A Copula-based random function model of sequential ground motions

J. Shen(1), J. Chen(2) (3)

(1) *Ph.D. candidate, College of Civil Engineering, Tongji University, shen_tju@163.com*
(2) *Professor, College of Civil Engineering, Tongji University, cejchen@tongji.edu.cn*
(3) *Professor, College of Civil Engineering and Architecture, Xinjiang University, cejchen@tongji.edu.cn*

Abstract

This paper proposes a random function model of sequential ground motions, which uses eight physical parameters to describe the time history, and the correlation between the mainshock and aftershock is represented by Copula functions. Firstly, this paper explains the correlation between the mainshock and aftershock under the physical mechanism of 'source-path-local site'. According to the physical mechanism, based on the point source model and the uniform isotropic medium model, random function models of mainshock and aftershock are given, both with eight basic physical parameters. Secondly, Copula theory is used to build joint cumulative distribution functions of the eight parameters. These Copula functions have established the correlation of the mainshock and aftershock. Using these Copula functions, the sequential ground motion can be reproduced by six two-dimensional random variables. Finally, using 1038 pairs of sequential ground motions collected from PEER and K-NET & KiK-net, the eight physical parameters of the mainshock and aftershock in random function model have been identified and statistically analyzed. These physical parameters are identified by the Fourier amplitude spectra and the Fourier phase spectra of as-recorded time histories. The least square method and genetic algorithm are used for model parameter identification. At the same time, combined with the Copula function, the probability distributions of each two-dimensional random variable are given. Moreover, for engineering applications, the as-recorded sequential ground motions are divided into four classes based on site conditions. Correspondingly, the probability distributions are also given for four different classes. According to the random function model of sequential ground motions and the probability distributions of random variables, sequential ground motions can be generated for different site classes in a random form. Finally, the simulated sequential ground motion time histories are compared with the as-recorded time histories, and the mean acceleration response spectra for four site classes are also compared. The results show that the simulated sequential ground motions can maintain a high consistency with as-recorded ground motions from the perspective of time history and mean acceleration response spectra.

Keywords: sequential ground motions; Copula theory; Fourier spectra; random variable

1. Introduction

After decades of development, the resistance of buildings designed in accordance with current seismic codes have significantly improved when subjected to a single earthquake. However, historical data shows that after a large earthquake, buildings often experience aftershocks in a short time. Buildings damaged by the mainshock will subjected to strong aftershocks without reinforcement and repair. As a result, additional damage will be caused, leading to more severe structural damage or even collapse. The additional damage arose from aftershocks will cause serious casualties and property damage. Researchers often refer to the mainshock and its accompanying aftershocks as sequential ground motions. The most typical example of sequential ground motions is the earthquakes in New Zealand that occurred in 2010 and 2011 [1]. The mainshock caused severe damage to a small part of the city buildings, and a high number of buildings suffered slight damage or moderate damage. A few months later, a strong aftershock caused further damage to the damaged buildings, thereby leading to greater casualties and economic losses in cities than the mainshock. Therefore, it is necessary to research the sequential ground motion and put forward its time history model to ensure urban safety.

In the field of seismology, the study of sequential earthquakes can date back to 1894 when Omori [2] summarized the law of frequency attenuation of aftershocks. After more than 100 years of exploration, the research on sequential earthquakes has achieved fruitful results, including the famous Gutenberg-Richter law [3] and Bath law [4]. In general, researchers in seismology strive to truly reflect the relationship between mainshock and aftershock in the mechanism of epicenter. However, in the field of earthquake engineering, researchers pay significant attention to factors such as the amplitude, spectrum, and duration of a ground motion, which have a tremendous impact on structural response. Regarding the simulation of sequential ground motions in earthquake engineering, early attempts duplicate the mainshock (with a scaled amplitude) or randomly combine time histories of different mainshocks [5]. However, Ruiz García proposed that these methods would mispresent the structural response [6]. The commonly used methods in current research are establishing statistical relationships between the mainshock and aftershock ground motion characteristics (e.g., magnitude, PGA, PGV). After selecting a mainshock in earthquake database or generating a mainshock through a ground motion model (e.g., the Kanai–Tajimi model, evolutionary spectra method), the characteristics of aftershock are obtained according to statistical relationships. Though this method has a clear process and easy to apply, the sequential ground motion model established by this method considers actually only the randomness of mainshocks. The accompanying aftershock are uniquely determined based on a deterministic statistical relationship. Actually, the mainshock and aftershock are closely related in terms of seismic intensity, spectrum, and duration. Hence, the stochastic relationship between the mainshock and aftershock cannot be accurately expressed by a deterministic relationship. Moreover, the random model used in this method is mostly based on the assumption of stationarity, which are inconsistent with non-stationarity characteristics of ground motions. Therefore, the sequential ground motion model based on a deterministic relationship has great limitations.

