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A STUDY TO ESTIMATE SHAPE OF ENGINEERING BASEMENT ON A BASIS OF MODAL PROPERTIES OF SEDIMENTARY BASIN

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Abstract

In a case of constructing structures such as bridges and buildings, information of stratum profile at the construction site is indispensable. Although a boring survey is commonly conducted to obtain ground information, the cost cannot be negligible. It is significant to develop a method to estimate the detailed ground structure, particularly, the 3D shape of supporting layer without excavating the ground.

Various methods have been proposed to identify a ground structure, e.g., the Spatial Auto Correlation (SPAC) method, the Centerless Circular Array (CCA) method. Most of the methods assume horizontally layered media, and the applicability to an irregular-shaped stratum is limited. A latest technique can identify slopes of inclined bedrock, though upper boundary of it cannot obtain good estimation for local sedimentary basins where a wave tends to be trapped. As a different approach, we focused on the modal properties of sedimentary basins. Although a few studies have reported that a mode is formed within sedimentary basins, the relationship between the estimated mode shapes and the shape of the basin have not been discussed in detail.

To examine the relationship between modal properties and shape of a sedimentary basin, we observed microtremors at 19 sites along a transverse line on the sedimentary basin of 83 m transversal width (x axis) and 15 m depth (z axis), where the detailed boundary between the soft stratum and bearing stratum is known. The frequency domain decomposition (FDD) technique is applied to identify modal parameters. Several predominant frequencies and the corresponding singular vectors are identified, which are possibly the modal properties of the basin. The 3D finite element model (FE model) is also created, and eigenvalue analysis is performed using the FE model.

According to the results of FDD technique, the fundamental frequencies and the corresponding mode shapes show good agreement with those for the FE model. Notably, the identified fundamental mode shapes and the theoretical ones are almost overlapped both for transverse and for orthogonal directions. To confirm the validity of these results, a parameter study using 2-D FDM and FDD technique was performed using various two-dimensional convex shape, and the relationship between the convex shape and the modal properties is examined for both in-plane and out-of-plane directions. The results indicate that the modal characteristics obtained through the microtremor records related to the basin shape and suggest the possibility of estimating the boundary shape from the vibration mode characteristics.

Keywords: sediment-filled valley; sedimentary basin; resonance; mode shape; frequency domain decomposition

1. Introduction

The shape of an earthquake waveform is varied during its propagation from the source to observation sites. It is significantly affected by boundary shape and layered media of shallow sediments. Therefore, it is important to identify velocity structures of sediments.

A borehole survey is widely used for exploration of ground structures. It takes high cost and time to find where the ground structures change remarkably. To reduce the cost, various kind of alternative geophysical exploration techniques have been proposed by many researchers: e.g., the Spatial Auto Correlation (SPAC) method [1], the Centerless Circular Array (CCA) method [2]. Most of the methods assume horizontally layered media, and the applicability to an irregular-shaped stratum is limited. A latest technique proposed by Zhang and Morikawa [3] can identify slopes of inclined bedrock, though upper boundary of it cannot obtain good estimation for local sedimentary basins where a wave tends to be trapped. As a different approach, we focused on the modal properties of sedimentary basins. Although a few studies have reported that a mode is formed within sedimentary basins [4], the relationship between the estimated mode shapes and the shape of the basin have not been discussed in detail.

Importance of 2-D resonances in sediment-filled valley has been investigated through numerical study [4], and Elmart *et al.* [5] and Poggi *et al.* [6] indicated that 2-D resonance frequencies and mode shapes of valley can be estimated applying the frequency domain decomposition (FDD) technique to observed microtremor records.

In this study, we firstly observed the microtremor on the surface of sediment-filled valley, and subsequently estimate dominant frequencies and mode shapes through the FDD technique. Eigenvalue analysis based on the finite element model (FE model) is also conducted and the modal properties obtained through these two different techniques are compared to demonstrate the applicability of the FDD technique. Moreover, we conduct parameter studies for sedimental valley models with various boundary shapes using two-dimensional finite deferent method (2D-FDM). The relationships between the boundary shape and the modal properties obtained numerically are discussed to explore the possibility for estimating the detailed shape of supporting layer from the FDD results.

