NEW VARIATIONAL MODE DESCOMPOSITION ALGORITHM APPLIED TO IMPROVED EARTHQUAKE ACCELEROGRAMS ANALYSIS

F. Yépez[(1)], F. Granda[(2)], D. Benítez[(1)]

[(1)] *Professor of Civil Enginering, Colegio de Ciencias e Ingenierias, Universidad San Francisco de Quito, fyepez@usfq.edu.ec*
[(2)] *Research assistant, Colegio de Ciencias e Ingenierias, Universidad San Francisco de Quito, fyepez@usfq.edu.ec*
[(3)] *Professor-Electronic Engineering, Colegio de Ciencias e Ingenierias, Universidad San Francisco de Quito, dbenitez@usfq.edu.ec*

Abstract

Strong motion earthquake records are a great source of information regarding the local soil dynamic response and the seismic input acting to the structures located around the device location. In addition, a set of those signals recorded in several locations around the epicenter of the same earthquake event could give important information regarding seismic wave propagation and attenuation in the region, useful for engineering design purposes. Many time-frequency domain algorithms have been developed in order to perform signal processing and wave characterization analyses. Among them, Fourier transform has been one of the most used algorithms for decomposing signals into its constituent frequency components. Decomposition methods have also appeared later as improved techniques that decompose signals in intrinsic mode functions (IMF), which are amplitude-modulated-frequency-modulated signals easier to analyze, demonstrating some advantages regarding resolution and adaptability to different type of signals, over traditional approaches. In this work, a novelty technique, the variational-mode decomposition (VMD) has been applied to the analysis of the 2016 Mw7.8 Pedernales earthquake records (the most powerful earthquake ever recorded in Ecuador). For the first time in the country, multi-directional earthquake accelerograms from 20 stations located between 30 to 500 km far from the epicenter of a great earthquake are available for the analysis. The VMD method decomposes a signal into an ensemble of band-limited modes. The band-limited intrinsic modes about a center frequency are obtained by using the Hilbert Transform, shifting and normalizing the signal data to create a frequency translated analytic signal. This process permits to have an accurate analysis of the signal peaks in different frequency windows and its distribution in the space covered by the device array. Spatial analysis permitted to create peak heat maps of the region where records were obtained in order to show those results graphically. Analysis of the results after applying this technique to the mentioned earthquake records were compared with traditional Fourier spectra results in terms of amplitude-frequency peaks, while amplification and attenuation of seismic waves in distance were also mapped, depending on the propagation direction. Results showed different patterns of wave attenuation in the region and corroborated soil dynamic amplification of some of the recording locations, demonstrated that the VMD signal analysis algorithm is more robust than other approaches for earthquake records, especially for interpretation of multi-component signals. This algorithm also improved signal noise reduction and gave amplitude peaks in the frequency and time domain with higher resolution.

Keywords: variational mode deconposition; earthquake accelerograms; intrinsic mode functions; signal analysis

1. Introduction

Strong motion records are the way to obtain real data regarding earthquake effects in the recording device locations. Variation of amplitudes, frequency content and record duration are typical parameters that are analyzed in order to understand the seismic input acting over the soil and the structures located in the surrounding area of the recording devices. If a set of records of the same earthquake event obtained at different distances from the epicenter is available, seismic wave propagation and attenuation could also be analyzed in an entire region, and could contribute to understand the earthquake action over such region for future engineering design purposes.

In order to analyze frequency content of earthquake accelerograms, time and frequency-based analysis methods have been widely used for signal processing of such non-stationary signals [1]. Fourier analysis has been used for decades as a traditional way to perform frequency analysis. The procedure decomposes the signal into its constituent frequency components; however, it cannot identify their temporal localization. In addition, it has an important limitation when it can only provide an average spectral decomposition of the signal. Availability of more powerful computers and recent advances in applied mathematics have permitted the development of new and fast algorithms and techniques that improve frequency analysis of transient earthquake signals [2]. Some of them, the time-frequency methods, map a one-dimensional signal into a two-dimensional function of time and frequency, describing how the spectral content of the signal changes with time.