In this study, Copula theory is introduced. Based on the existing physical models of ground motions, the correlation between mainshock and aftershock is investigated in the physical mechanism of earthquake. With Copula theory, a random function model of sequential ground motions for engineering is established. Then, according to the earthquake databases of PEER and K-NET & KiK-net, 1038 pairs of mainshocks and corresponding aftershocks from 14 earthquake events are used to determine the probability distributions of the 6 two-dimensional random variables and 2 one-dimensional random variables in this model. Finally, the proposed model was validated through comparisons of response spectra and time histories.

2. Copula theory

Copula theory was first proposed by Sklar M. [7], that is, any multivariate joint probability distribution can be separated into corresponding multiple marginal distributions and a Copula function, which uniquely determines the correlation between variables. After the development by many researchers, Copula theory has become an effective method to solve the problem of how to determine the joint probability distribution of high-dimensional random variables. The advantage of Copula theory is that the estimation of the variable marginal distribution function and the selection of the Copula function are performed independently. Moreover, the rank correlation coefficient describing the nonlinear relationship between the variables is introduced, which can more accurately describe the correlation of the variables.

Copula theory was first applied in the fields of finance and insurance [8], and then widely used in hydrology [9]. Relatively speaking, its application in earthquake engineering is still in its infancy. Only a few researchers have introduced Copula theory into the study of ground motion and structural seismic analysis. Goda and Salami [10] established the relationship between the maximum deformation and the residual deformation of the structure based on Copula function. Zhu and Lu [11] analyzed the correlation between the 34 intensity parameters of the mainshock and corresponding aftershock using Copula functions, and proposed a conditional mean spectrum model of the aftershocks based on Copula functions.

Copula theory provides an effective and convenient way to determine the joint probability distribution function between two or more random variables. As mentioned earlier, this theory introduces correlation coefficients that describe the correlation of variables. Among them, Pearson linear correlation coefficient, Kendall rank correlation coefficient, and Spearman rank correlation coefficient are widely used, which are expressed separately as:

$$\rho = \frac{\sum_{i=1}^{N}(x_{1i}-\bar{x}_1)(x_{2i}-\bar{x}_2)}{\sqrt{\sum_{i=1}^{N}(x_{1i}-\bar{x}_1)^2}\sqrt{\sum_{i=1}^{N}(x_{2i}-\bar{x}_2)^2}} \tag{1}$$

$$\rho_S = \frac{\sum_{i=1}^{N}(r_i-\bar{r})(s_i-\bar{s})}{\sqrt{\sum_{i=1}^{N}(r_i-\bar{r})^2}\sqrt{\sum_{i=1}^{N}(s_i-\bar{s})^2}} \tag{2}$$

$$\tau = \frac{\sum_{i<j}\mathrm{sign}[(x_{1i}-x_{1j})(x_{2i}-x_{2j})]}{0.5N(N-1)} \tag{3}$$

where x_{1i} and x_{2i} are the i th observations of two random variables; $\bar{x}_1$ and $\bar{x}_2$ are their mean values; N is the sample size; sign[·] is a sign function, where when $(x_{1i}-x_{1j})(x_{2i}-x_{2j})>0$, sign = 1, otherwise sign = 0; r_i and s_i are the ranks of the random variables, and r and s are their mean values.

In this paper, the Kendall rank correlation coefficient that can reflect the nonlinear relationship between variables is selected as a measure of the correlation between variables. According to the Kendall rank correlation coefficient and the marginal distribution of the random variables, the joint probability distribution function between variables can be established with the help of the Copula function. Taking a two-dimensional random variable as an example, its joint cumulative distribution functions (CDF) can be expressed as:

$$F(x_1,x_2)=C[F_1(x_1),F_2(x_2);\theta]=C(u_1,u_2;\theta) \tag{4}$$

where $F_1(x_1)$ and $F_2(x_2)$ are the marginal CDF of the variables x_1 and x_2; $u_1=F_1(x_1)$, $u_2=F_2(x_2)$, it can be proved that u_1 and u_2 are both obey uniform distribution with interval [0,1]; $C(u_1,u_2)$ is the Copula distribution function, it is easy to know that its variable interval is also [0,1]; θ is the relevant parameter corresponding to the Copula distribution function, which can be determined by different Copula distribution functions and Kendall rank correlation coefficients of random variables.

