DIMENSION-REDUCTION SIMULATION OF MULTI-SUPPORT AND MULTI-COMPONENT NON-STATIONARY GROUND MOTION FIELDS

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Abstract

Researches have shown that the spatial variability of earthquake ground motions and multi-component seismic action have a significant impact on energy dissipation characteristics and failure mechanism of large-span complex spatial engineering structures, such as large-scale spatial reticulated shells, dams and long-span bridges. Therefore, for refined seismic response analysis and seismic optimization design of such structures, it is more reasonable to adopt spatially correlated multi-support and multi-component ground motion input which can take into account both spatial variability and multi-component seismic action. Nowadays, the Monte Carlo simulation method (MC scheme) has been developed rapidly in recent years, and among which, the spectral representation method (SRM) and proper orthogonal composition (POD), owing to their accuracy and simplicity, appear to be the most widely used ones. However, the sample functions generated by the conventional SRM and POD can't describe the probability information of stochastic field in probability density level, which gives rise to an inadequate quantification of probability propagation from the external excitations to structural dynamic responses. Therefore, though the simulation efficiency of the SRM and the POD has been improved dramatically, the extremely high-dimensional randomness degree (the number of random variables) involved in the MC scheme still remains a principal challenge for it being applied in probability density evolution analysis and reliability assessment of large-scale complex engineering structures.

For that, in this study, firstly, based on the stochastic model of multi-support and one-component fully non-stationary ground motion fields, the unified spectral decomposition representation of the multi-support and multi-component fully non-stationary ground motion stochastic fields on the basis of the coherence function matrix is derived. Furthermore, by introducing random function form serving as constraint for the orthogonal random variables, the dimension-reduction simulation of multi-support and multi-component fully non-stationary ground motion fields is realized. Then, according to the regression results of the correlation involved in the intensity envelope function parameters of the multi-component ground motions, the four-segment intensity envelope function parameter values of the three-component ground motions for soil site I_0 in "Seismic ground motion parameters zonation map of China" are suggested. Finally, based on the Matsushima's model, four-segment intensity modulation function, the Clough-Penzien time-varying power spectrum and the composite coherence function model, the representative samples of the multi-support and multi-component fully non-stationary ground motions acceleration processes are generated. Numerical examples adequately verify the effectiveness of the proposed method in engineering practices. Benefitting from the proposed scheme, the extremely high randomness degree can be effectively reduced to merely two. Thus, the number-theoretical method can be conveniently applied to select the representative point sets with regard to the elementary random variables. As a result, the complete probability characteristics of the multi-support and multi-component stochastic ground motions can thus be reflected with just two elementary random variables, and the total number of sample functions is just several hundred, making it possible for the proposed scheme being combined with the probability density evolutionary method (PDEM) to implement the accurate stochastic dynamic response analysis and dynamic reliability assessment of complex engineering structures subjected to earthquake disasters.

Keywords: multi-component ground motions, non-stationary process, spectral decomposition, dimension reduction

1. Introduction

Studies have shown that earthquake ground motion has significant spatial variability, which is mainly caused by lagged coherence effect, wave-passage effect, attenuation effect, and local site effect during the propagation of seismic waves. In addition, ground motion is a very complex multi-component motion, with two horizontal components and one vertical component usually being obtained in seismic observations. In fact, it is shown that the spatial variability and multi-component action of earthquake ground motions can significantly affect the energy consumption characteristics and failure mechanisms of large-span complex spatial structures (such as large spatial reticulated shell structures) during seismic excitation [1, 2]. Therefore, for the seismic response analysis and seismic optimization design of large-span complex spatial structures, it is more reasonable to use spatially correlated multi-support and multi-component ground motion inputs that can consider both the spatial variability and multi-component action of ground motions simultaneously [3].