2. Microtremor observations and data processing

2.1 Geographical settings of target site

Fig.1(a) shows the target sites in Namiita, Kesennuma, Miyagi, Japan. As shown in the figure, the east side of the site is surrounded by low hills with sediments among them. The sediments are spread toward the river in front of west side. In the construction process of a new road for north-to-south direction, the soil improvement work was conducted. Through the soil improvement work, the detailed depth to bedrock was investigated at a few meter intervals along the A-A’ line of Fig. 1(a) and 2-D shape of the basement is shown in Fig. 1(b). Fig.1(c) shows the N value at the borehole survey point as indicated in Fig.1(b). Sharp velocity contrast is identified between the sediment and the bedrock through both the soil improvement work and the borehole data.

2.2 Observation of the microtremor data

Before the soil improvement work, we carried out multi-point simultaneous observations of microtremors for three cases (Cases 1 to 3) along the A-A’ line shown in Fig. 1(a), using seven force-balance-type triaxial accelerometers (Titan, by Nanometrics Inc.) and seven data loggers with 24-bit resolution and 200-Hz sampling rate for each case. The sensor arrangement is shown in Fig. 1(b), representing the case number *n* and the sensor number *m* as *n-m* in the figure. About 50-minute length of microtremors were recorded for each case during the middle of the nights from June 22nd and 23rd, 2017. Microtremor records were synchronized in the accuracy of 1 millisecond using GPS clock system.

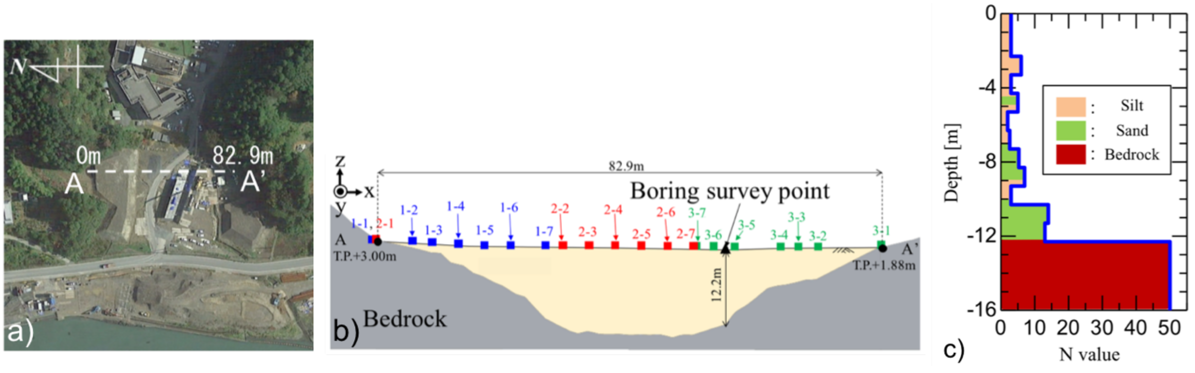

Fig. 1 – Geographical setting. (a) Target site (Google Earth). (b) Array stations and bedrock shape. (c) N value at the boring survey point.

2.3 Outline of FDD technique

The FDD technique [7] is one of methods to identify modal properties. It uses the singular value decomposition (SVD) of the power spectral density (PSD) matrix among all receivers, assuming that a system is linear, input is white noise, and the object structure is lightly damping. Under these assumptions, the eigenvalues of the PSD matrix are approximately replaced by auto-PSD functions of single degree of freedom systems. Thus, the largest eigenvalues of the PSD matrix for all the frequencies give peaks at resonance frequencies.