3. Random function model of sequential ground motions

In this section, this paper briefly review the random function model of single ground motions proposed by Wang and Li [12], and then extend it to sequential ground motions. In a continuous form, according to the physical mechanism, source-path-local site shown in Fig. 1, the Wang and Li's model defines the synthetic ground acceleration time history $a(t)$ as:

$$a(t) = -\frac{1}{2\pi}\int_{-\infty}^{+\infty} A(\boldsymbol{\lambda},\omega,R)\cdot\cos[\omega t + \Phi(\boldsymbol{\lambda},\omega,R)]\,\mathrm{d}\omega \tag{5}$$

where $\boldsymbol{\lambda}=[A_0,\tau,a,b,c,d,\xi_g,\omega_g]$ is the random vector of physical parameters, R is the epicentral distance, ω is the circular frequency, and $A(\boldsymbol{\lambda},\omega,R)$ is the Fourier amplitude spectrum model of the acceleration time history of the ground motion:

$$A(\boldsymbol{\lambda},\omega,R)=\frac{A_0\cdot\omega\cdot\mathrm{e}^{-K\omega R}}{\sqrt{\omega^2+(1/\tau)^2}}\cdot\sqrt{\frac{1+4\xi_g^{\,2}(\omega/\omega_g)^2}{[1-(\omega/\omega_g)^2]^2+4\xi_g^{\,2}(\omega/\omega_g)^2}} \tag{6}$$

$\Phi(\boldsymbol{\lambda},\omega,R)$ is the Fourier phase spectrum of the acceleration time history of ground motion:

$$\Phi(\boldsymbol{\lambda},\omega,R)=\arctan(\frac{1}{\tau\omega})-R\cdot\ln[a\omega+1000b+0.1323\sin(3.78\omega)+c\cos(d\omega)] \tag{7}$$

where A_0 is the amplitude coefficient that represents the influence of the source intensity; τ is the source coefficient in Brune's model [13], which expresses the time characteristics of the source; a, b, c, and d are the empirical parameters; ξ_g is the equivalent damping ratio and ω_g is the equivalent predominant circular frequency of the local site.

Different from commonly used ground motion models in the field of earthquake engineering, Wang's model considers not only local site, but also the whole physical mechanism of the "source-path-local site", and have the non-stationary characteristics that are in full agreement with the realistic ground motion records. In addition, the randomness characteristics of ground motion was included through several random variables, which are called physical parameters in Wang's model.

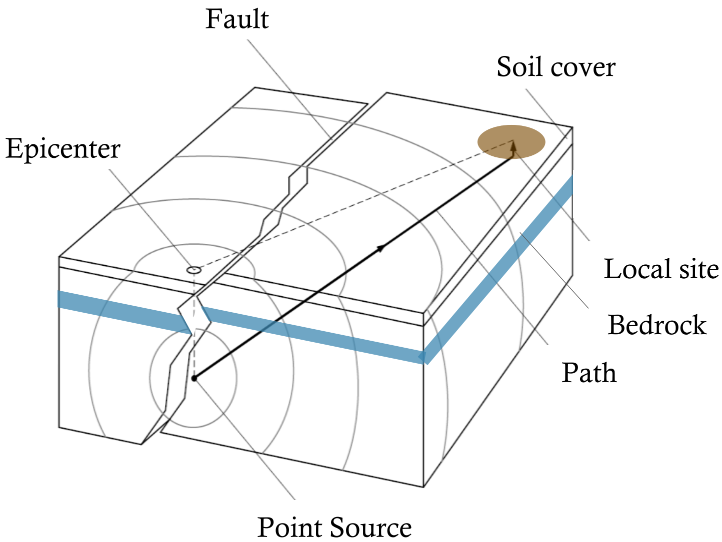


Fig. 1 – Physical mechanism of "source-path-local site"

In the view of physical mechanism of sequential ground motion, although the mainshock and aftershock show many differences at parameters, response spectra and time histories, they all follow the same mechanism of "source-path-local site". In this process, the physical factors (i.e., physical parameters in

Wang's model) which determine the ground motion cannot be fully observed or controlled. Thus, the ground motion must exhibit significant randomness whether it is mainshock or aftershock. In addition, the spatial correlation of the epicenter and propagation route of mainshock and aftershock determines the correlation of their time histories. Therefore, the key to modeling sequential ground motions is determining the correlation between the physical factors of the mainshock and aftershock.