Due to the strong randomness of earthquake ground motions, the adoption of stochastic methods to generate ground motions has always attracted much attention. The multi-component stochastic ground motion model can be generated based on the one-component seismic model by means of further considering the correlation between different components in different directions. From this point of view, the multi-component ground motion model can be regarded as an extension of the one-component ground motion model. At present, the simulation methods of multi-support and multi-component non-stationary stochastic ground motion fields mainly include linear filtering method, wavelet analysis method, SRM [4-6], and POD [7-9]. These methods are all developed based on the second-order statistics of the stochastic ground motion fields (evolutionary power spectral density matrix or coherence function matrix), and their essence is realized by MC scheme for a large number of a series of random variables. Among them, the SRM and POD have been widely used in the simulation of stochastic processes and stochastic fields due to their complete theory, simple calculation, and easy implementation [10-12]. However, in case that the conventional MC scheme is used to simulate the stochastic ground motion field, a large number of sampling for a series of high-dimensional random variables are usually required to ensure the simulation accuracy. This treatment not only greatly increases the calculation amount of the simulation, but also limits the application of this method in the analysis of stochastic seismic response and seismic reliability of large and complex spatial structures to a large extent.

In view of the above research status, firstly, based on the SRM and POD of multi-support and one-component fully non-stationary stochastic ground motion field, the unified spectral decomposition expression of spatial correlation multi-support and multi-component fully non-stationary stochastic field is derived on the basis of the coherence function matrix. Secondly, by introducing the idea of random function [13-15], the dimension-reduction representation of spatial correlation multi-support and multi-component fully non-stationary stochastic ground motion field is established, which can accurately describe the stochastic ground motion field on the probability density level with merely two elementary random variables. Finally, based on the Matsushima's model of multi-component ground motion, a representative sample set of multi-support and multi-component fully non-stationary ground motion field is generated. Due to the probability information of the representative sample set is complete, it can be naturally combined with the PDEM [16-17] to realize the refined dynamic response analysis and seismic reliability evaluation of large-scale complex engineering structures induced by multi-support and multi-component earthquake actions.

2. Stochastic model of multi-support and multi-component fully non-stationary ground motion stochastic fields

The two-sided evolutionary power spectral density matrix of a multi-support and multi-component fully non-stationary stochastic ground motion field is expressed as follows:

$$\boldsymbol{S}(t,\omega)=\begin{bmatrix} \boldsymbol{S}_{11}(t,\omega) & \boldsymbol{S}_{12}(t,\omega) & \cdots & \boldsymbol{S}_{1n}(t,\omega) \\ \boldsymbol{S}_{21}(t,\omega) & \boldsymbol{S}_{22}(t,\omega) & \cdots & \boldsymbol{S}_{2n}(t,\omega) \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{S}_{n1}(t,\omega) & \boldsymbol{S}_{n2}(t,\omega) & \cdots & \boldsymbol{S}_{nn}(t,\omega) \end{bmatrix} \tag{1}$$

where n denotes the number of support nodes during the seismic action. Diagonal element $\boldsymbol{S}_{ii}(t,\omega)$ $(i=1,2,\cdots,n)$ is the evolutionary power spectral density matrix upon three components at the same support node, while non diagonal element $\boldsymbol{S}_{ij}(t,\omega)$ $(j\neq i)$ is the cross evolutionary power spectral density matrix upon three components at different support nodes.

Meanwhile, $\boldsymbol{S}_{ii}(t,\omega)$ and $\boldsymbol{S}_{ij}(t,\omega)$ can be unified as follows:

$$\boldsymbol{S}_{ij}(t,\omega)=\begin{bmatrix} S_{ix,jx}(t,\omega) & S_{ix,jy}(t,\omega) & S_{ix,jz}(t,\omega) \\ S_{iy,jx}(t,\omega) & S_{iy,jy}(t,\omega) & S_{iy,jz}(t,\omega) \\ S_{iz,jx}(t,\omega) & S_{iz,jy}(t,\omega) & S_{iz,jz}(t,\omega) \end{bmatrix},\ i,j=1,2,\cdots,n \tag{2}$$

where the subscripts x and y represent the two components in the horizontal direction, and z represents the vertical component, respectively. The diagonal elements are the cross-evolutionary power spectrum of the i-th and j-th support nodes in the three directions, while the non-diagonal elements are the cross-evolutionary power spectrum of the i-th and j-th support nodes in different directions.

