TIME VARYING SPECTRAL ANALYSIS OF GROUND MOTIONS USING EMD AND VARMA MODEL

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Abstract

For ground motions, nonstationary property in intensity and frequency content is one of the most important factors that can significantly influence the dynamic response of structures caused by them. Considerable attention has been paid to the method to describe this nonstationary property recently and the time varying spectrum is thought as a suitable tool used for this purpose which can intuitively represent the energy distribution of ground motions in time and frequency plane. However, to get the time varying spectrum in an efficient way is still an open issue. In this paper, a new method for time varying spectrum analysis is proposed. First, the ground motion record is decomposed into several components called intrinsic mode functions (IMFs) by using the empirical mode decomposition (EMD) method which is a part of the Hilbert–Huang transform (HHT). Then, the IMFs are represented as a time varying vector autoregressive moving average (VARMA) model and the Kalman filter is used to estimate the time varying model parameters. Finally, the time varying spectrum is directly obtained based on the time varying parameters. In the examples presented, the time varying spectra obtained by using the proposed method and other frequently used methods are compared and discussed. Also, some suggestions for future research are provided.

Keywords: earthquake, ground motion, time varying spectrum, EMD, VARMA model

1. Introduction

It's known that besides the conventional properties of earthquake ground motions (e.g. the properties of amplitude, frequency and duration), the nonstationary properties in amplitude and frequency contents are also important factors that can significantly influence the dynamic response of structures caused by them [1]. To describe these nonstationary properties efficiently, some notions have been proposed, e.g., the zero-crossing rate is used to describe the nonstationarity in frequency content and the time varying spectrum is used to describe the nonstationarity both in amplitude and frequency contents. The time varying spectrum is thought as the most suitable one, for it can provide a proper description of energy distribution in time and frequency domains. The modeling and estimation of time varying spectrum has been an open issue for the characterization of earthquake ground motions.

Since earthquake ground motions in the real world are nonlinear and nonstationary signals in nature and their length is often limited, which makes the estimation of time varying spectrum a difficult task for us. The classical Fourier spectral analysis method and the traditional time-frequency analysis methods such as short time Fourier transform, spectrogram, evolutionary spectrum, Wigner-Ville distribution and wavelet transform, have been proved to be deficient and limited for processing nonlinear and nonstationary signals [1-2]. As a breakthrough to the existing methods, a novel method, Hilbert-Huang transform (HHT), developed by Huang et al. [3] is thought as a promising and powerful tool for processing nonlinear and nonstationary signals. This method has been used in many research fields since its introduction. In earthquake engineering, HHT has been successfully used in many applications such as time frequency analysis of earthquake ground motions, structural vibration signals, structural damage detection and health monitoring. These applications have shown that HHT can give much finer energy-time-frequency distribution properties and extract more valuable nonlinear and nonstationary features than the traditional signal processing methods [3].

HHT consists of two parts: empirical mode decomposition (EMD) and Hilbert spectral analysis (HSA). EMD is used to decompose a signal into a finite set of intrinsic mode functions (IMFs) which admit well-behaved Hilbert transforms [2-3]. HSA is designated to calculate the instantaneous frequencies and amplitude of IMFs through the Hilbert transform and obtain the time-frequency distribution of amplitude called Hilbert spectrum. For its advantages in signal processing, HHT has caught many researchers' attention. In recent years, considerable research has been reported on the applications of HHT and the algorithms of EMD [1]. However few work has been done on HSA and the limitations of HSA are often ignored. It is to be noted that there exist at least two limitations to HSA. First, to get a meaningful instantaneous frequency for a signal through Hilbert transform, the definitions of an IMF is only the necessary condition. Two additional requirements according to the Bedrosian theorem and Nuttall theorem are need to be satisfied as well [4-5]. Though Huang et al. [3] suggested the normalized Hilbert transform to circumvent the restriction of Bedrosian theorem, the difficulty associated with the Nuttall theorem was still kept unresolved. Second, the characteristic scales of IMFs are designated to increase from the smallest to the largest, so according to the uncertainty principle the smaller scale IMFs give much wider frequency spreading than the larger scale IMFs, i.e., the energy-time-frequency distribution for the smaller scale IMFs cover much wider frequency bands than for the larger scale IMFs. This may lead to larger covariance and deteriorated readability of the Hilbert spectrum at higher frequencies. Huang et al. [3] suggested the spatial smoothing to resolve the difficulty, but the improvement was limited.

To address above problems, in our previous work [6] a time varying vector autoregressive moving average (VARMA) model-based method is proposed to calculate the instantaneous frequencies of IMFs, then the instantaneous frequencies and the envelopes derived from the cubic spline interpolation of the maxima of IMFs are used to obtain the Hilbert spectrum. Without using the Hilbert transform, the method circumvents the limitations of Bedrosian theorem and Nuttall theorem. Since the variance of the model parameters can be properly tuned when they are estimated using the Kalman filter, the method can improve the readability of Hilbert spectrum at higher frequencies. Moreover, using a time varying VARMA model to represent the

IMFs as the outputs of a time varying system with white noise input and taking the time varying eigenfrequencies of the system as the instantaneous frequencies of IMFs, the method is more physically meaningful, and it can also reduce the effect of the noise which is yielded during the EMD procedure on the instantaneous frequencies.