The cross-PSD matrix $[G_{yy}]$ is expressed as follows:

$$[G_{yy}] = \begin{bmatrix} < y_1^* y_1 > & \cdots & < y_1^* y_n > \\ \vdots & \ddots & \vdots \\ < y_n^* y_1 > & \cdots & < y_n^* y_n > \end{bmatrix}, \tag{1}$$

where [ ] stands for the $n \times n$ matrix (n for the number of receivers), y for output in the frequency domain, $< >$ for the ensemble average, superscript * for the complex conjugate. Under the assumptions, $[G_{yy}]$ can be written as

$$[G_{yy}]|_{\omega=\omega_k} \approx \alpha_k \{\phi_k^*\}\{\phi_k\}^T, \tag{2}$$

in which { } represents the $n \times 1$ column vector, ω_k the k-th resonance circular frequency, $\{\phi_k\}$ the k-th mode vector of the system, α_k its corresponding coefficient, superscript T indicates transpose. On the other hand, $[G_{yy}]$ is a Hermitian matrix and it can be decomposed through SVD as follows:

$$[G_{yy}] = \sum_{i=1}^{n} s_i \{u_i^*\}\{u_i\}^T, \tag{3}$$

where s_i is the i-th singular value and $\{u_i\}$ is the i-th singular vector. From Eqs. (2) and (3), α_k and $\{\phi_k\}$ can be determined from s_i and $\{u_i\}$, respectively [7].

2.4 Combining local mode shapes

Simultaneous observation covering the whole of the target site with enough spatial resolution is desirable to estimate mode shapes of the site. However, the number of sensors for the observation is generally limited. The practical way to estimate detailed mode shapes of the whole target site is to repeat observations several times dividing the site into a few parts, and combining mode shapes identified for each part. We obtained mode shapes at three different parts (Cases 1 to 3) and combined them by adjusting the absolute values and phase angles to be equivalent at the adjacent or same sites; Stations 1-7 (x=27.9 m) and 2-2 (x=30.7 m), and Stations 2-7 and 3-7 (x=51.2 m).

3. Modal analysis using observed microtremors and FE model

3.1 Frequency domain decomposition (FDD) analysis

The observed records of microtremors are processed as follows: the time series of horizontal components is divided into short blocks of 20-second length, and the Fourier spectra of each block were calculated with 0.2-20 Hz of band pass filter and smoothed using the Hanning window three times. The cross-PSD matrix is calculated the ensemble average of auto- and cross-spectra among the stations by averaging over the different time blocks. Subsequently, the SVD is carried out for the cross-PSD matrix and the predominant frequencies are identified by peak picking. Here, we make the cross-PSD matrices for two directions separately using the records for each horizontal direction.

The upper portions in Fig. 2 show the first singular values for each direction. Although the FDD technique requires to pick up the peaks related to the modes (of the target system) from the first singular value spectrum (hereafter, $s_1(f)$), the peaks of $s_1(f)$ are unapparent. Such unapparent peaks are possibly caused not only by a large number of degrees of freedom of a target system but by the non-white noise inputs. We, thus have to avoid to select such the peaks which relate to non-white inputs. Supposing that the free-field motions on the bedrock outcrop approximate the input motions. We focus on the Fourier spectrum (hereafter, $F_b(f)$) at the bedrock Station1-1 for Case 1, 2-1for Case2, 3-1 for Case3. $F_b(f)$ is show in the lower portions of Fig. 2. There are two types of peaks in Fig. 2: one is peaks found both in $s_1(f)$ and in $F_b(f)$, indicated by blue vertical lines, and the other is ones found only in $s_1(f)$, indicated by red lines. Considering the former type of peaks reflects characteristics of non-white inputs, that is motions at bedrock, we select frequencies of 3.81 Hz in the x direction and 3.76 Hz in the y direction as the fundamental frequencies identified through the FDD technique. Consequently, the first singular vectors for the x- and y-directions (hereafter, $u_{x1}(f)$ and $u_{y1}(f)$) at the frequencies of 3.81 Hz and 3.76 Hz are identified as the representative mode shapes for the x- and y-directions, respectively. The frequencies for k-direction, hereafter, are represented as f_{FDDk}, where k is x or y.

However, this way of selecting frequencies still does not seem to be reliable due to the complexity of $s_1(\omega)$. To confirm the reliability of the FDD results, we employ other numerical-based analysis and perform eigenvalue analysis in the next section. The comparison between the singular vector and the mode shapes is discussed later.

