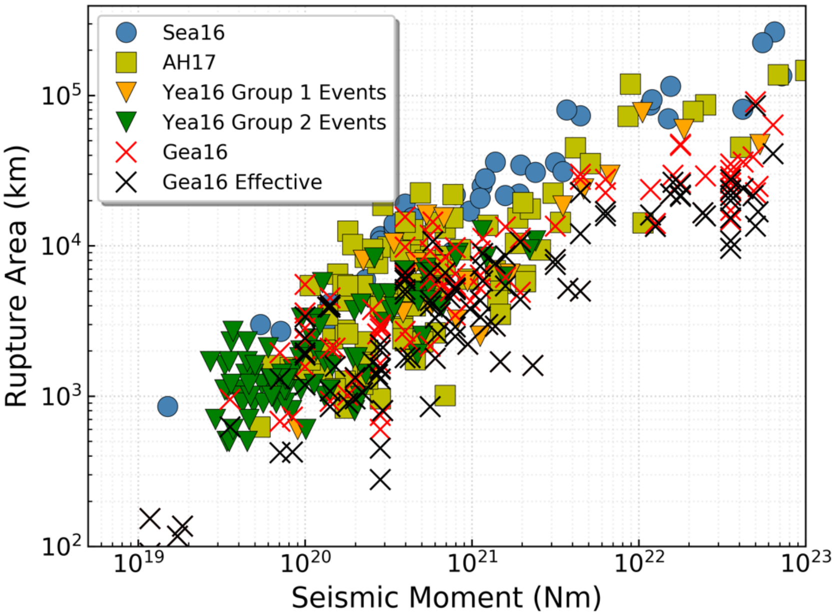

Fig. 2 – Rupture area (m) as reported in the four databases that are used in the analysis plotted against seismic moment. In the figure legend: Sea16:[5], AH17:[6], Yea16:[10] and Gea16:[11]

Specifically, in Fig 1. we plot the average slip (m) and in Fig 2. the rupture area (km^2) vs seismic moment. The distribution in both figures shows the range of differences between the different datasets and allows to identify specific trends between them. One thing to notice is that the [11] dataset shows lower rupture area values than the other datasets, both for the original and effective dimensions analysis. The average slip for [11] is also lower than the other datasets but the trend is not as clear as for the rupture area and is more prominent at small seismic moment values.

[10] split their database into 2 groups, Group 1 (18 events) has independent constraints from prior detailed rupture analyses that used similar basic rupture model formulation as in [10] and the preferred values of V_r from those studies have been adopted by [10]. Group 2 (96 events), for which independent constraints are lacking, was analyzed by [10] with a suite of models for each event with different V_r. We test separately these groups. For [11] we test the dataset with the original values of the source parameters published for each event and with the source parameters estimated through effective dimension analysis [12].

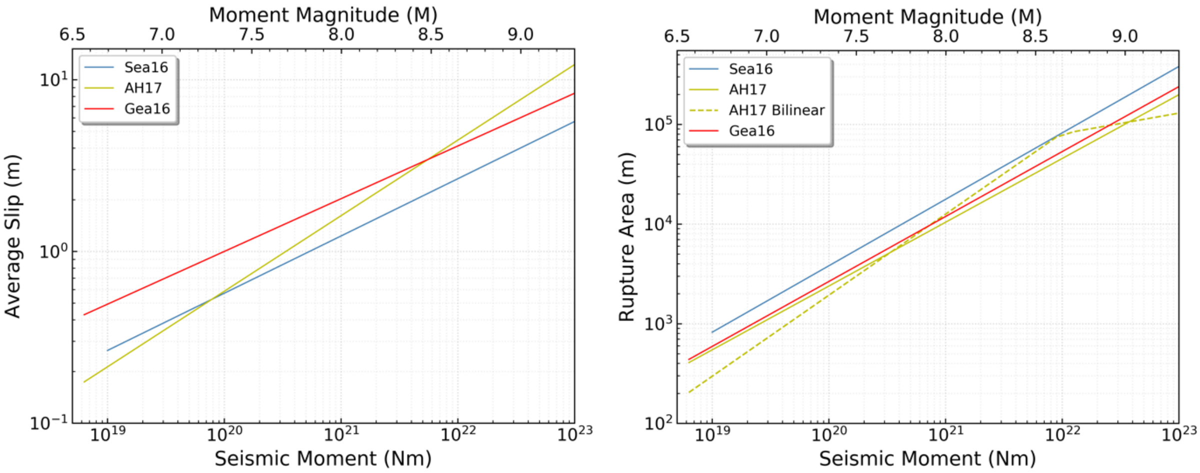

Fig. 3 – Comparison of the average slip (left) and rupture area (right) scaling relations developed by Sea16:[5], AH17:[6] and Gea16:[11]