In Eq. 7, eight physical parameters determine the acceleration time history of ground motion. In the earthquake generation and propagation process, the correlation between the spatial locations of the source and path results in correlations between the eight physical parameters of the mainshock and aftershock. Moreover, it is generally believed that the dynamic characteristics of a local site, the station of which records the mainshock and aftershock, are constant. Therefore, the physical parameters describing the dynamic characteristics of the local site (i.e., ξ_g and ω_g) are considered equal during the mainshock and aftershock. The spatial correlation of the mainshock and aftershock depends on the six physical parameters describing the source and propagation path. The acceleration time history of sequential ground motion can be expressed as follows:

$$a(t) = -\frac{1}{2\pi}\int_{-\infty}^{+\infty} A(\boldsymbol{Z},\omega,R)\cdot\cos[\omega t + \boldsymbol{\Phi}(\boldsymbol{Z},\omega,R)]\mathrm{d}\omega \tag{8}$$

where $\boldsymbol{Z} = [\boldsymbol{\lambda}_{\mathrm{M}}, \boldsymbol{\lambda}_{\mathrm{A}}]$ is the random vector of physical parameters of mainshock and aftershock, in which $\boldsymbol{\lambda}_{\mathrm{M}} = [A_{0\mathrm{M}}, \tau_{\mathrm{M}}, a_{\mathrm{M}}, b_{\mathrm{M}}, c_{\mathrm{M}}, d_{\mathrm{M}}, \xi_{\mathrm{gM}}, \omega_{\mathrm{gM}}]$ and $\boldsymbol{\lambda}_{\mathrm{A}} = [A_{0\mathrm{A}}, \tau_{\mathrm{A}}, a_{\mathrm{A}}, b_{\mathrm{A}}, c_{\mathrm{A}}, d_{\mathrm{A}}, \xi_{\mathrm{gA}}, \omega_{\mathrm{gA}}]$. As has been elaborated above, $\boldsymbol{Z}$ can be rewritten as follows:

$$\boldsymbol{Z} = [\boldsymbol{A}_0, \boldsymbol{\tau}, \boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}, \boldsymbol{d}, \xi_{\mathrm{g}}, \omega_{\mathrm{g}}] \tag{9}$$

where $\boldsymbol{A}_0$, $\boldsymbol{\tau}$, $\boldsymbol{a}$, $\boldsymbol{b}$, $\boldsymbol{c}$, and $\boldsymbol{d}$ are six two-dimensional random variables used to describe the correlation of the mainshock and aftershock. The values and the probability distributions of these variables are detailed in the subsequent section.

4. Parameter identification and fitting

Two-sequence type ground motion records (i.e., mainshock and the following biggest aftershock) from the Pacific Earthquake Engineering Research Center (PEER) and Strong-Motion Seismograph Networks (KiK-net, K-NET) were selected according to the following criteria:

1) Both the mainshock records and the corresponding aftershock records belong to the same station.

2) When there are multiple aftershocks after mainshock, the one with the largest magnitude was selected.

3) Both the mainshock and the aftershock occur in shallow crustal.

4) To avoid the effect of the soil–structure interaction, the recording station must be in a free site or on the ground floor of low-rise buildings.

5) The fault distance should be larger than 10 km to reduce the influence of near-field effect.

Based on the selection criteria, 1038 as-recorded sequential ground motions were selected from 14 earthquakes, as shown in Table 1. These ground motions are divided into four classes according to Chinese seismic code, including 130 of site Class I, 636 of site Class II, 238 of site Class III and 34 of site Class IV.

According to the random function model of sequential ground motions proposed in Section 3, the process of identifying physical parameters is shown as Fig. 2. By means of least square method and genetic algorithms, physical parameters of sequential ground motions are identified. Then, this paper investigated their statistical distributions, and used Bayesian Information Criterion to obtain the optimal probability model of each physical parameter. The corresponding probability density function (PDF) statistics are shown in Tables 2 and 3. Among them, when the PDF is Normal distribution, P_1 and P_2 are mean and standard deviation, respectively; when the PDF is Lognormal distribution, P_1 and P_2 are log mean and standard deviation, respectively; when the PDF is Weibull distribution, P_1 and P_2 are shape parameters and scale parameters, respectively.