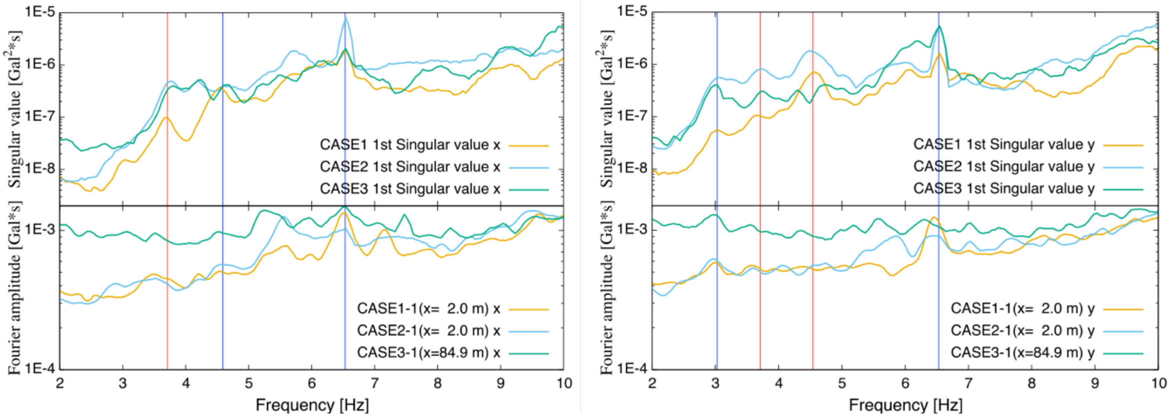

Fig. 2 – The results of FDD analysis. The upper portion shows singular value spectra obtained from microtremor data. The left and right panels show x and y direction, respectively. The lower portion shows Fourier spectra at the bedrock outcrop.

3.2 Finite element modeling

The target site is modeled by three-dimensional (3D) FE model as shown in Fig. 3, using finite element structural analysis system FINAS/STAR (ITOCHU Techno-Solutions Corp.). The boundary shape between the sediment and the bedrock is determined based on the detailed depth shown in Fig. 1(b). The model parameters are determined based on the boring survey (Fig.1(b)) and laboratory measurement of shear wave velocity test of sample peaces. The shear wave velocity V_S and the unit weight γ are 0.16 km/s and 18 kN/m^3 for sedimentary layer, and 2.8 km/s and 25 kN/m^3 for bedrock, respectively. Poisson's ratio ν is assumed as 0.33 for both layers. The mesh size for the FE analysis is determined as less than 1 m to give assurance of vibration up to 30 Hz. The bottom and the side are fixed boundary in the x-direction, and the surface and the side boundaries are free in the y-direction.

Table 1 lists the eigenfrequencies and the participation factors, and Fig.4 shows the mode shapes of the FE model. According to Table 1, six modes exist within the narrow frequency range from 3.48 to 6.29 Hz. This means that $s_1(\omega)$ has many peaks within limited frequency range. Such the jammed peaks make it difficult to identify the eigenmodes through the FDD techniques. On the other hand, the participation factor is larger for the fundamental mode than for the higher mode. It indicates that modal estimation for the fundamental mode might be easier than that for the higher mode in this system. Hereafter, the fundamental frequency in the x- and y-directions are denoted as f_{FEx} and f_{FEy}, respectively. It is noted that, the participation factor of the fundamental mode for the z-direction is also large. This means that we should employ the observation records not only of the horizontal directions but of the vertical direction. The analysis for z-direction is left for future works.

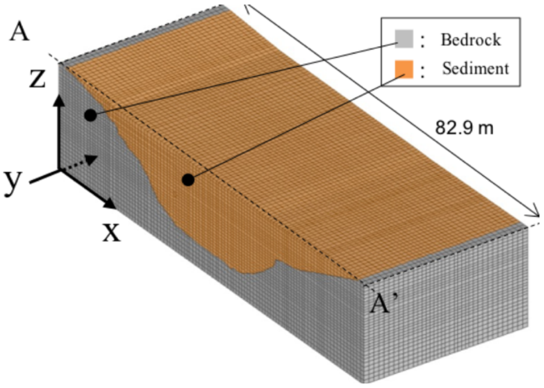

Fig. 3 – The FE model

Table 1 – Result of eigenvalue analysis using the FE model

Frequency (Hz)	Participation factor		
	x	y	z
3.48	0.000	1.637	0.000
4.19	1.404	0.000	-0.006
5.72	0.008	0.000	1.580
6.24	0.230	0.000	0.041
6.26	0.000	0.698	0.000
6.29	0.016	0.000	0.502