For the simulation purpose, this paper adopts the cross-power spectrum model of multi-dimensional ground motion proposed by Japanese scholar Matsushima in 1974 [18]:

$$S_{ix,jx}(t,\omega)=S_{iy,jy}(t,\omega)=\gamma_{ij}\sqrt{S_{ix}(t,\omega)\cdot S_{jx}(t,\omega)} \tag{3a}$$

$$S_{iz,jz}(t,\omega)=\gamma_{ij}\sqrt{S_{iz}(t,\omega)\cdot S_{jz}(t,\omega)} \tag{3b}$$

$$S_{ix,jy}(t,\omega)=S_{jy,ix}(t,\omega)=\gamma_{ij}\sqrt{S_{ix}(t,\omega)\cdot S_{jy}(t,\omega)} \tag{3c}$$

$$S_{ix,jz}(t,\omega)=S_{jz,ix}(t,\omega)=S_{iy,jz}(t,\omega)=S_{jz,iy}(t,\omega)=0.6\sqrt{S_{ix}(t,\omega)\cdot S_{jz}(t,\omega)} \tag{3d}$$

Furthermore, the evolutionary power spectral density matrix of multi-support and multi-component fully non-stationary stochastic ground motion field can be decomposed into the following form [19]:

$$\boldsymbol{S}(t,\omega)=\boldsymbol{D}(t,\omega)\boldsymbol{\gamma}(\omega)\boldsymbol{D}^{\mathrm{T}}(t,\omega) \tag{4}$$

where the diagonal matrix $\boldsymbol{D}_i(t,\omega)=\mathrm{diag}\left[\sqrt{S_{ix}(t,\omega)},\sqrt{S_{iy}(t,\omega)},\sqrt{S_{iz}(t,\omega)}\right]$ $(i=1,2,\cdots,n)$.

In Eq. (4), $\boldsymbol{\gamma}(\omega)$ denoets the coherenence function matrix, given by:

$$\boldsymbol{\gamma}(\omega)=\begin{bmatrix} \boldsymbol{\gamma}_{11}(\omega) & \boldsymbol{\gamma}_{12}(\omega) & \cdots & \boldsymbol{\gamma}_{1n}(\omega) \\ \boldsymbol{\gamma}_{21}(\omega) & \boldsymbol{\gamma}_{22}(\omega) & \cdots & \boldsymbol{\gamma}_{2n}(\omega) \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{\gamma}_{n1}(\omega) & \boldsymbol{\gamma}_{n2}(\omega) & \cdots & \boldsymbol{\gamma}_{nn}(\omega) \end{bmatrix} \tag{5}$$

where $\boldsymbol{\gamma}_{ij}(\omega)$ can be espressed as below:

$$\boldsymbol{\gamma}_{ij}(\omega)=\begin{bmatrix} \gamma_{ix,jx}(\omega) & \gamma_{ix,jy}(\omega) & \gamma_{ix,jz}(\omega) \\ \gamma_{iy,jx}(\omega) & \gamma_{iy,jy}(\omega) & \gamma_{iy,jz}(\omega) \\ \gamma_{iz,jx}(\omega) & \gamma_{iz,jy}(\omega) & \gamma_{iz,jz}(\omega) \end{bmatrix},\ i,j=1,2,\cdots,n \tag{6}$$

3 Dimension-reduction representation of multi-support and multi-component fully non-stationary ground motion stochastic fields