Table 1 – Selected sequential ground motions

Earthquake events	Mainshocks			Aftershocks		
	Time	Mw	Quantity	Time	Mw	Quantity
Mammoth Lakes	1980.05.25 16:34	6.1	6	1980.05.25 16:49	5.7	6
Livermore	1980.01.24 19:00	5.8	12	1980.01.27 02:33	5.4	12
L'Aquila	2009.04.06 01:33	6.3	64	2009.04.06 17:47	5.6	64
Irpinia	1980.11.23 19:34	6.9	20	1980.11.23 19:35	6.2	20
Northridge	1994.01.17 12:31	6.7	62	1994.03.20 21:20	5.3	62
Umbria Marche	1997.09.26 09:40	6	18	1997.10.06 23:24	5.5	18
Alaska	2002.10.23 03:30	6.7	38	2002.11.03 22:12	7.9	38
East Japan	2011.03.11 14:46	9	24	2011.03.11 15:15	7.7	24
Whittier Narrows	1987.10.01 14:42	6	134	1987.10.04 10:59	5.3	134
Imperial Valley	1979.10.15 23:16	6.5	28	1979.10.15 23:19	5	28
New Zealand	2010.09.03 16:35	7	154	2011.02.21 23:51	6.2	154
Chi-Chi	1999.09.20 17:47	7.6	480	1999.09.20 17:57	5.9	480
Kumamoto	2016.04.14 21:26	6.2	8	2016.04.16 01:25	7	8