Though the improved method has shown its valuable applicable potential in time varying spectrum estimation of nonstationary signals, its imperfection is still obvious. Since the instantaneous frequencies of IMFs are calculated based on VARMA model parameters, the computational-complexity is considerably high. As the further development of the improved method, in this paper an innovative procedure is introduced in which the time varying spectrum is directly calculated using the time varying VARMA parameters. In the following examples presented, the time varying spectra obtained by using the proposed method and other frequently used methods are compared and discussed, and some suggestions for future research are also provided.

2. HHT method

HHT is a two-step method which consists of EMD and HSA. To clarify the difference between the improved and the original HHT method, the EMD and HSA processes are briefly described in the following sections.

2.1 Empirical mode decomposition

As discussed by Huang et al. [3], the EMD method is based on the simple assumption that any data consists of different simple intrinsic modes of oscillations. Using this method, a signal can be decomposed in a finite set of oscillatory modes. Each of these oscillatory modes is represented by an intrinsic mode function (IMF) with the following definitions: (1) in the whole dataset of a signal, the number of extrema and the number of zero crossings must either equal or differ at most by one, and (2) at any point, the mean value of the envelope defined by the local maxima and the envelope defined by the local minima is zero.

Besides above definitions, a signal must have at least two extrema – one maximum and one minimum to be successfully decomposed into IMFs. To extract IMFs from a given signal, a process called sifting is used in EMD [1,3]. In practical implementation of the sifting process to extract an IMF, the second definition of an IMF often requires too many sifting steps which could obliterate the intrinsic amplitude variations and render the results physically less meaningful. To preserve the natural amplitude variations of the oscillations, sifting must be limited to as few steps as are mathematically permissible. Here, we use the S-number criterion, where the sifting is stopped when the number of zero crossings and extrema is the same number for *S* successive sifting steps [3].

Once all IMFs are sifted from a given signal x(t), it can be expressed as follows

$$x(t) = \sum_{j=1}^{n} c_j(t) + r_n(t), \tag{1}$$

where n is the total number of IMFs, and $r_n(t)$ is the final residue which represents the DC component containing the overall trend. The $c_j(t)$ are nearly orthogonal to each other, and all have nearly zero means.

2.2 Hilbert spectral analysis

The Hilbert spectral analysis is designated to calculate the instantaneous frequencies and amplitude of IMFs through the Hilbert transform and obtain the time-frequency distribution of amplitude called Hilbert spectrum.

For each IMF $c(t)$ of the L^p class, if its Hilbert transform is $y(t)$, then the analytic signal corresponding to $c(t)$ is defined as [7]

$$z(t) = c(t) + iy(t) = A(t)e^{i\theta(t)}, \tag{2}$$

where

$$A(t)=\sqrt{c^2(t)+y^2(t)} \quad \text{and} \quad \theta(t)=\tan^{-1}\left(\frac{y(t)}{c(t)}\right). \tag{3}$$

Here $A(t)$ is the instantaneous amplitude (envelope), and $\theta(t)$ is the phase function, then $c(t)$ is expressed as

$$c(t)=\Re\left(A(t)e^{i\theta(t)}\right)=A(t)\cos\theta(t), \tag{4}$$

where $\Re(\cdot)$ indicates the real part of a complex number. For the analytic signal $z(t)$, the instantaneous frequency $f(t)$ is defined as

$$f(t)=\frac{1}{2\pi}\frac{d\theta(t)}{dt}. \tag{5}$$

After performing the Hilbert transform on each IMF component, the original signal can be expressed as the real part in the following form

$$x(t)=\Re\left\{\sum_{j=1}^{n}A_j(t)\exp\left[i\int 2\pi f_j(t)dt\right]\right\}. \tag{6}$$

In Eq. (6), the residue $r_n(t)$ is purposely omitted, since it is either a monotonic function or a constant. Eq. (6) also allows represent the amplitude and frequency as functions as time in a three-dimensional plot, in which the amplitude can be contoured on the time-frequency plane. This time-frequency distribution of the amplitude is called Hilbert spectrum $h(f,t)$, and the corresponding square form $h^2(f,t)$ is called Hilbert energy spectrum.

3. The improved method

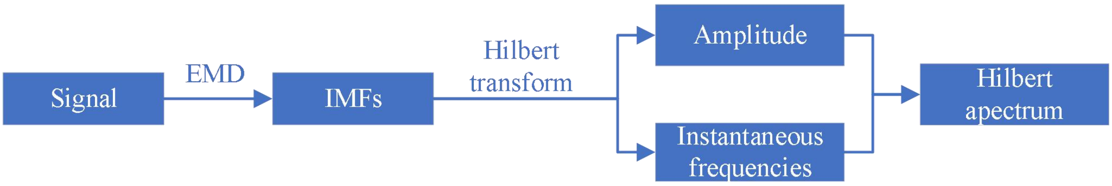
As has been mentioned, in the improved method, ground motion is first decomposed into several IMFs by using EMD. Then, the IMFs are represented as a time varying vector autoregressive moving average (VARMA) model and the Kalman filter is used to estimate the time varying model parameters. Finally, the time varying spectrum is directly calculated using the time varying VARMA parameters. In Fig. 1, the flowcharts of the HHT method, improved method in previous work and the method in this paper are presented in which the difference of three methods is highlighted. Briefly, the method in this paper consists of flowing steps:

(1) Use the EMD to get all the n IMFs which is the same as the original HHT method.