Generally, the coherence function matrix $\boldsymbol{\gamma}(\omega)$ is a nonnegative definite Hermitian matrix, which can be eigen decomposed as follows:

$$\boldsymbol{\gamma}(\omega)=\boldsymbol{\Psi}(\omega)\boldsymbol{\Lambda}(\omega)\boldsymbol{\Psi}^{*\mathrm{T}}(\omega),\ \boldsymbol{\Psi}^{*\mathrm{T}}(\omega)\boldsymbol{\Psi}(\omega)=\boldsymbol{I} \tag{7}$$

where $\boldsymbol{\Psi}(\omega)=\left[\boldsymbol{\psi}_1(\omega),\boldsymbol{\psi}_2(\omega),\cdots,\boldsymbol{\psi}_M(\omega)\right]$ denotes eigenvector matrix, in which $\boldsymbol{\psi}_r(\omega)=\left[\phi_{1r}(\omega),\phi_{2r}(\omega),\cdots,\phi_{Mr}(\omega)\right]^{\mathrm{T}}$ is the r-th eigenvector of the coherence function matrix $\boldsymbol{\gamma}(\omega)$. $\boldsymbol{\Lambda}(\omega)=\mathrm{diag}\left[\lambda_1(\omega),\lambda_2(\omega),\cdots,\lambda_M(\omega)\right]$ denotes eigenvalue diagonal matrix. $\boldsymbol{I}$ is $M\times M$ -order unit matrix. In general, element $\phi_{mr}(\omega)$ is a complex function of ω, thus it can be defined that so $\phi_{mr}(\omega)=\chi_{mr}(\omega)+\mathrm{i}\kappa_{mr}(\omega)$.

Furthermore, if the coherence function matrix $\boldsymbol{\gamma}(\omega)$ is a positive definite Hermitian matrix, it can be decomposed by Cholesky methology:

$$\boldsymbol{\gamma}(\omega)=\boldsymbol{B}(\omega)\boldsymbol{B}^{*\mathrm{T}}(\omega) \tag{8}$$

where $\boldsymbol{B}(\omega)$ is a lower triangular matrix, given by:

$$\boldsymbol{B}(\omega)=\begin{bmatrix} B_{11}(\omega) & 0 & \cdots & 0 \\ B_{21}(\omega) & B_{22}(\omega) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ B_{M1}(\omega) & B_{M2}(\omega) & \cdots & B_{MM}(\omega) \end{bmatrix} \tag{9}$$

Here, the elements in Eq. (9) can also be defined as $B_{mr}(\omega)=\rho_{mr}(\omega)+\mathrm{i}\eta_{mr}(\omega)\ \ (m=1,2,\cdots,M)$.

Based on the above-mentioned eigen decomposition and Cholesky decomposition of the coherence function matrix $\boldsymbol{\gamma}(\omega)$, a unified spectral decomposition expression of the multi-support and multi-component fully non-stationary stochastic ground motion field can be obtained:

$$\begin{aligned} X_{iv}(t)=2\sum_{r=1}^{M}\sum_{k=1}^{N}\sqrt{S_{iv}(t,\omega_k)\Delta\omega}G_r(\omega_k)\cdot\big[&C_{mr}(\omega_k)\left(R_{rk}\cos\omega_k t-I_{rk}\sin\omega_k t\right) \\ &-D_{mr}(\omega_k)\left(R_{rk}\sin\omega_k t+I_{rk}\cos\omega_k t\right)\big] \end{aligned} \tag{10}$$

where $X_{iv}(t)$ and $S_{iv}(t,\omega_k)$ $(i=1,2,\cdots,n\ ;\ v=x,y,z)$ denote seismic acceleration component and evolutionary power spectral density function, respectively.