Those advances in computers and applied math also contributed to the development of the decomposition methods. These new techniques decompose a seismic signal into a sum of intrinsic oscillatory components called intrinsic mode functions (IMF) [3], having advantages in resolution, implementation and adaptability to different type of signals [4]. Latter, variational-mode decomposition (VMD) methods arise and presented improved and robust algorithms for interpretation of multi-components signals. Their improvements in signal noise reduction and the high resolution (more defined amplitude peaks in the frequency and time-frequency domain) of the time-frequency decomposition make suitable their application for studying seismic signals [5] [6], as this paper will demonstrate through the analysis of the Pedernales-Ecuador Mw 7.8 earthquake recorded in April 2016.

2. Variational Mode Decomposition (VMD) Method

Several intrinsic modes IMF can be extracted front a transient signal using the Eq. (1), where u_k is the k-th IM of the original signal, w_k is the center frequency of the k-th corresponding IM, and N is the total number of modes to be obtained in the decomposition. The expression is obtained with the use of the alternative direction method of multipliers (ADMM) applied to a Lagrangian form of Eq. (1), that is represented by Eq. (2), as follow:

$$\underset{\{u_k\},\{w_k\}}{min}\left\{\sum_{k=1}^{N}\left\|\delta_t\left[\left(\delta_t+\frac{j}{\pi t}\right)*u_k(t)\right]e^{-jw_kt}\right\|_2^2\right\}\ s.t.\sum_{k=1}^{N}u_k=f \qquad (1)$$

$$\mathcal{L}(\{u_k\},\{w_k\},\lambda):=\alpha\left\{\sum_{k=1}^{N}\left\|\delta_t\left[\left(\delta_t+\frac{j}{\pi t}\right)*u_k(t)\right]e^{-jw_kt}\right\|_2^2\right\}+\left\|f(t)-\sum_{k=1}^{N}u_k(t)\right\|_2^2 + \langle\lambda(t),f(t)-\sum_{k=1}^{N}u_k(t)\rangle \qquad (2)$$

It is necessary to iterates the ADMM method over terms described by Eq. (3), (4) and (5) until convergence condition expressed by Eq. (6) is obtained.

$$\hat{u}_k^{n+1}(w) \leftarrow \frac{f(w) - \sum_{i<k} \hat{u}_i^n(w) - \sum_{i>k} \hat{u}_i^n(w) + \frac{\lambda^n(w)}{2}}{1 + 2\alpha(w - w_k^n)^2} \tag{3}$$

$$w_k^{n+1} \leftarrow \frac{\int_0^\infty w\left|\hat{u}_k^{n+1}\right|^2 dw}{\int_0^\infty \left|\hat{u}_k^{n+1}\right|^2 dw} \tag{4}$$

$$\lambda^{n+1}(w) \leftarrow \lambda^n(w) + \tau\left(f(w) - \sum_k \hat{u}_k^{n+1}(w)\right) \tag{5}$$

$$\frac{\sum_k \left\|\hat{u}_k^{n+1} - \hat{u}_k^n\right\|_2^2}{\left\|\hat{u}_k^n\right\|_2^2} < \epsilon \tag{6}$$

The band-limited intrinsic modes about a center frequency are obtained by using the Hilbert Transform, shifting and normalizing the signal data to create a frequency translated analytical signal [6] [7]. Reader could refers to [6] for a detailed explanation about the terms used in Eqs. (1) to (6).

3. Strong ground motion database

A set of three-components earthquake ground motion records corresponding to the Pedernales – Ecuador Mw7.8 earthquake (April 2016) and registered in the local Geophysical Institute RENAC network (installed in 2014), has been selected in order to apply the variational mode decomposition -VMD- methodology described before and to perform the signal analysis with special focus on the changes in propagation patterns according to the distance from the epicenter. The seismic event was the strongest earthquake felt in Ecuador in many decades, causing around 600 casualties and more than 7000 buildings destroyed [8]. It was the first time that strong ground motion accelerograms of an important earthquake were recorded in the country. Fig. 1 shows location of the Ecuadorian RENAC network, while Table 1 shows recording station names, city, epicentral distance and recorded peak ground acceleration for each component. When available, V_{s30} values measured in the recording location was also reported in Table 1.