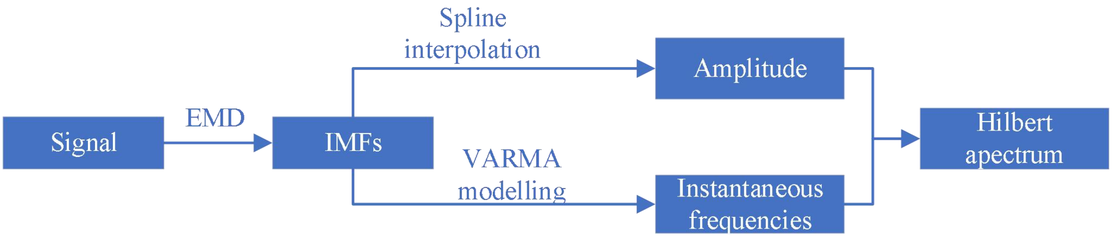
(2) Represent the IMFs as the time varying VARMA (p, q) model

$$\mathbf{y}(k)=\sum_{i=1}^{p}\mathbf{\Phi}_i(k)\mathbf{y}(k-i)+\sum_{i=1}^{q}\mathbf{\Theta}_i(k)\mathbf{u}(k-i)+\mathbf{u}(k), \tag{7}$$

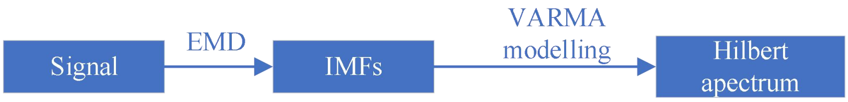
where $\mathbf{y}(k)$ consists of the n IMFs, i.e., $\mathbf{y}(k)=\{c_i(k), i=1,2,\cdots,n\}^{\mathrm{T}}$ and $c_i(k)$ is the i^{th} IMF. The autoregressive coefficients $\mathbf{\Phi}_i(k)$ and the moving average coefficients $\mathbf{\Theta}_i(k)$ are $n\times n$ matrices. The vector $\mathbf{u}(k)$ is an n-dimensional zero-mean Gaussian white noise process with covariance matrix $R(k)\delta_{i,j}$, where $\delta_{i,j}$ is the Kronecker delta function and $R(k)>0$. The variable k means the time instant $t=k\cdot\Delta t$ with Δt as the sampling interval. It has been shown that a time varying n-dimensional VARMA (2m, 2m-1) model is equivalent to a time varying system with nm degrees of freedom [8-10]. Since all the n IMFs are orthogonal monocomponent signals, they can be regarded as the n outputs of a time varying n-DOF system. Consequently, we can use the time varying n-dimensional VARMA (2, 1) model to represent the n IMFs.



(a) HHT method



(b) Improved method in previous work



(c) Method in this paper

Fig. 1 – Flowcharts of three methods

(3) Recast the time varying VARMA (p, q) model into state space form

$$\begin{cases} \mathbf{x}(k+1) = \mathbf{A}(k)\mathbf{x}(k) + \mathbf{B}(k)\mathbf{v}(k) \\ \mathbf{y}(k) = \mathbf{C}(k)\mathbf{x}(k) + \mathbf{D}(k)\mathbf{v}(k) \end{cases}, \tag{8}$$

where

$$\mathbf{x}(k) = \begin{bmatrix} \mathbf{y}(k) \\ \mathbf{y}(k-1) \\ \vdots \\ \mathbf{y}(k-p+1) \end{bmatrix}_{np\times 1} \text{ and } \mathbf{v}(k) = \begin{bmatrix} \mathbf{u}(k) \\ \mathbf{u}(k-1) \\ \vdots \\ \mathbf{u}(k-q+1) \end{bmatrix}_{nq\times 1}. \tag{9}$$

The matrices $\mathbf{A}(k)$, $\mathbf{B}(k)$, $\mathbf{C}(k)$ and $\mathbf{D}(k)$, called model parameters, are designated to

$$\mathbf{A}(k) = \begin{bmatrix} \mathbf{\Phi}_1(k) & \mathbf{\Phi}_2(k) & \cdots & \mathbf{\Phi}_p(k) \\ \mathbf{I} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & & \mathbf{0} \\ \vdots & \vdots & & \vdots \\ \mathbf{0} & \cdots & \mathbf{I} & \mathbf{0} \end{bmatrix}_{np\times np}, \mathbf{B}(k) = \begin{bmatrix} \mathbf{\Theta}_1(k) & \mathbf{\Theta}_2(k) & \cdots & \mathbf{\Theta}_q(k) \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \end{bmatrix}_{np\times nq},$$

$$\mathbf{C}(k) = [\mathbf{I}\ 0\ 0\ 0]_{n\times np} \text{ and } \mathbf{D}(k) = [\mathbf{I}\ 0\ 0\ 0]_{n\times nq}.$$