In Eq. (10), R_{rk} and I_{rk} $(r=1,2,\cdots,M\ ;\ k=1,2,\cdots,N)$ are two orthogonal random variables with zero mean, satisfying the basic conditions as follows:

$$E\left[R_{rk}\right]=E\left[I_{rk}\right]=0,\ E\left[R_{rk}I_{sl}\right]=0,\ E\left[R_{rk}R_{sl}\right]=E\left[I_{rk}I_{sl}\right]=\frac{1}{2}\delta_{rs}\delta_{kl} \tag{11}$$

Obviously, in case that $G_r(\omega_k)=\sqrt{\lambda_r(\omega_k)}$, $C_{mr}(\omega_k)=\chi_{mr}(\omega_k)$ and $D_{mr}(\omega_k)=\kappa_{mr}(\omega_k)$, Eq. (10) is regarded as the POD simulation function based on the orthogonal random variables. On the other hand, in case that $G_r(\omega_k)=1$, $C_{mr}(\omega_k)=\rho_{mr}(\omega_k)$ and $D_{mr}(\omega_k)=\eta_{mr}(\omega_k)$, Eq. (10) is regarded as the SRM simulation function based on the orthogonal random variables. Therefore, Eq. (10) is referred as the unified spectral decomposition expression based on orthogonal random variables.

However, the conventional MC scheme employed to implement the realization of random variables involved in Eq. (10) still faces two main challenges cased by the high-dimensional random variables (usually as high as millions). To this end, the idea of random function proposed by Liu et al. [13-15] is introduced herein to effectively reduce the number of random variables and perform the dimension-reduction representation of multi-support and multi-component fully non-stationary stochastic ground motion fields. Specifically, the random function form discussed in the present paper is defined as follows:

$$\begin{cases} \bar{R}_{sl} = \cos(s \times \Theta_1 + l \times \Theta_2) \\ \bar{I}_{sl} = \sqrt{2}\sin\left(s \times \Theta_1\right) \times \cos\left(l \times \Theta_2\right) \end{cases}, \; s = 1, 2, \cdots, M \;;\; l = 1, 2, \cdots, N \tag{12}$$

where the elementary random variables Θ_1 and Θ_2 are mutually independent and subjected to uniform distribution over the intervall $(0, 2\pi)$.

It is a handy work to demonstrate that the constructed random function, i.e., Eq. (12), can completely satisfy the basic conditions defined in Eq. (11). Thus, the multi-support and multi-component fully non-stationary stochastic ground motion field can be represented by merely two elementary random variables with the aid of the random functions, which effectively bypasses the challenges faced by the MC scheme.

4. Numerical investigations of multi-support and multi-component fully non-stationary stochastic ground motion fields

In the present paper, the following evolutionary power spectral density function [20] is adopted, given by:

$$S_{iv}(t,\omega) = \left[\varphi_{iv}(t)\right]^2 \bar{S}_{iv}(t,\omega) \tag{13}$$

where $\varphi_{iv}(t)$ denotes the intensity modulation function of the *v*-component at the *i*-th support node, and the four-segment intensity modulation function proposed by Li et al. [21] is adopted herein to sufficiently considering the intensity differences among different seismic components. $\bar{S}_{iv}(t,\omega)$ presents the time-varing power spectrum of the *v*-component at the *i*-th support node, and the Clough-Penzien time-varing power spectrum proposed by Deodatis [5] is employed, given by:

$$\bar{S}_{iv}(t,\omega) = \frac{\omega_{\mathrm{g},iv}^4(t) + 4\xi_{\mathrm{g},iv}^2(t)\omega_{\mathrm{g},iv}^2(t)\omega^2}{\left[\omega^2 - \omega_{\mathrm{g},iv}^2(t)\right]^2 + 4\xi_{\mathrm{g},iv}^2(t)\omega_{\mathrm{g},iv}^2(t)\omega^2} \times \frac{\omega^4}{\left[\omega^2 - \omega_{\mathrm{f},iv}^2(t)\right]^2 + 4\xi_{\mathrm{f},iv}^2(t)\omega_{\mathrm{f},iv}^2(t)\omega^2} \times S_{0,iv}(t) \tag{14}$$