Fig. 1 – Ecuadorian RENAC strong ground motion stations. Nearest station to the Pedernales 2016 Mw7.8 earthquake epicenter is highlighted by a blue square

Table 1 – RENAC recording stations, city, epicentral distance, V_{s30} values and PGA (3 components)

Station	City	Epicentral distance (Km)	V_{s30} (m/s)	PGA(g) EW	PGA(g)NS	PGA(g) Z
APED	Pedernales	36	342	1.408	0.829	0.742
AES2	Esmeraldas	76	-	0.154	0.110	0.044
ASDO	Sto. Domingo	115	-	0.206	0.111	0.051
ACHN	Chone	120	200	0.330	0.370	0.173
ALOR	San Lorenzo	159	-	0.026	0.027	0.015
APO1	Portoviejo	167	224	0.318	0.380	0.104
AMNT	Manta	171	496	0.405	0.524	0.165
EPNL	Quito	174	-	0.027	0.020	0.013
AOTA	Otavalo	188	-	0.043	0.035	0.019
AIB1	Ibarra	202	-	0.049	0.058	0.012
AIB2	Ibarra	204	-	0.021	0.033	0.009
ALAT	Latacunga	206	-	0.032	0.028	0.012
AMM2	Ambato	235	-	0.026	0.035	0.015
ATUL	Tulcán	251	-	0.016	0.021	0.007
AGYE	Guayaquil	270	1800	0.019	0.023	0.015
AMIL	Milagro	288	-	0.052	0.046	0.019
ALIB	La Libertad	308	429	0.042	0.040	0.021
ACUE	Cuenca	381	-	0.036	0.030	0.018
ACH1	Machala	407	-	0.025	0.024	0.008
ALJ1	Loja	492	-	0.015	0.016	0.009

As an example, Fig. 2 shows the AMNT record -EW component- and its Fourier amplitude spectrum. This record is further decomposed using the VMD method setting to four IMF sequences. Fig. 3 show a time-domain representation of each mode, where noise reduction is obtained in some sequences as an advantage of the applied method. After, spectrum analysis was applied to IMF signals obtained.

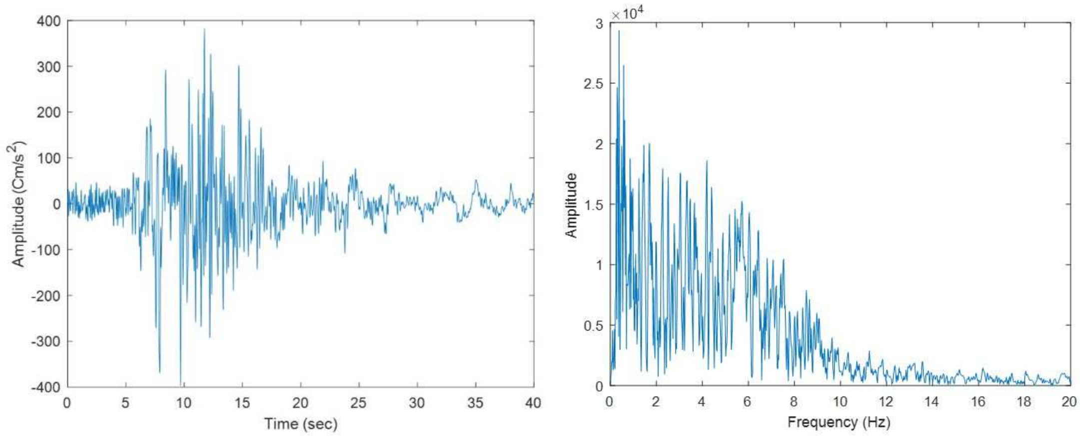

Fig. 2 – AMNT record -EW component- and Fourier amplitude spectrum of the signal

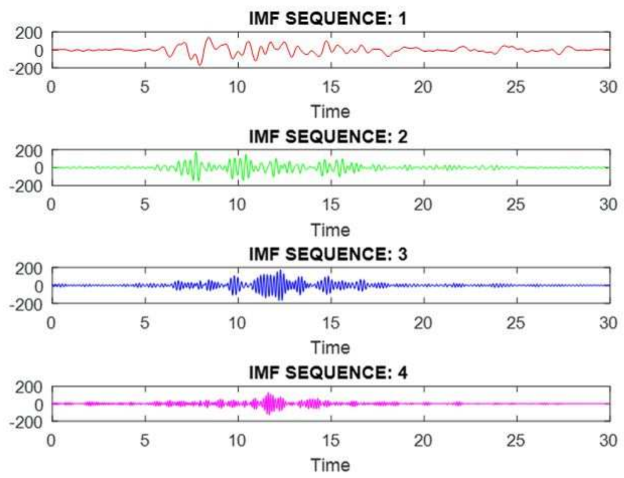

Fig. 3 – VMD results of the AMNT record showing IMF modes with less noise contamination, especially in sequences 2, 3 and 4