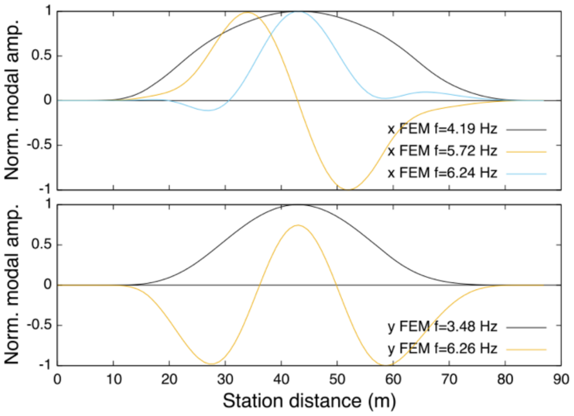

Fig. 4 – Mode shapes of the FE model

3.3 Comparison of results from observation and the FE analysis

The mode shapes of the fundamental mode for x- and y-directions are apparently different as shown in Fig. 4: the convex shape for y-direction is sharper than that for x-direction. We focus on the difference of the convex shapes for the fundamental mode in this section, and compare modal properties obtained through two methods.

The upper portion of Fig. 5 shows the analytical fundamental mode shapes for the x- and y-directions shown in Fig. 4 (hereafter, φ_{x1} and φ_{y1}), and the singular vectors $u_{x1}(f)$ and $u_{y1}(f)$ obtained through the microtremor observation. The bedrock shape is shown in the lower portion of Fig. 5. In this figure, the singular vectors are drawn using real values of the vector elements.

The frequencies of the fundamental mode are listed in the legends of Fig. 5, and the frequencies f_{FDDx} and f_{FDDy} do not coincide with f_{FEx} and f_{FEy}, respectively. Their differences, however, are less than 10 %, namely, f_{FEx}/f_{FDDx} = 0.91, and f_{FEy} / f_{FDDy} = 1.08. Although the detailed shapes of $u_{x1}(f)$ and $u_{y1}(f)$ are not as smooth as the analytical mode shapes, the outlines of them agree with φ_{x1} and φ_{y1}. These results reveal that the FDD technique can be applied possibly to sedimental valley with high velocity contrast as a modal identification method.

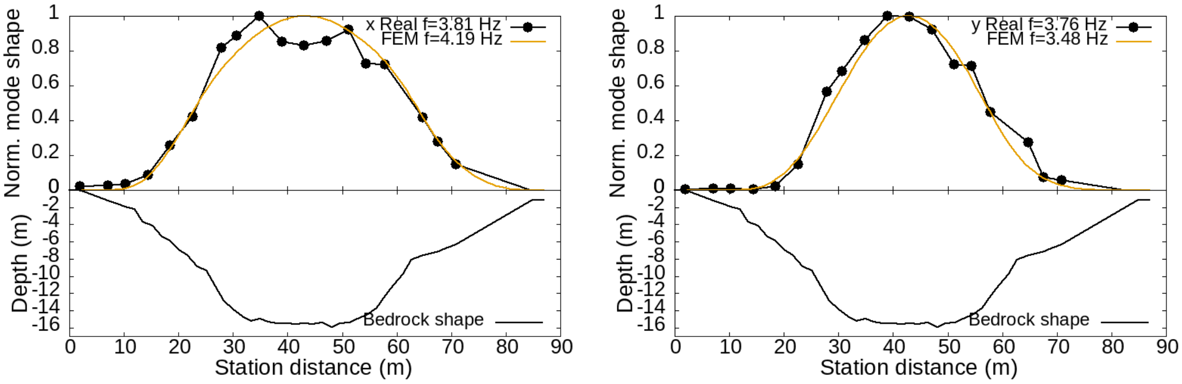

Fig.5 – Comparison of estimated modal shapes between FDD and FE analysis. The upper portion shows mode shapes (The left and right panels show x- and y-direction, respectively). The lower portion is the bedrock shape shown in Fig. 1(b).