where

$$\omega_{\mathrm{g},iv}(t) = \omega_{0,iv} - d_i \times (t/T)^2, \; \xi_{\mathrm{g},iv}(t) = \xi_{0,iv} + e_i \times (t/T)^2 \tag{15a}$$

$$\omega_{0,iv} = \bar{\omega}_{\mathrm{g},iv} + d_i / 2, \; \xi_{0,iv} = \bar{\xi}_{\mathrm{g},iv} - e_i / 2 \tag{15b}$$

$$\omega_{\mathrm{f},iv}(t) = 0.1\omega_{\mathrm{g},iv}(t), \; \xi_{\mathrm{f},iv}(t) = \xi_{\mathrm{g},iv}(t) \tag{15c}$$

in which T is the duration of earthguake, $\bar{\omega}_{\mathrm{g},iv}$, $\bar{\xi}_{\mathrm{g},iv}$, d_i and e_i are earthquake parameters, respectively. $S_{0,iv}(t)$ is the spectral intensity parameter, expressed by:

$$S_{0,iv}(t) = \frac{\bar{a}_{\max}^2}{r_{iv}^2 \left[\pi\omega_{\mathrm{g},iv}(t)\left(2\xi_{\mathrm{g},iv}(t) + \frac{1}{2\xi_{\mathrm{g},iv}(t)}\right)\right]} \tag{16}$$

where $\bar{a}_{\max}$ is the mean value of peak ground motion acceleration (PGA), and r_{iv} denotes the equivalent peak factor of the *v*-component at the *i*-th support node. For the purpose of reflecting the differences among different seismic components, the relationship of the parameters for the three components are defined as follows [22]:

$$\bar{\omega}_{g,iz} = 1.58\bar{\omega}_{g,iy} = 1.58\bar{\omega}_{g,ix} \tag{17a}$$

$$\bar{\xi}_{g,iz} = \bar{\xi}_{g,iy} = \bar{\xi}_{g,ix} \tag{17b}$$

$$r_{iz} = 1.5r_{iy} = 1.5r_{ix} \tag{17c}$$

This paper adopts a composite coherence function model that can comprehensively consider the lagged coherence effect, wave-passage effect, and local site effect, to describe the spatial variability of the ground motion field, such that:

$$\gamma_{ij}(\omega) = \left|\gamma_{ij}(\omega)\right| \exp\left\{ \mathrm{i}\left[\theta_{ij}^{\text{w-p}}(\omega) + \theta_{ij}^{\text{s-r}}(\omega)\right]\right\} \tag{18}$$

where $\left|\gamma_{ij}(\omega)\right|$ denotes the lagged coherence effect. Phase angles $\theta_{ij}^{\text{w-p}}(\omega)$ and $\theta_{ij}^{\text{s-r}}(\omega)$ denote the wave-passage effect, and local site effect, respectively.

Fig.1 shows the representative time-histories of a multi-support and multi-component fully non-stationary stochastic ground motion field generated by the proposed dimension-reduction method. It can be seen from the figure that due to the ground motion components in the two horizontal directions adopt the same time-varying power spectrum model and corresponding parameters, the shapes of the ground motion acceleration time-history curves in the two horizontal directions are basically the same. At the same time, since the time-varying power spectrum model parameters of the vertical ground motion component and the difference between the intensity modulation function and the horizontal ground motion component are taken into account, the generated vertical ground motion acceleration process has significant differences from the horizontal ground motion component in amplitude, the time when the peak value of the intensity reaches, and the duration of the plateau. What's more, the duration of the segment in terms of the vertical ground motion acceleration process is significantly different from the two horizontal ground motion components.