4. Spectrum analysis of the database

Fourier amplitude spectrum of the record showed in Fig. 2 is compared with FFT based spectrum of each IMF sequence considered before for the same record (Fig. 4). Peak values and their corresponding frequencies for each of the four decomposed modes were recorded for further analysis. This procedure was performed for each record of the data set described in Table 1. After, a geographical distribution based on the amplitude of the peak values of each record extracted with the VMD method were obtained in the form of heat maps of the country using a GIS tool, in order to analyze variation of the record parameters with the distance, in an effort to get a better understanding of the seismic waves propagation and attenuation in the territory. Peak amplitude of IMF1, 2, 3 and 4 values of all Z-axis records are showed in Fig. 5, observing a distribution mainly in an east-south direction of the country from the epicenter. Although, IMF1 and 2 showed also small distribution in north-east direction. This is congruent with the failure mode of the earthquake rupture, with highest energy to the south-east direction from the epicenter. A higher value concentration in APED station (nearest to epicenter) in IMF1 (3.18Hz), IMF2 (7.39Hz) and IMF3 (11.1Hz) modes is detected. On the other hand, farthest stations show peak values around 0.5Hz. Table 2 shows stations with highest peak values in IMF1 and 2.

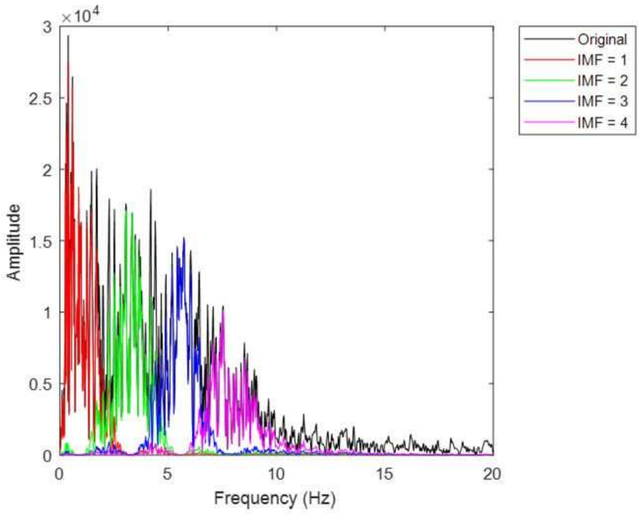

Fig. 4 – Fourier amplitude spectrum of the AMNT –EW component- original signal and comparison with the FFT based spectra of the decomposed signal (for the four IMF sequences considered). Noise reduction effect in the decomposed modes can be appreciated