A comparison of the scaling relations for slip and rupture area that were developed from [5], [6] and [11] datasets, is shown in Fig. 3, reflecting the epistemic uncertainty in the estimated source parameters for each study.

As a final validation for the datasets that are used in our analysis, we plotted the product of the published values of the average slip and area against seismic moment. As it can be seen in Fig. 4 all datasets show a linear trend as expected. Similar to what was observed the [11] dataset exhibits lower values than the other datasets, both for the original and effective dimensions analysis source parameters.

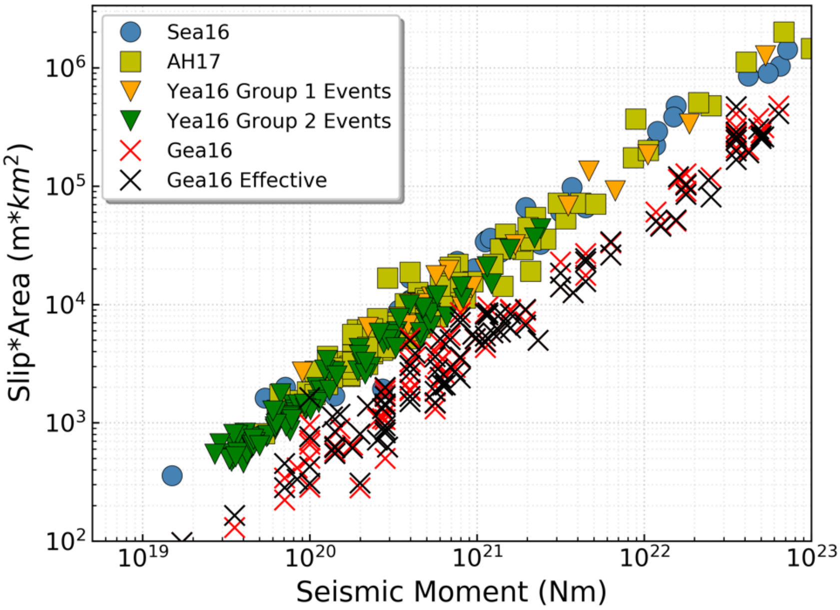

Fig. 4 – Product of the published values of the average slip x rupture area plotted against seismic moment. In the figure legend: Sea16:[5], AH17:[6], Yea16:[10] and Gea16:[11]

3. Methodology

The methodology to test the self-consistency of the three sets of scaling relations ([5], [6] and [11]) is described in the following steps:

1) For each event in a database, we use the rupture area scaling relation to estimate the seismic moment (or magnitude) from the corresponding rupture area value.
2) For the same event, we estimate the average slip (from the average slip scaling relation) using the moment (or magnitude) calculated in the previous step.
3) For each database we use at least two different sets of scaling relations to check if the self-consistency of the scaling relations depends on the specific dataset or if it is a property of the scaling relations.

Ideally, in a plot of the estimated slip in step 2 vs the reported slip, for all events of a database, all data points would follow the identity line and that would mean that the scaling relations are fully self-consistent. In Fig. 5 we plot the estimated slip vs the report slip for the [5], [6], [10] and [11] databases. Square symbols show the datapoints from the analysis using the scaling relations that was derived with the specific dataset, while the triangles the datapoints derived with an alternative set of scaling relations. For the [10] database (middle panel plots) we test scaling relations derived by [5] and [11]. For [6] we test both the linear (squares) and the bilinear models (diamonds) developed for the rupture area. The combination of database-scaling relation set used in the analysis is shown in the figure legends (ScRel field for the scaling relations). The identity line is also shown as a reference.