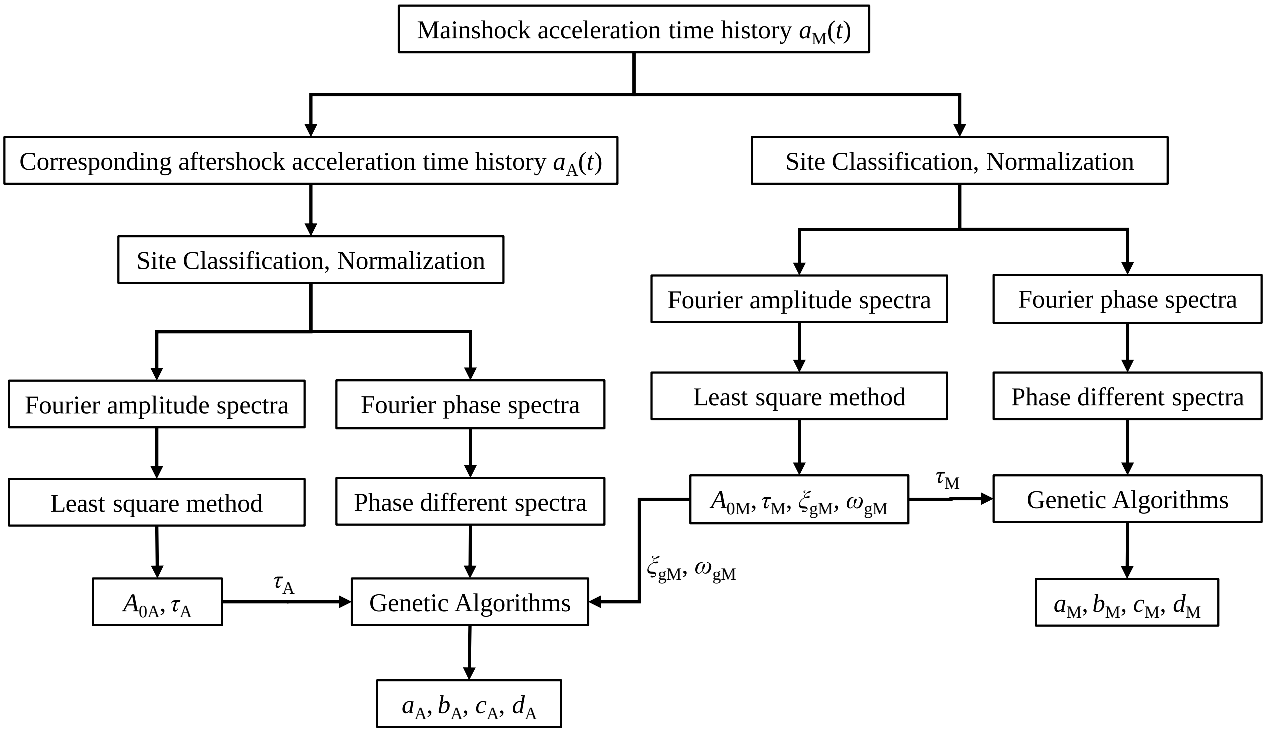


Fig. 2 – Flowchart of parameter identifications

Table 2 – PDFs of two-dimensional physical parameters

Physical parameters	Site	Mainshock			Aftershock		
		Distributions	P_1	P_2	Distributions	P_1	P_2
A_0	I	Lognormal	-3.043	1.270	Lognormal	-4.660	1.082
	II		-2.852	1.259		-4.537	1.300
	III		-1.797	0.936		-3.427	1.005
	IV		-1.606	0.805		-2.196	0.997
τ	I	Weibull	0.719	0.685	Weibull	0.669	0.479
	II		0.661	0.359		0.634	0.380
	III		0.726	0.389		0.848	0.392
	IV		0.811	0.483		0.738	0.160
a	I	Lognormal	1.886	0.550	Lognormal	2.150	0.462
	II		1.812	0.659		2.054	0.544
	III		1.461	0.721		1.834	0.643
	IV		1.046	0.739		1.774	0.638
b	I	Lognormal	2.012	0.498	Lognormal	2.312	0.458
	II		1.933	0.557		2.220	0.528
	III		1.746	0.631		1.940	0.587
	IV		1.421	0.514		1.790	0.637
c	I	Weibull	1.272	1.231	Weibull	1.512	1.582
	II		1.315	1.271		1.659	1.700
	III		1.443	1.351		1.587	1.661
	IV		1.674	1.243		1.749	1.858
d	I	Weibull	1.682	1.591	Weibull	1.729	1.782
	II		1.565	1.615		1.844	1.788
	III		1.625	1.607		1.895	1.835
	IV		1.631	1.684		1.896	2.016

Table 3 – PDFs of one-dimensional physical parameters

Physical parameters	Site	Distributions	P_1	P_2
ξ_g	I	Weibull	0.524	2.688
	II		0.484	2.709
	III		0.494	2.595
	IV		0.356	1.928
ω_g	I	Weibull	11.261	1.130
	II		12.043	1.224
	III		5.610	0.851
	IV		11.409	2.827

5. Correlations of physical parameters

As mentioned above, the key to build random function model of sequential ground motion is how to determine the correlations of the 6 two-dimensional physical parameters. This key is essentially a problem of how to determine the joint probability distribution for two-dimensional random variables. This paper considers introducing Copula theory to describe it. Total 7 commonly used Copula functions are selected as alternative functions: Independent, Gaussian, t, Gumbel, Plackett, Frank, and Clayton functions. Among them, the Independent function indicates that the parameters are independent of each other. The other six functions describe the correlation between parameters from different aspects.

Fig. 3 shows the flowchart to determine the optimal Copula functions. The relevant parameters θ can be calculated by Eq. (10):

$$\tau_K = 4\int_0^1\int_0^1 C(F_1(x_1), F_2(x_2);\theta)\,\mathrm{d}C(F_1(x_1), F_2(x_2);\theta) - 1 \tag{10}$$

where τ_K is Kendall rank correlation coefficient tau; $C(u_1, u_2;\theta)$ is alternative Copula function.

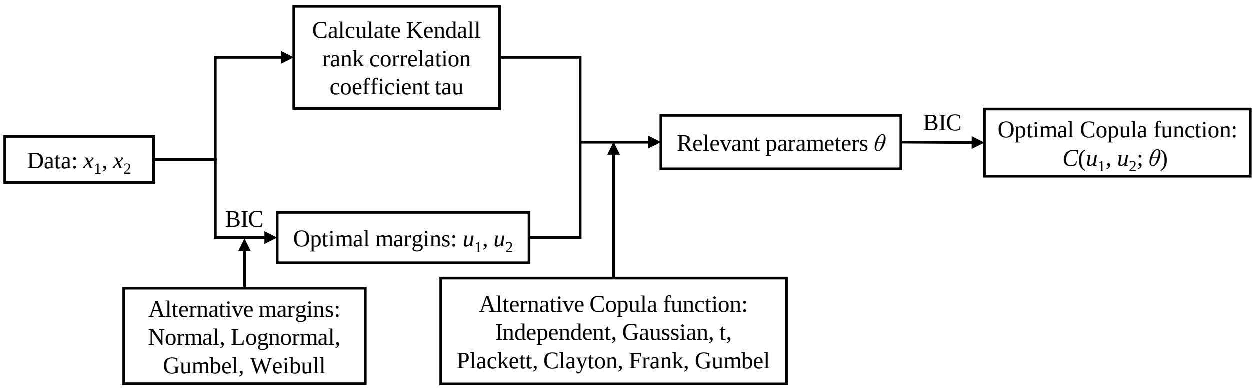


Fig. 3 – Flowchart to determine the optimal Copula functions

According to the process shown in Fig. 3, the optimal Copula function of each physical parameter is determined, and shown in Table 4. So far, a random function model of sequential ground motions and its parameter distributions have been established. The process to generate stochastic ground motions is as follows:

1) Choose corresponding parameter distributions (i.e., P_1 and P_2) and relevant parameter θ according to engineering local site;

2) Generate a group of samples of 8 physical parameters of mainshock;

3) Determine the conditional Copula functions of the 6 two-dimensional physical parameters of aftershock based on mainshock parameter samples;

4) Generate a group of samples of 6 two-dimensional parameters of aftershock, and 2 one-dimensional parameters (i.e., ξ_g and ω_g) of aftershock is equal to them of mainshock;

5) Input the two groups of samples into Eq. 8, and the mainshock and aftershock acceleration time histories is obtained by inverse Fourier transform, respectively;

6) Combine the mainshock and aftershock acceleration time histories with a time interval (such as 20 seconds), then the sequential ground motion is generated.

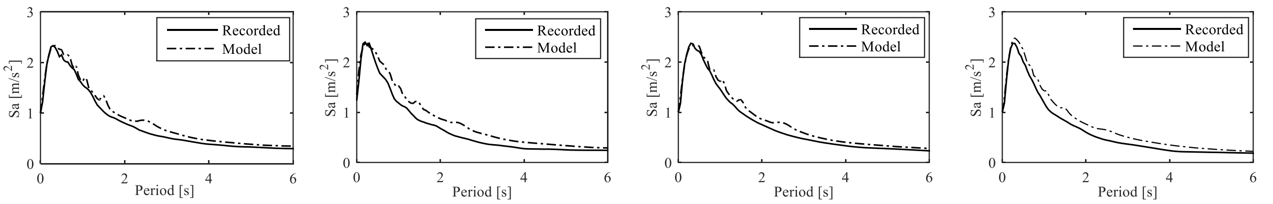
Table 4 – Optimal Copula functions of two-dimensional physical parameters

Physical parameters	**Site**	**Functions**	**Relevant parameters θ**	**Physical parameters**	**Site**	**Functions**	**Relevant parameters θ**
A_0	I	Plackett	9.896	b	I	Clayton	0.916
	II		18.808		II		1.230
	III		26.833		III		1.262
	IV		7.156		IV		1.442
τ	I	Clayton	2.214	c	I	Clayton	0.467
	II		2.601		II		0.493
	III		1.287		III		0.407
	IV		1.951		IV		1.400
a	I	Plackett	2.710	d	I	t (ν=4)	0.211
	II		3.592		II		0.255
	III		2.840		III		0.271
	IV		3.057		IV		0.659

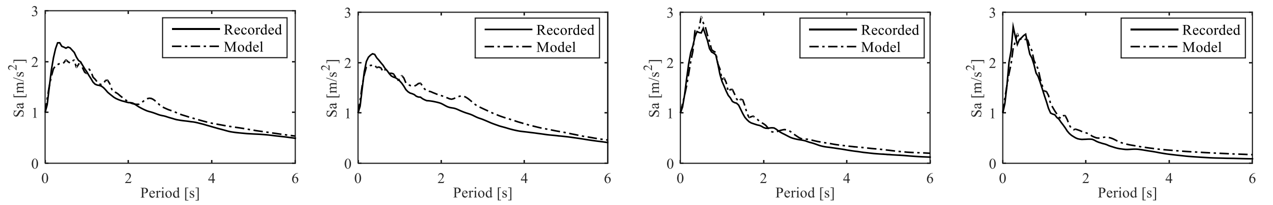
6. Model verifications

In order to verify the validity of the random function model of sequential ground motion, this paper simulates and reconstructs the recorded acceleration time histories of selected sequential ground motions. Total 1038 mainshocks and 1038 aftershocks are generated.

Fig. 4 compares the mean acceleration response spectra of the mainshocks and aftershocks under different site conditions, with a damping ratio $\xi = 0.05$. it can be seen that the model and the recorded mean acceleration response spectra are basically match, and the statistical results have high reliability.



(a) Mainshock of Site I (b) Aftershock of Site I (c) Mainshock of Site II (d) Aftershock of Site II



(e) Mainshock of Site III (f) Aftershock of Site III (g) Mainshock of Site IV (h) Aftershock of Site IV

Fig. 4 – Comparisons of mean spectra of recorded and model

Fig. 5-6 shows two simulated acceleration time histories and their corresponding recorded time histories. It should be noted that in order to ensure that the structure is stationary after the mainshock, a common method is to insert a certain time interval between the mainshock and aftershock. The comparisons show that the model proposed in this paper can maintain a high consistency with the recorded time histories. This model can well reconstruct the ground motion time histories. Moreover, the non-stationary characteristics of ground motion can also be reproduced.