(4) Use the Kalman filter to estimate $\mathbf{A}(k)$ and $\mathbf{B}(k)$ based on above state space model. The iterative estimation process using Kalman filter is given by following set of equations [1,11]

$$\begin{cases} \hat{\mathbf{P}}(k+1) = \mathbf{P}(k) + \mathbf{Q}(k+1) \\ \mathbf{K}(k+1) = \hat{\mathbf{P}}(k+1)\mathbf{H}(k+1)^T[\mathbf{H}(k+1)\hat{\mathbf{P}}(k+1)\mathbf{H}(k+1)^T + \mathbf{R}(k+1)]^{-1} \\ \xi(k+1) = \xi(k) + \mathbf{K}(k+1)\{\mathbf{y}(k+1)^T - [\mathbf{H}(k+1)\xi(k)]\} \\ \mathbf{P}(k+1) = [\mathbf{I} - \mathbf{K}(k+1)\mathbf{H}(k+1)]\hat{\mathbf{P}}(k+1) \end{cases}, \tag{10}$$

where $\xi(k) = [\mathbf{\Phi}_1(k), \cdots, \mathbf{\Phi}_p(k), \mathbf{\Theta}_1(k), \cdots, \mathbf{\Theta}_q(k)]^T$ is called state vector, and $\mathbf{H}(k) = [\mathbf{x}(k-1)^T \ \mathbf{v}(k-1)^T]$ is called measurement vector. $\hat{\mathbf{P}}_{k+1}$ is the prediction covariance of state and $\mathbf{P}_{k+1}$ is the posterior covariance of state. K_{k+1} is called Kalman gain. $\mathbf{Q}(k)$ is covariance of process noise. The notation $\mathbf{e}(k+1) = \mathbf{y}(k+1) - [\mathbf{H}(k+1)\xi(k)]^T$ is called prediction error and $\hat{\mathbf{e}}(k+1) = \mathbf{y}(k+1) - [\mathbf{H}(k+1)\xi(k+1)]^T$, calculated according to $\xi(k+1)$, is called residue. The covariance matrix $\mathbf{R}(k)$ can be calculated according to $\mathbf{e}(k)$, i.e., $\mathbf{R}(k) = [(k-1)\cdot\mathbf{R}(k-1) + \mathbf{e}(k)^T \cdot \mathbf{e}(k)]/k$ where $\mathbf{R}(0)$ can be set 10^{-4} to 10^{-2}. As has been pointed out, by tuning the value of $\mathbf{Q}(k)$ properly, the time resolution and readability of Hilbert spectrum at higher frequencies can be improved. In [26-28], a method for choosing $\mathbf{Q}_k$ was set forth. In this study, we find the value of $10^{-4}\mathbf{I}$ for $\mathbf{Q}_k$ is optimal for most cases.

To initiate the Kalman filter, the p initial values of $\boldsymbol{y}(k)$ and q zeros are used to construct the measurement vector $\mathbf{H}(0)$, the state vector $\mathbf{X}(0)$ is estimated based on the corresponding time invariant VARMA model, and $\mathbf{P}(0)$ can be chosen as a large positive definite matrix (e.g. $10^4\mathbf{I}$). Then, all the state vectors can be estimated step by step using Eq. (10). After each iteration, the noise $\mathbf{u}(k)$ in $\mathbf{H}(k+1)$ is substituted by the residue $\hat{\mathbf{e}}(k)$. The iteration is stopped when the difference between the successive estimation results of $\|\hat{\mathbf{e}}(k)\|^2$ is smaller than a preset limit.

For the improved method in previous work, the instantaneous eigenfrequencies of IMFs are calculated by taking the eigenvalue decomposition of $\mathbf{A}(k)$

$$\mathbf{A}(k) = \mathbf{\Psi}(k)\boldsymbol{\lambda}(k)\mathbf{\Psi}(k)^{-1}, \tag{11}$$

where $\boldsymbol{\lambda}(k) = diag[\lambda_i(k)]$, for $i = 1, 2, \cdots, np$. Then, the instantaneous eigenfrequencies of IMFs can be calculated according to the instantaneous eigenvalues $\lambda_i(k)$

$$f_i(k) = \left\| \ln[\lambda_i(k)] \right\| / 2\pi\Delta t. \tag{12}$$

Because the instantaneous eigenvalues $\lambda_i(k)$ appear in complex conjugate pairs, one pair for each degree of freedom, we can get n different instantaneous eigenfrequencies, i.e., the instantaneous frequencies of n IMFs. For each IMF, define the envelope by a cubic spline through all the maxima. Then all the envelopes and instantaneous frequencies are used to obtain the Hilbert spectrum. Above steps describe the main calculating procedure of the improved method in previous work.