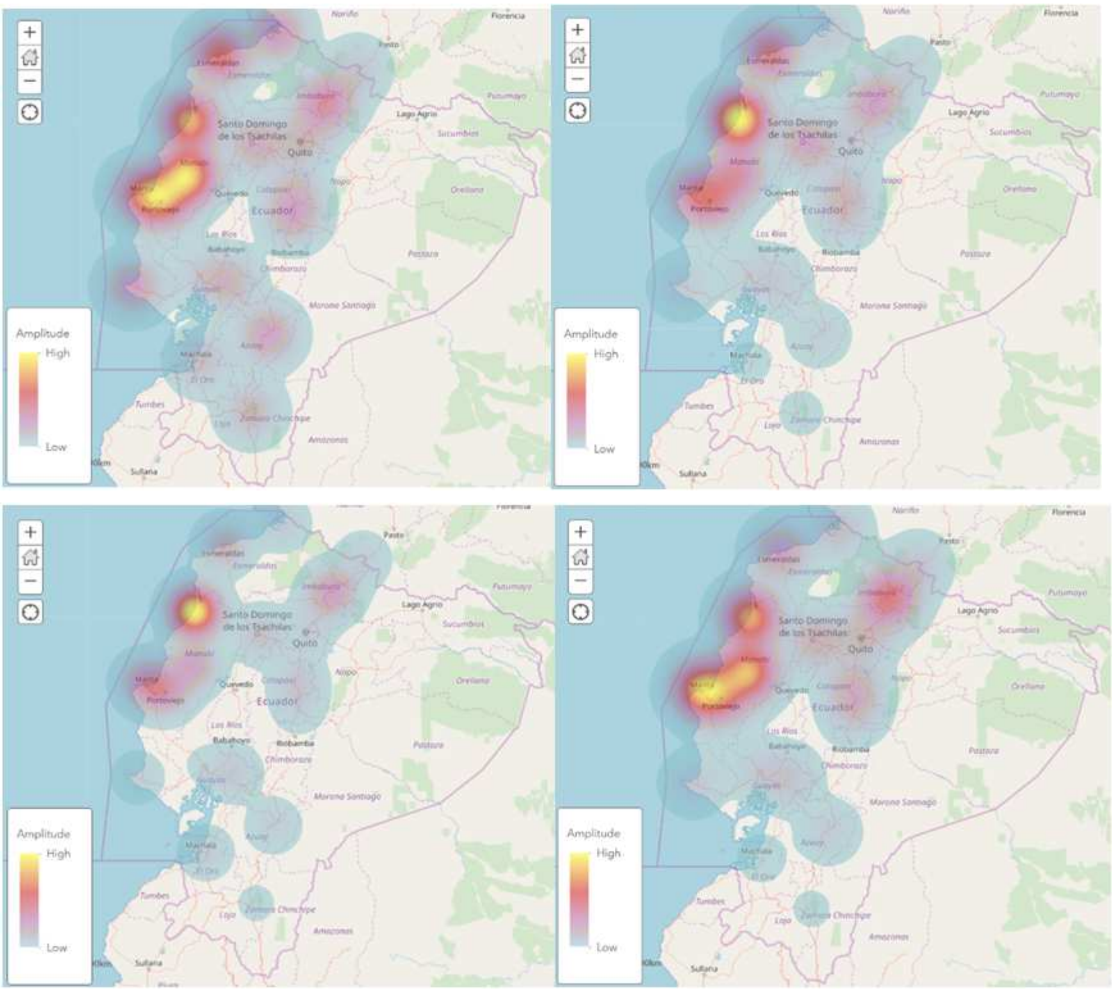

Fig. 5 – Distribution of IMF1, 2, 3 and 4 values of the records, Z-axis

Table 2 – Highest peak values in IMF1 (left) and IMF2 (right) of relevant records (Z-Axis)

Station	City	Frequency (Hz)	Station	City	Frequency (Hz)
APED	Pedernales	3.18	APED	Pedernales	7.39
ASDO	Santo Domingo	3.04	ASDO	Santo Domingo	5.19
AIB2	Ibarra	1.34	AIB2	Ibarra	2.08
ALAT	Latacunga	1.29	AES2	Esmeraldas	1.88
AIB1	Ibarra	0.92	ATUL	Tulcán	1.63
AMM2	Ambato	0.72	AOTA	Otavalo	1.46
EPNL	Quito	0.68			
ACHN	Chone	1.30			
AGYE	Guayaquil	1.32			
ACUE	Cuenca	0.56			
ALJ1	Loja	0.55			
ACH1	Machala	0.55			
AMIL	Milagro	0.54			
ALIB	La Libertad	0.49			