4. Parameter study using 2-D finite-difference method

It is still a task to estimate the bedrock shape using the identified modal characteristics. To explore the possibility, the relationship between the bedrock shape and the modal characteristics are investigated through numerical study using the two-dimensional finite difference method (2D-FDM).

The detailed relationship between the bedrock shape and the fundamental mode shape on artificial sedimental valleys is still unclear. Considering the apparent difference of the fundamental mode shapes between for x- and y-direction, that is, in-plane and out-of-plane directions, we employ the two-dimensional finite-difference method (2-D FDM) for both SH (out-of-plane) and P-SV (in-plane) waves with the second order staggered-grid scheme. Resonance modes are obtained through the FDD technique, which is applied to motions on the surface generated by certain input motions to the bedrock.

4.1 Synthetic microtremor recordings

A target model is shown in Fig. 6 and its parameters are listed in Table 2. The model size is 200 m × 100 m with a grid interval of 0.5 m. The number of grid points corresponding to one S wave length in sedimentary layer is 23, which is larger than the minimum requirements of the spatial sampling rate [8,9]. The basin-edge is set at a distance of 60 m from the outermost boundary of the model and the basin width is constant value of 80 m at the surface. Two simple basin shapes are employed: rectangular and triangle. The maximum depth of the basin (hereafter, D_{max}) is varied from 5 m to 25 m, with 5-meter increment.

For the absorbing boundary, the perfect matched layer (PML) [10] of 10 m width is introduced. As an artificial microtremor simulation, the source time functions are generated by randomly synthesizing Ricker wavelets with a central frequency of 1 to 15 Hz and inputted into the 19 point sources located at a depth of 78 m with 10 m intervals. The receivers, at which the mode shapes are evaluated, are located on the surface with 5 m intervals, shown in Fig. 6 as the red inverted triangles. The durations of signals for each case are 16.384 sec with a time increment of 0.0002 s, and the motions on the surface are saved after resampling with interval of 0.002 s at the receivers. From the motions on the surface, the singular values and the singular vectors are obtained applying the FDD technique.

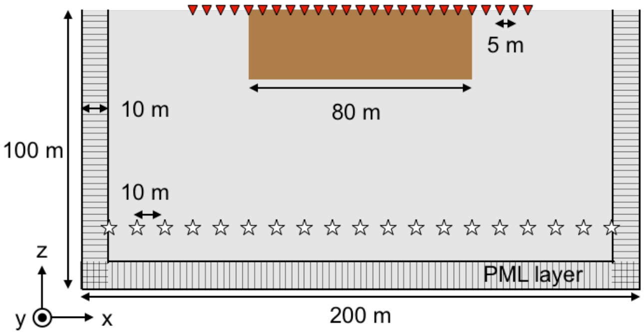

Fig. 6 – Basic model for 2-D FDM analysis. 19 sources are presented with open stars and 25 receivers with red inverted triangles.

Table 2 – Parameters for the FDM model.

	S velocity (m/s)	P velocity (m/s)	Density (kg/m^3)	Poisson ratio
Sedimentary layer	160	318	1800	0.33
Bedrock	600	1039	2500	0.25

4.2 Results of parameter study

The first singular value spectra ($s_{FDM1}(f)$) and the selected first singular vectors (u_{FDM1}) for the two models of basin are shown in Fig. 7. The selected frequencies for corresponding to the fundamental mode are shown as the black closed circles in the left panels. The frequencies for the fundamental mode are selected considering the acceptable shape of u_{FDM1} as a fundamental mode shape, and we consider them as the mode shapes. The numbers in the legend of left panels represent value of D_{max}. Although both $s_{FDM1}(f)$ and $u_{FDM1}(f)$ do not seem to be stable enough, the following aspects are considered.