(5) In this paper, the time varying spectrum is calculated in a different way, i.e., the time varying VARMA model parameters obtained by using Kalman filter are directly used to calculate the time varying spectrum. Using the time varying VARMA parameters, the time varying spectrum matrix of IMFs $\mathbf{P}_{n\times n}(f, \mathrm{t})$ is given by

$$\mathbf{P}_{n\times n}(f,\mathrm{t}) = 2\mathbf{R}(k)\Delta t \frac{\left| \mathbf{I} - \mathbf{\Theta}_{1,k}\mathbf{e}^{-i2\pi f\Delta t} - \cdots - \mathbf{\Theta}_{q,k}\mathbf{e}^{-i2\pi qf\Delta t} \right|^2}{\left| \mathbf{I} - \mathbf{\Phi}_{1,k}\mathbf{e}^{-i2\pi f\Delta t} - \cdots - \mathbf{\Phi}_{p,k}\mathbf{e}^{-i2\pi pf\Delta t} \right|^2}. \tag{13}$$

Since all the n IMFs are orthogonal monocomponent signals, the time varying spectrum of signal $p(f, \mathrm{t})$ can be represented as the sum of the diagonal elements of $\mathbf{P}_{n\times n}(f, \mathrm{t})$, i.e.,

$$p(f,\mathrm{t})=\sum_{i=1}^{n}\mathbf{P}_{i,i}(f,t)\ . \tag{14}$$

It's to be noted that the time varying spectrum here is bilinear spectrum which is the analogy to the Hilbert energy spectrum or the power spectrum in stationary signal case, and its square root is linear spectrum which is the analogy to the Hilbert spectrum or the Fourier spectrum in stationary signal case.

3. Results and discussions

To investigate the efficiency of the proposed method in this paper, two examples are provided. In the first example, acceleration record El Centro (1940, N-S) used in Huang et al. [3] is analyzed using above three methods respectively. In the second example, ground accelerations recorded during the 2011 Tohoku earthquake at station MYG004 where the largest peak ground acceleration (about 27m/s^2) is recorded. The components in three directions, i.e., north-south (NS), east-west (EW) and up-down (UD) directions are analyzed and the corresponding time varying H/V spectrum ratio of this site is also estimated and discussed.

In Fig. 2, acceleration time histories of ground motion El Centro (1940, N-S) and its corresponding IMFs are presented and the tendency that the intrinsic scales of IMFs increase from the smallest to the largest gradually and the first 6 IMFs carry the majority of the seismic energy is clearly shown.

Fig. 3 gives the time varying spectra obtained by using the original HHT method, the method in previous work and the improved method in this paper. It can be seen the energy distribution presented in the results of above three methods are similar as a whole and the seismic energy mainly distributes along the horizontal belts below 5 Hz in the first 10 seconds. For the result of the original HHT method, the energy of all the IMFs are blended with each other and badly localized, the energy for each IMF cannot be identified from corresponding Hilbert spectrum. But for the results of both improved methods, the energy of all the IMFs are clearly separated and well localized, the decreasing tendency in frequency distinctively indicates the nonstationarity in frequency contents. It is also clearly indicated in Figs. 3(b) and 3(c) that the first 6 IMFs share the majority of the seismic energy; however, it cannot be seen from Fig. 3(a). By comparing the results of the method in previous work and the method in this paper, it can be seen that the frequency resolution of the method in previous work is better but the time resolution is worse than the method in this paper, and the latter method achieves a better compromise in time and frequency resolution. Also, the method in previous work to some extent overestimates the energy in higher frequency band above 5Hz.

In the second example, ground accelerations recorded during the 2011 Tohoku earthquake at the station MYG004 are presented in Fig. 4. The corresponding time varying spectra of the three ground motion components are shown in Figs. 5 to 7 respectively. Similar to the first example, it can be seen that for the result of the original HHT method the energy distribution in time-frequency plane is blurred and badly localized. The frequency resolution of the method in previous work [6] is best, but the time resolution is worse than the method in this paper and overestimation of energy distribution in high frequency band above 10 Hz. The method of this paper achieves a better compromise in time and frequency resolution.

As the further application of the method in this paper, and for the state-space function in Eq. (8), if the IMFs of horizontal component is used to construct the output vector $\mathbf{y}(k)$ and the IMFs of vertical component is used to build the input vector $\mathbf{u}(k)$, the corresponding time varying VARMA parameters can be used to calculate the time varying H/V spectral ratio of the site where the station locates. The time varying H/V spectral ratio $S_r(f,t)$ is given by

$$\mathbf{S}_{n\times n}(f,\mathrm{t})=2\Delta t\frac{\left|\mathbf{I}-\mathbf{\Theta}_{1,k}\mathbf{e}^{-i2\pi f\Delta t}-\cdots-\mathbf{\Theta}_{q,k}\mathbf{e}^{-i2\pi qf\Delta t}\right|}{\left|\mathbf{I}-\mathbf{\Phi}_{1,k}\mathbf{e}^{-i2\pi f\Delta t}-\cdots-\mathbf{\Phi}_{p,k}\mathbf{e}^{-i2\pi pf\Delta t}\right|}\ . \tag{13}$$