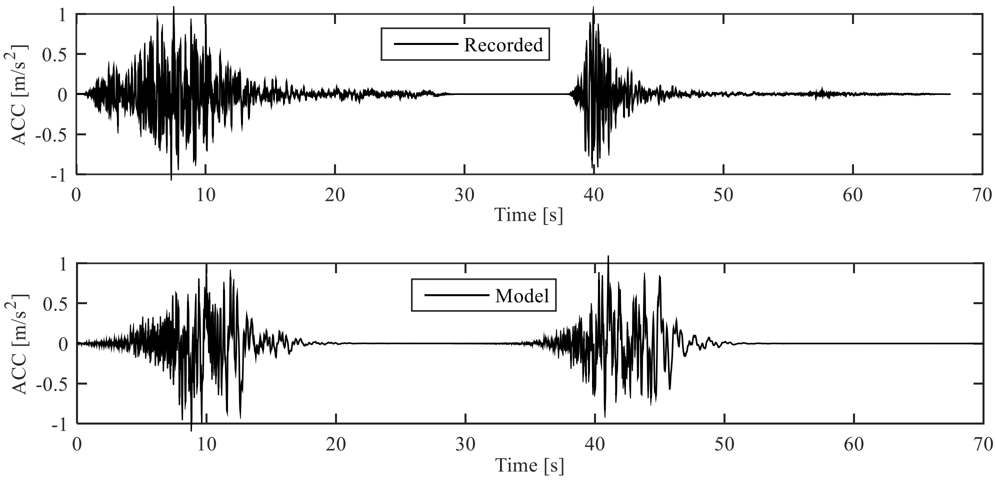


Fig. 5 – Comparisons of time histories (Mammoth Lakes MLS-N)

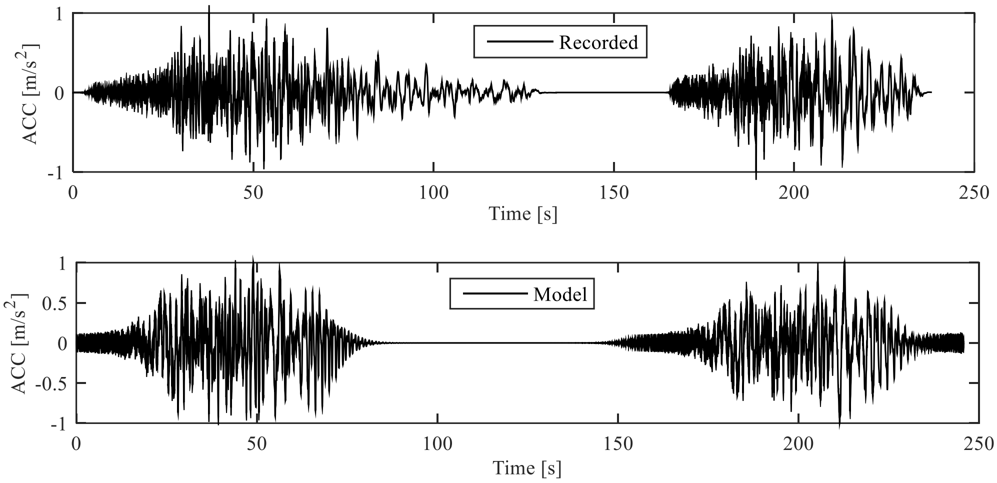


Fig. 6 – Comparisons of time histories (Chi-Chi KAU088-N)

6. Conclusion

This paper has put forward a random function model of sequential ground motions based on Copula theory. With the help of Copula theory, the spatial correlation between mainshock and aftershock are illustrated. By means of Least Square Method and Genetic Algorithms, physical parameters of each sequential ground motion are identified and fitted with 1038 pairs of as-recorded sequential ground motions. Finally, the simulated ground motion is compared with the measured ground motion and response spectra, which verifies the feasibility of this paper's model. Actually, the process of generating and propagating of ground motions is extremely complicated. The model proposed in this paper still lacks in describing the phase of ground motion, that need to be further studied.

7. Acknowledgements

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8. Copyrights

Full version of this conference paper including more detailed research results have been submitted to the journal of *Structural Safety*.

9. References

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