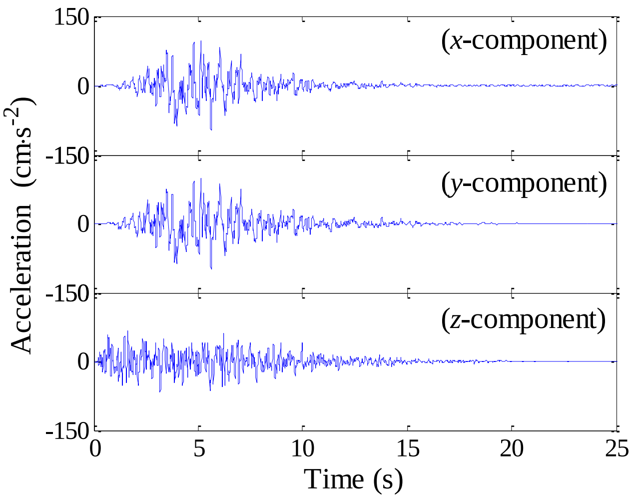

Fig. 1 Representive time-histories for the ground motions at one typical soil site

Fig.2 presents the comparison of the evolutionary power spectrum density of the simulated ground motion acceleration process with the corresponding target value. As shown in the figure, the comparison between the evolution power spectrum density of the representative time-history and its corresponding target value has a commendable fitting result, which fully verifies the accuracy and effectiveness of the method suggested in this paper.

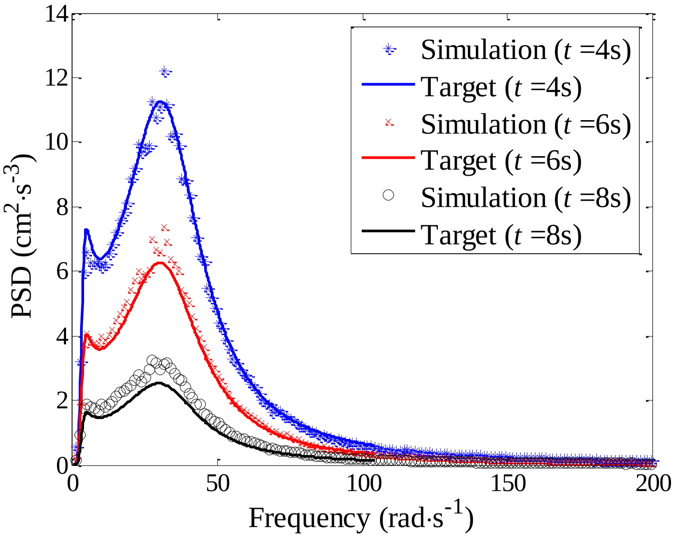

Fig. 2 Comparison of the PSD of the simulated ground motion acceleration process with the corresponding target value at three typical time instants

5. Conclusions

Based on the theory of spectral decomposition (SRM and POD) of multi-support and one-component fully non-stationary stochastic ground motion random fields, this paper derives a unified spectral decomposition representation of multi-support and multi-component fully non-stationary stochastic ground motion fields on the basis of the coherence function matrix. At the same time, by means of constructing a random function form of the orthogonal random variables, the dimension-reduction representation of a spatially correlated multi-support and multi-component non-stationary stochastic ground motion field is achieved. Research shows that by introducing a constraint form of the random function, the randomness degree (the number of elementary random variables) of a multi-support and multi-component fully non-stationary stochastic ground motion field can be effectively reduced. As a result, with merely two elementary random variables, the probability characteristics of multi-support and multi-component fully non-stationary stochastic fields can be accurately represented in the second-order statistical sense.

Numerical analysis shows that with the help of the proposed dimension-reduction representation method, just a few hundred representative samples are required to obtain the satisfactory accuracy. Meanwhile, due to the probabilistic information of the representative sample set generated by the method proposed in this paper is complete, it can be naturally combined with the PDEM, which provides an effective way for the refined seismic response analysis, seismic reliability evaluation and seismic optimization design of large-scale complex structures based on the behavior and life cycle.

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7. References

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