$$S_r(f,t)=\sum_{i=1}^{n}\mathbf{S}_{i,i}(f,\mathrm{t}) \tag{14}$$

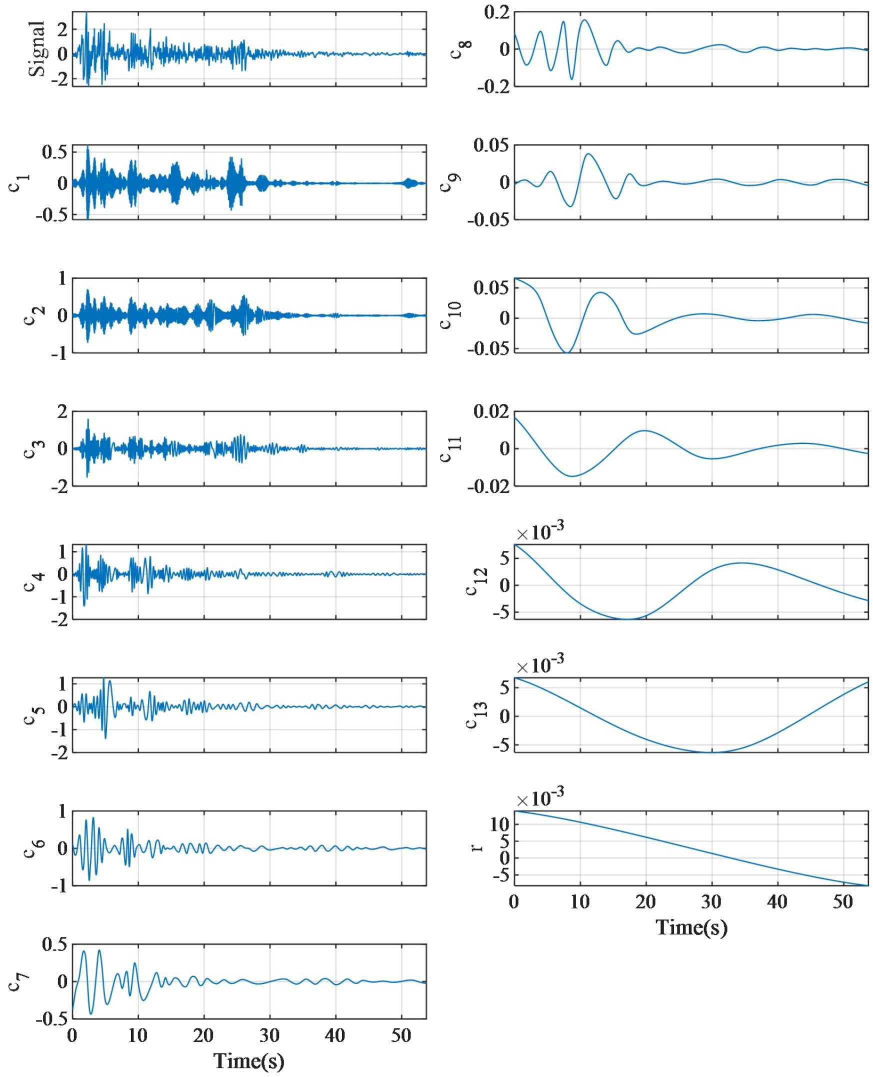
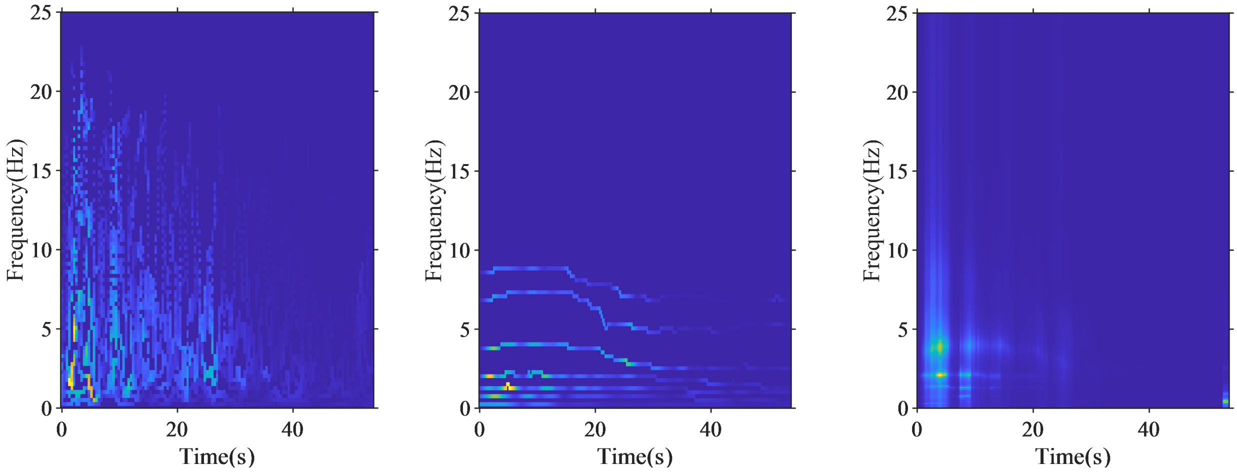