Regardless of the models, the frequencies for the fundamental mode is lower as the basin is deeper. On the other hand, the mode shape is not sensitive to the maximum depth D_{max}, except for the in-plane direction of the triangle basin (Fig. 7(c)). This indicates only the frequencies corresponding to the fundamental mode is useful to estimate the depth for basin with flat bottom.

Regarding the difference in basin shape, the frequencies of the fundamental mode for the triangle basin are higher than those for the rectangular one and it is intuitive. On the other hand, the mode shapes for the triangle basin are totally sharper than those for the rectangular one regardless of directions of motions and it implies that the fundamental mode shapes reflect the basin shape.

Regarding the difference in directions of motions, the frequencies for each D_{max} for the in-plane mode are higher than those for the out-of-plane mode. This can be useful information to find the fundamental frequency from the complicated shapes of $s_{FDM1}(f)$. Comparison of the mode shapes between the in-plane and out-of-plane directions is shown in Fig. 8 for $D_{max} = 15$ m. In the case of rectangular basin, the mode shapes for both directions almost coincide. On the other hand, the apparent difference can be seen in the case of triangle basin. This characteristic supports the result obtained in Chapter 3.

These results indicate the possibility to use the modal information to estimate the bedrock shape. It is noted in particular that the difference of the modal characteristics between x- and y-direction plays an important role.