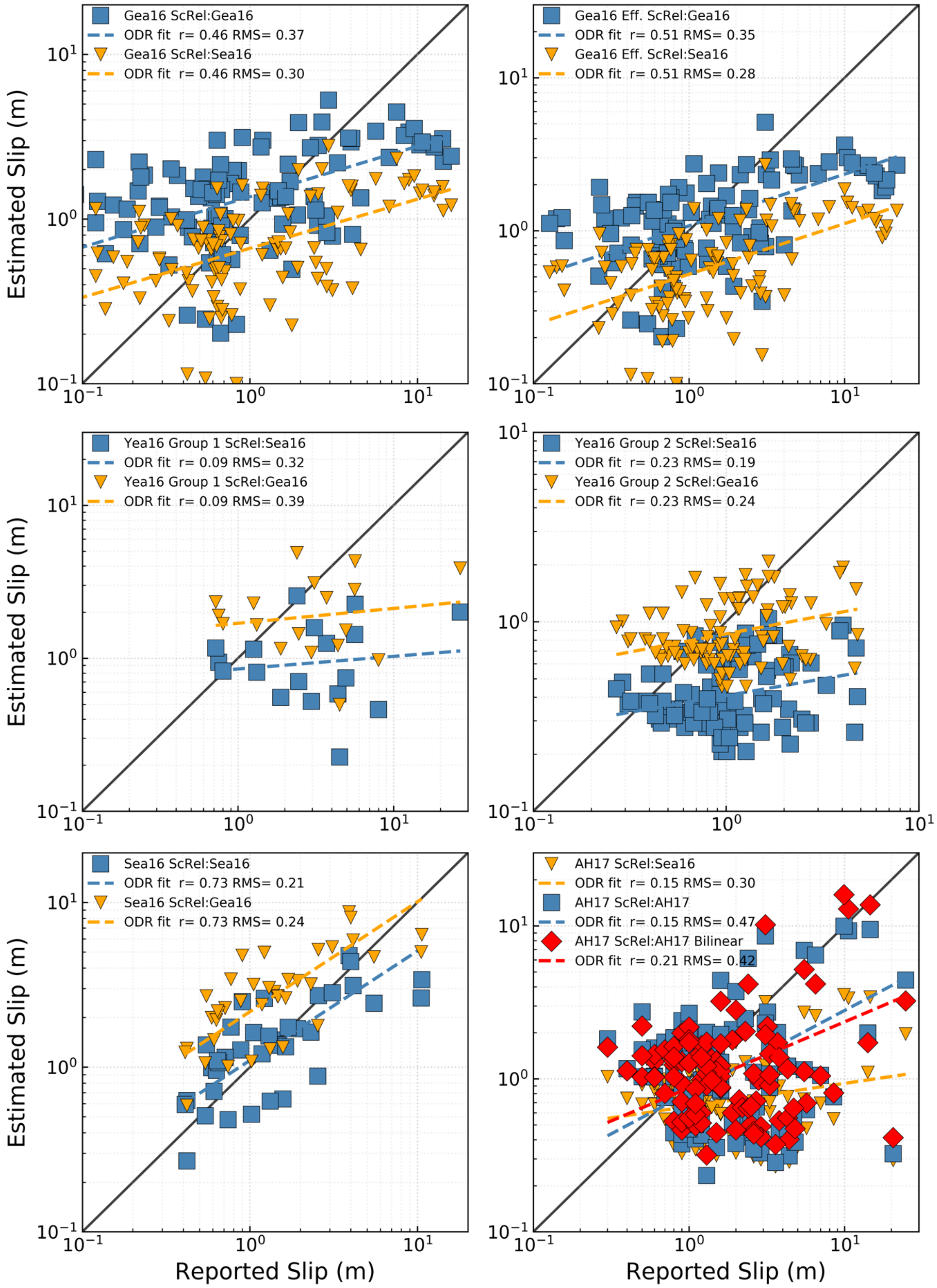

Fig. 5 – Comparison of the estimated slip vs the reported slip for the four databases studied. Square symbols show the datapoints from the analysis using the scaling relations that was derived with the specific dataset, while the triangles the datapoints derived with an alternative set of scaling relations. The least squares fits are shown with the dashed lines.

For quantifying the results we apply a regression fit to each dataset, shown in the figures with the dashed lines. Since each datapoint (reported slip, estimated slip) has errors in x and y variables we employed the orthogonal regression technique (7], [8], [9]) which accounts for errors in both variables. Standard deviations need to be assigned to the x and y variables, and these values are converted to regression weights by taking the inverse of their squares.