Fig. 2 – Time histories of ground motion El Centro (1940, N-S) and its IMFs



(a) Original HHT (b) Method in previous work (c) Method in this paper

Fig. 3 – Time varying spectra obtained by using three methods

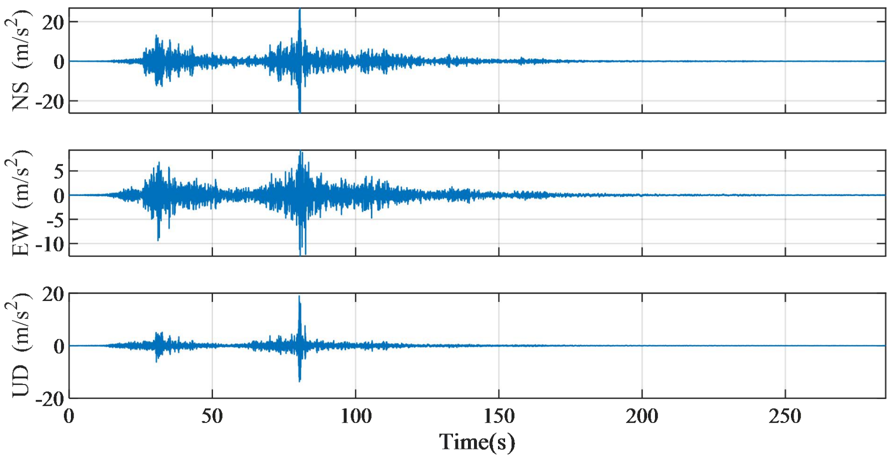
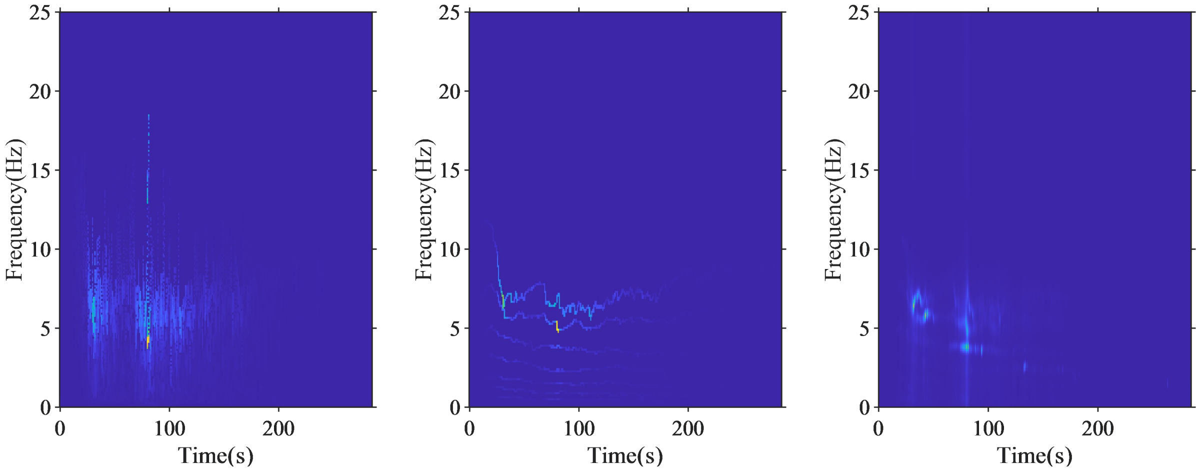
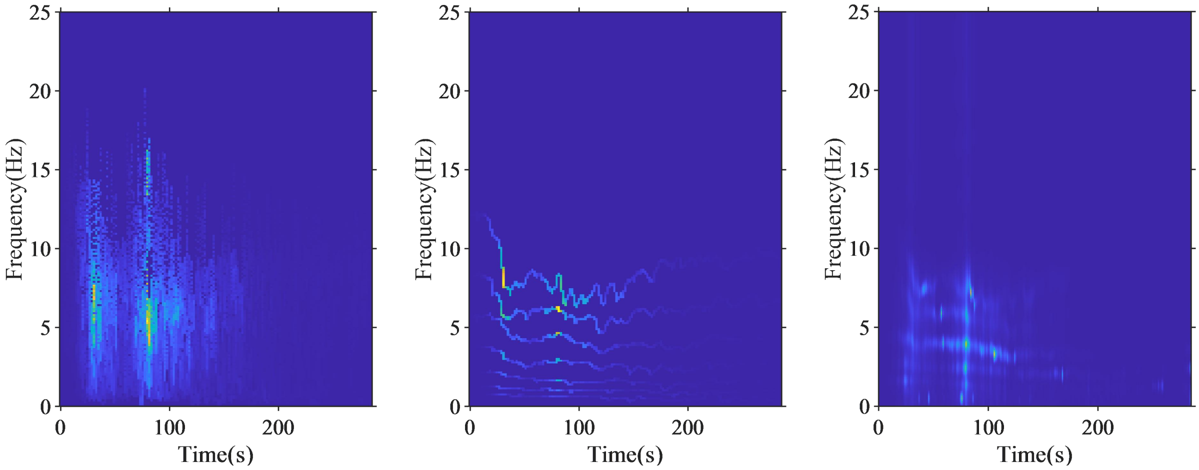


Fig. 4 – Ground motions recorded at the station MYG004 during the 2011 Tohoku earthquake