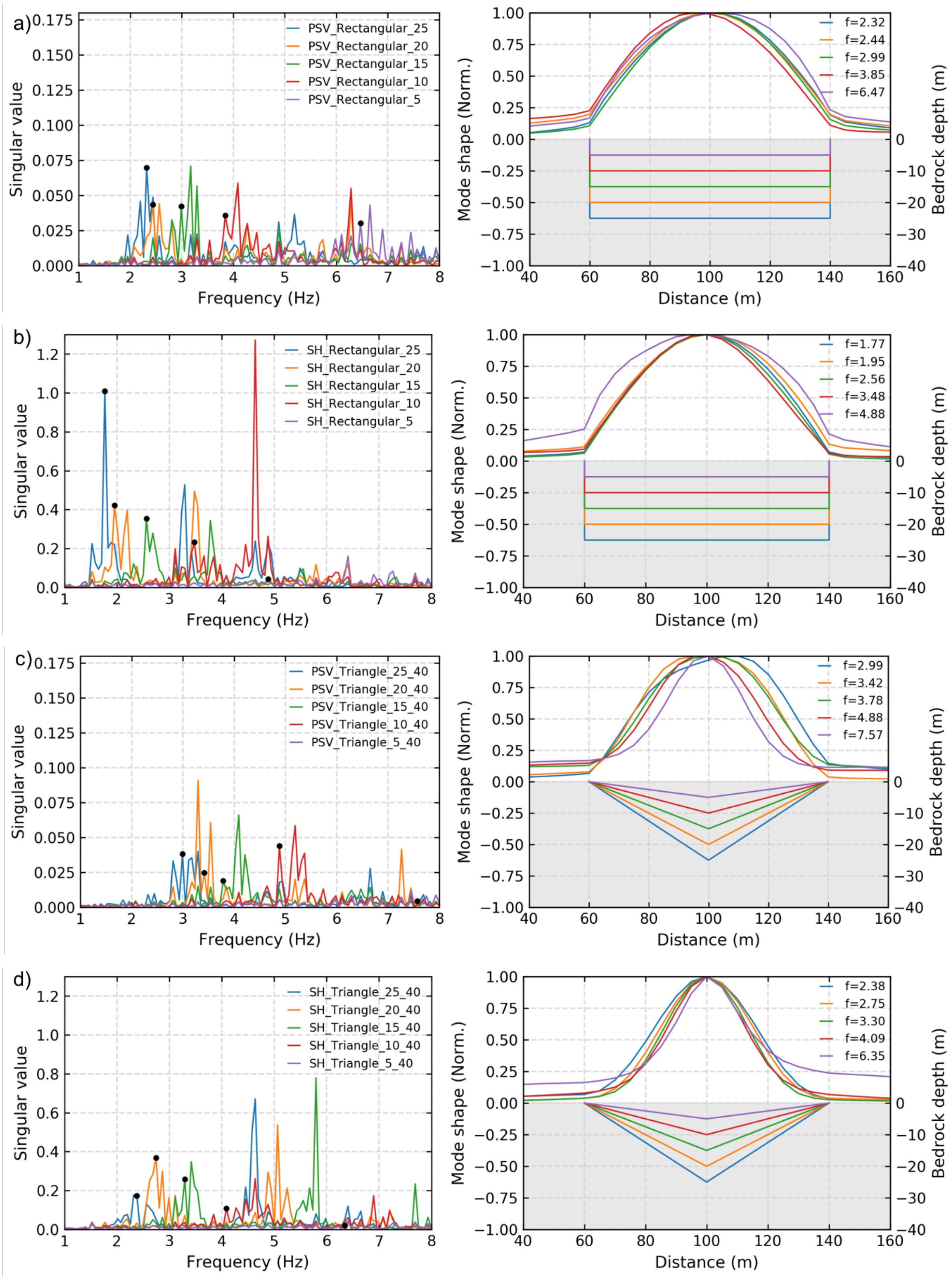

Fig. 7 – Comparison of singular value spectra (left) and mode shapes (right) for different basin shapes and depths. (a) Rectangular, in-plane, (b) Rectangular, out-of-plane, (c) Triangle, in-plane, (d) Triangle, out-of-plane. The black closed circles around the predominant peaks in singular value spectra indicate the selected fundamental modes.

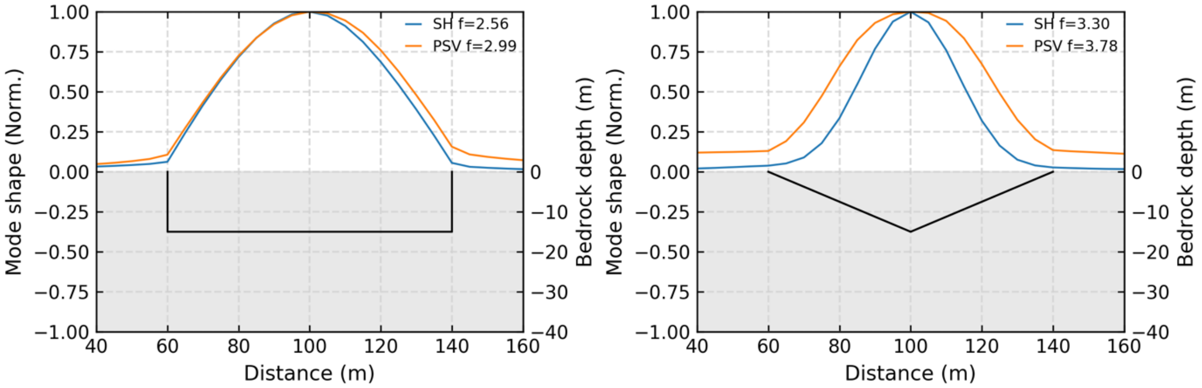

Fig. 8 – Comparison between in-plane and out-of-plane modes in the case of D_{max}=15. Left and right panels show results for rectangular and triangle basin, respectively.

5. Conclusion

This study attempted to identify the fundamental modal properties of the free-field motions on small-scale valley filled with sediment using microtremor records. We proposed the way to find the fundamental mode of the sediments using the FDD technique by focusing on the frequency property of motions at a site on the surface of bedrock outcrop, and verified the reliability of the fundamental mode identified through eigenvalue analysis using the three-dimensional finite element model.

To examine the relationship between the bedrock shape and the modal properties of motions on the surface more detail, we carried out parameter study through the numerical analysis for two models of different basin shapes with various depths to the bedrock. To simulate the microtremor field numerically, the dynamic response analysis using the two-dimensional FDM and the modal identification using the FDD technique are combined, and the fundamental mode of motions on the surface of the basin are identified for both the in-plane and out-of-plane modes. The characteristics of the fundamental mode coincide with those through the microtremor records. These results indicate the possibility to use the modal information to estimate the bedrock shape.

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