The standard deviation in the reported slip depends on the estimation method and theoretically is different for each event. However, we do not have this information for all databases, therefore we decided to adopt a uniform relative error of 0.4 for all reported slip values. The estimated slip has standard deviations that are tight to the standard deviations of the individual scaling relations. In the estimation of the slip we use a combination of the rupture area and average slip scaling relations and as the total error we assume the sum of the standard deviations of the two scaling relations. [6] performed an orthogonal regression to the dataset and they estimated standard deviations for both x and y. Since we use the rupture area scaling relations to estimate the magnitude, we account as standard deviation the σ_y from their Table 2. The standard deviation used in our analysis are listed in Table 1. The SciPy ODRPACK functions were used for the orthogonal regressions. The RMS and the correlation coefficient, r, for each fit are also estimated and listed in Table 2.

Table 1 – Standard deviations in the estimated slip used in the regression analysis.

Scaling relation	**Area**	**Slip**	**Total standard deviation**
Sea16	0.175	0.184	0.254
Gea16	0.2881	0.3250	0.434
AH17	0.266	0.209	0.338
AH17 Bilinear	0.267	0.209	0.339

Table 2 – RMS and correlation coefficients from the various regression fits.

Dataset	**Scaling relation**	**RMS**	**r**
Gea16	Gea16	0.37/0.35	0.46/0.51
	Sea16	0.30/0.28	0.46/0.51
Yea16	Gea16	0.39/0.24	0.09/0.23
	Sea16	0.32/0.19	0.09/0.23
Sea16	Gea16	0.24	0.73
	Sea16	0.21	0.73
AH17	AH17	0.47	0.15
	AH17 Bilinear	0.42	0.21
	Sea16	0.30	0.15

4. Discussion - Conclusions

The Gea16 scaling relations and database appear to have the smallest degree of self-consistency among all the models that were tested with the database that was used to develop them (top panels, Fig. 5). The scaling relations seem to produce better results for the intermediate slip values, however they do not exhibit enough scaling at the lowest and highest slip values of the database. For the slip dataset developed after the effective dimensions analysis, the goodness-of-fit metrics are slightly better but at larger slip values a strong weakening of the estimated slip is exhibited. The Sea16 scaling relations produce consistently lower results than the Gea16 ones and although their slopes are, in general parallel, the Sea16 relations show very small self-consistency with both Gea16 datasets (top panels, Fig. 5).

The Sea16 scaling relations show a high degree of self-consistency when tested with the Sea16 database (bottom left panel, Fig. 5). The have the smallest RMS and the highest r values in our analysis and are very close to the identity line. Although, the goodness-of-fit metrics for the Gea16 scaling relations are similar to the ones of Sea16, the Gea16 models result in consistently higher estimated values with the exception of the very large slip values were all combinations of scaling relations and database tend to underpredict.

The AH17 scaling relations exhibit moderate self-consistency when tested with the AH17 database (bottom right panel, Fig. 5). The goodness-of-fit metrics promote the bilinear model against the linear model. However, the linear model gives values that are closer to the identity line, exhibiting a higher degree of self-consistency compared to the bilinear model. Looking at the data it seems that there is a large number of events for which the estimated slip does not scale with the reported values and this is true for both the linear and bilinear models. The Sea16 scaling relations do not show self-consistency when tested with the Yea16 database.

The Gea16 and Sea16 scaling relations practically do not show any self-consistency when tested with the Yea16 database (middle panels, Fig. 5). The same trends are observed for both Group 1 and Group 2 datasets of the Yea16 database.

In the present study we tested for self-consistency three sets of scaling relations and four databases. The methodology we followed involved the use of rupture area and average slip scaling relations with the seismic moment (or magnitude) having the role of the connecting parameter between the two. The analysis showed that none of the studied scaling relations is 100% self-consistent. All scaling relations set seem to lose self-consistency at very large slip values (larger events). In order to quantify our analysis, we applied a least squares fit to the data, using an orthogonal regression technique to account for the errors in both x and y variables.

The results from the fits in general verify what can be visually observed. We do not propose a model and we also do not provide recommendations as to which model should adopted by the users. Each model has each one validity and different applications may require different scaling relations.

5. Acknowledgements

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7. References

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