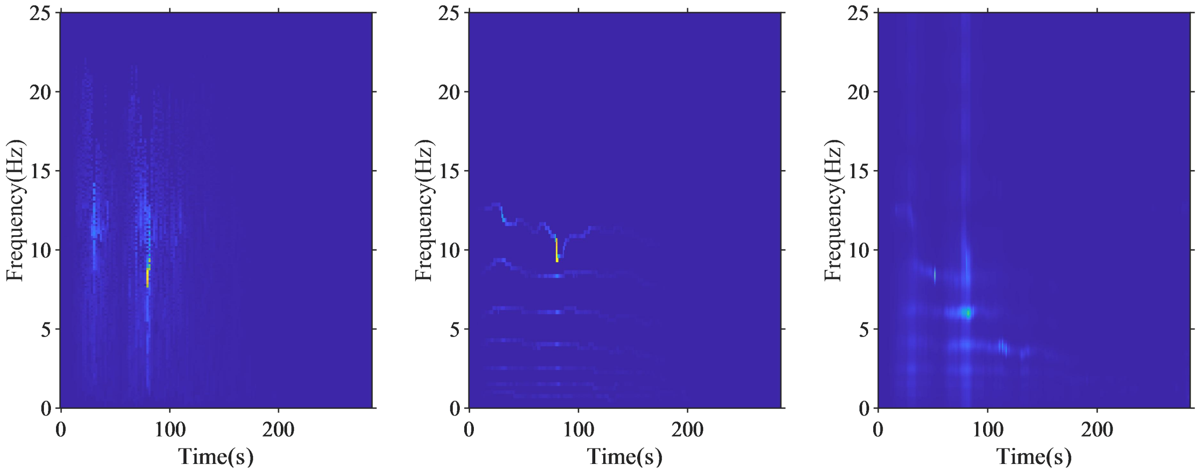
(a) Original HHT (b) Method in previous work (c) Method in this paper

Fig. 5 – Time varying spectra of NS component by using three methods



(a) Original HHT (b) Method in previous work (c) Method in this paper

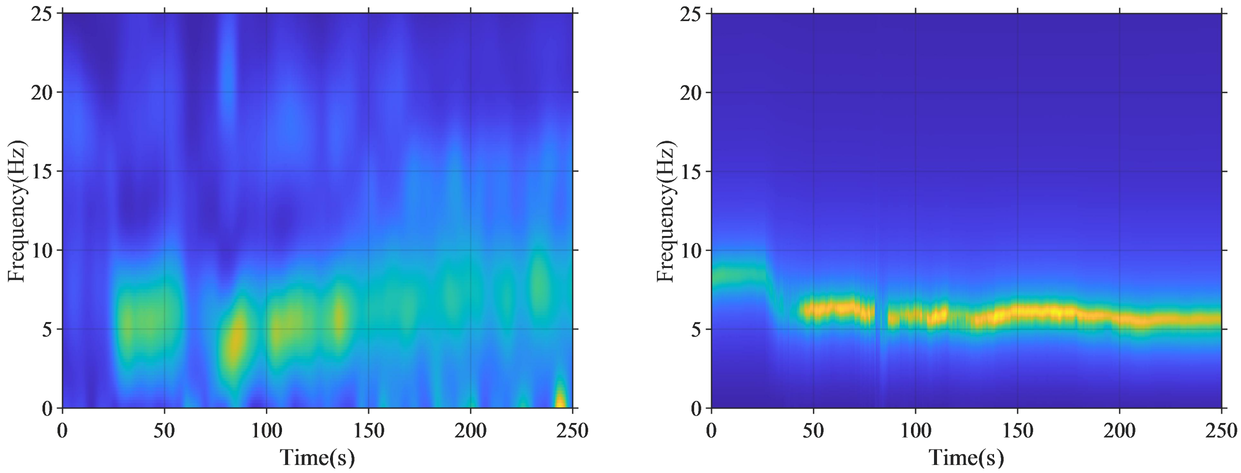
Fig. 6 – Time varying spectra of EW component by using three methods



(a) Original HHT (b) Method in previous work (c) Method in this paper

Fig. 7 – Time varying spectra of UD component by using three methods

As we know the H/V spectral ratio (i.e., the ratio between the Fourier amplitude spectra of the horizontal and the vertical component of ground motions) is simple and fast way to get properties of a site, both H/V peak frequency and amplitude are straightforward related to the soil transfer function (in term of fundamental resonance frequency and site amplification factor). In Fig. 8, the time varying H/V spectral ratio obtained by using the conventional moving window method and the method proposed in this paper are presented. For both methods, the tendency of H/V peak frequency is identical as a whole. For the moving window method, the H/V peak frequency spreads in a quite more wide range. While for the method in this paper, the spread of H/V peak frequency is quite small and its changing trend with time is clearly exhibited.



(a) Moving window method (c) Method in this paper

Fig. 8 – Time varying spectra ratios obtained by using two methods

4. Conclusions

In this paper, an innovative method based on EMD and VARMA model is proposed for time varying spectrum estimation of earthquake ground motions. The efficiency of the method is verified using two examples. The method also shows important potential in seismic site effects and structure damage detection for future research work.

5. Acknowledgements

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