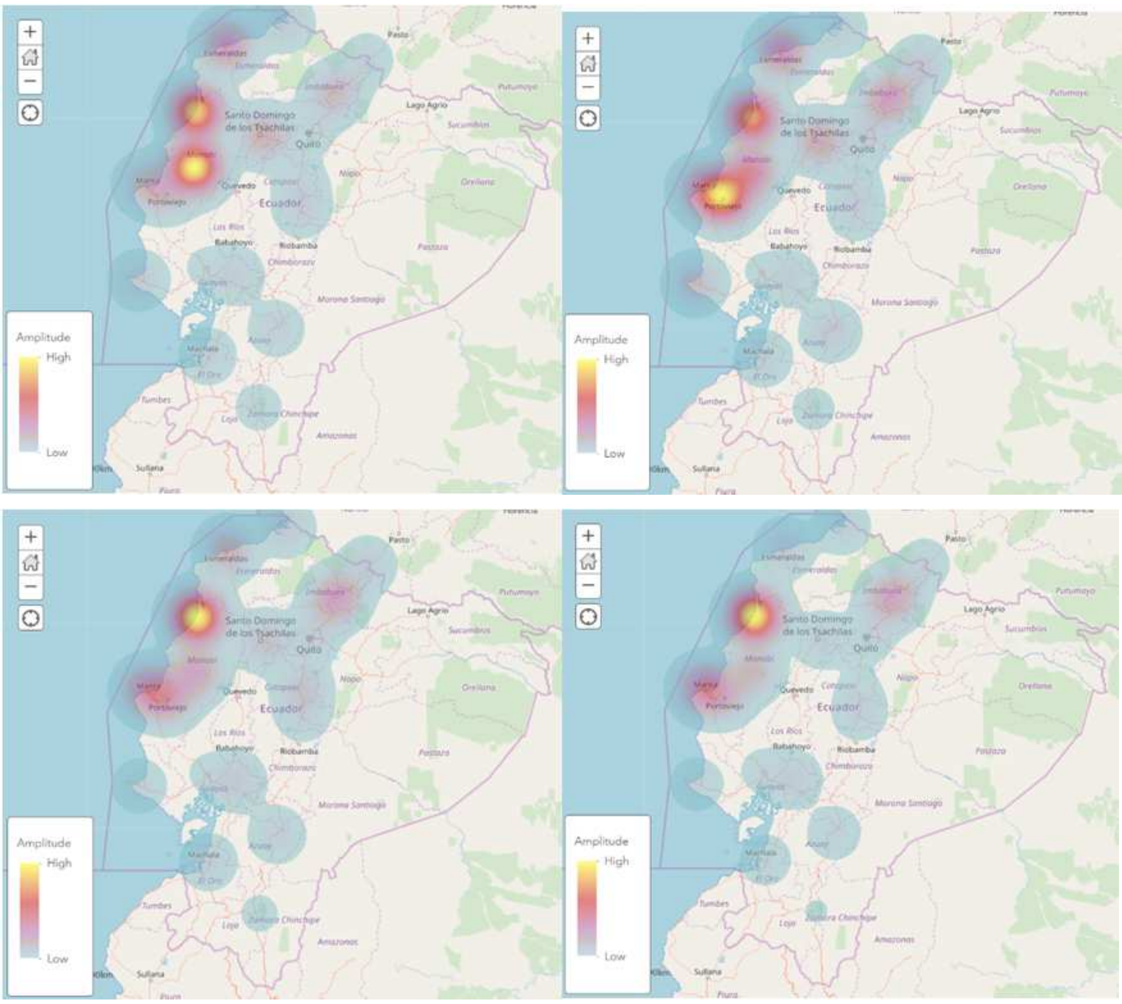

Fig. 6 – Distribution of IMF1, 2, 3 and 4 values of the records, NS-axis

On the other hand, stations APED, ACH, APO1 and AMNT showed deeper concentration of high peak values in IMF modes for NS-axis, even at considerable epicentral distance, behavior not observed when Z-axis components were analyzed (with the exception of the APED station, the nearest to the epicenter). As it is possible to observe in Table 1, those station locations had reported the lower V_{s30} values. It is clear that this behavior is congruent with an important soil dynamic amplification effect, creating energy concentration in specific frequency ranges even if records are far from the epicenter. Strong structural damage has been reported in those cities after the earthquake [8]. Superposition of the IMF1 and IMF2 results summarized in Table 3 show a more detailed distribution of the high peak values in records obtained in the mentioned stations, including AES2 station which showed similar behavior.

Table 3 – Peak values and frequency values with strong soil amplification (NS-Axis)

Station	City	Frequency (Hz)	Epicentral dist. (Km)
APO1	Portoviejo	2.08 IMF2	167
APED	Pedernales	2.04 IMF1	36
AMNT	Manta	1.83 IMF2	171
ACHN	Chone	0.67 IMF1	120
AES2	Esmeraldas	0.66 IMF1	76

5. Conclusions

VMD method has demonstrated to be very efficient in providing a deep insight regarding peak amplitude and frequency values of strong ground motion records. The analysis of different IMF modes permitted an improved signal process and analysis, compared to the classical Fourier amplitude spectrum analysis. An independent mode analysis permitted to obtain results that can be reported in table format and in geographic heat maps that can show attenuation patters of seismic energy in distance. An independent organization of results can highlight similarities in terms of peak amplitudes and frequencies that can be related to geological, geotechnical and geographic site conditions. The selection of a bigger number of intrinsic-mode functions and their impact on the analysis quality of seismic records belonging to a most crowded network is an interesting area of future research.

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7. References

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