NON-LINEAR AND NON-STATIONARY SEISMIC RESPONSE IN OAXACA CITY, MEXICO DURING THE 20170908 (Mw=8.2) EARTHQUAKE

L. Alcántara [(1)], S. R. García [(2)], M. P. Romo [(3)] M. G. Gómez [(4)]

[(1)] *Researcher, Instituto de Ingeniería, Universidad Nacional Autónoma de México, lalcantaran@iingen.unam.mx*
[(2)] *Researcher, Instituto de Ingeniería, Universidad Nacional Autónoma de México, sgab@pumas.iingen.unam.mx*
[(3)] *Researcher, Instituto de Ingeniería, Universidad Nacional Autónoma de México, mromo@pumas.iingen.unam.mx*
[(4)] *Undergraduate student, ESIA-Zacatenco, Instituto Politécnico Nacional, marylu199313@gmail.com*

Abstract

A normal-faulting earthquake occurred on September 8, 2017, in the Mexican subducted Cocos Plate. The National Seismological Service (SSN) of Geophysics Institute at Universidad Nacional Autónoma de México (UNAM) reported this intraplate event with an 8.2 magnitude and an epicentral location in the Mexican Gulf of Tehuantepec, some 137 km SW Pijijiapan, Chiapas. This great earthquake caused more than 100 fatalities and important damages in the infrastructure located along the Oaxaca Coast and inland. The event was recorded both in the SSN broadband seismological network and in the strong ground motion network operated by the Engineering Institute of UNAM (IIUNAM). In this regard, IIUNAM has implemented an instrumental seismic program that has coverage along the Pacific coast where Cocos Plate subducts beneath the North-American Plate. Those seismic arrangements have also been deployed in some important populations such as the city of Oaxaca where the mentioned Chiapas earthquake was recorded. The purpose of this paper is to analyze the ground motion seismic response in some soils of Oaxaca City in Mexico but considering a non-linear and non-stationary methodology using the Hilbert-Huang transform (HHT). HHT has been used to explore temporal-frequency-energy dissemination and is based on two phases an empirical mode decomposition (EMD) and Hilbert spectral analysis. In the study, we use accelerogram records gotten on the accelerograph seismic network of Oaxaca City during the large earthquake of September 8, 2017 (M8.2). These time-series recordings come from different subsoil conditions.

Keywords: Hilbert-Huang transform, empirical mode decomposition, seismic ground motions, fundamental frequency.

1. Introduction

The implementation of accelerographic networks in Mexico has allowed, in recent years, the monitoring of seismic activity in the country and the generation of an important database of accelerograms [1]. The collection of seismic data and especially of large earthquakes is one of the main tasks for the study of the seismic phenomenon undoubtedly. Once this information has been obtained its analysis is the next activity, and for that, the tool used in this regard is of vital importance. To perform a frequency analysis of time-acceleration recordings is a common practice to use tools developed for linear and/or stationary signals [2-4].

Unfortunately, the seismic phenomenon presents a non-linear and non-stationary behavior among many other natural processes and therefore the use of such analysis tools is not appropriate. Several studies have demonstrated that non-linearity and non-stationarity affect the seismic response of either soils or structures concerning the condition of linearity and stationarity. According to the previous, in this study, we consider the Hilbert-Huang transformation (HHT) as an alternative tool given its rationality to cope with these nonstationary random events [5-9]. HHT analysis is composed of two phases: the empirical mode decomposition method (EMD) and the Hilbert spectral analysis (HS).

The analysis considers the strong ground motions recordings gotten in the city of Oaxaca in Mexico. The recordings or accelerograms were captured during the occurrence of the large earthquake of September 8, 2017 (M8.2). This earthquake together with the event of Jalisco in 1932 (M8.2) is the largest event instrumentally recorded in the country.

2. Hilbert-Huang transform

Two processes integrate Hilbert-Huang transform (HHT): 1) the Hilbert spectral analysis and 2) the Empirical Mode Decomposition (EMD).

2.1 Hilbert spectral analysis

The Hilbert-Huang transform (HHT) is a recently developed semi-empirical method, which arises from the need to analyze systems whose data are most likely nonlinear and nonstationary. HHT is an adaptive basis method that has the advantage of analyzing time-frequency-energy representations with a physical meaning. Nevertheless, it has the disadvantage of being data-dependent and thus its basic definition is different from the established mathematical hypothesis for data analysis. According to Duffing Eq. (1) represents, a simple nonlinear system where γ represents the amplitude of a periodic forcing function associated with a frequency ω and ϵ is a parameter. If ϵ=0 then the system would be linear, otherwise if $\epsilon \neq 0$ the system would be nonlinear. In the past, to overcome this inconvenience ϵ was assumed too small and the problem solved using perturbation methods. Nevertheless, if ϵ is not small, the perturbation methods are not an option, because the system becomes a highly nonlinear one.

$$\frac{d^2x}{dt^2} + x + \epsilon x^3 = \gamma \cos(\omega t) \tag{1}$$

Considering the importance of the calculation of instantaneous frequencies and their corresponding approximations in the analysis of signals, the Hilbert transform offers a simple means to do so using Eq. 2, where $\hat{y}(t)$ can be written for any function *x (t)* of Lp class [10] and the PV refers to Cauchy's principle value integral.

$$H[x(t)] \equiv \hat{y}(t) = \frac{1}{\pi} PV \int_{-\infty}^{\infty} \frac{x(\tau)}{t-\tau} d\tau \tag{2}$$

Some authors [6, 11] have considered that an analytic function z(t) can be integrated using the Hilbert transform pair as is establish in Eq. (3). It is important to point out that z(t) is the mathematical approximation to the original signal x(t).

$$z(t) = x(t) + i\hat{y}(t) = A(t)e^{i\theta(t)} \tag{3}$$

Where

$$A(t) = \text{Instantaneous amplitud function} = (x^2 + \hat{y}^2)^{\frac{1}{2}}$$

$$\theta(t) = \text{Phase function} = tan^{-1}\left(\frac{\hat{y}}{x}\right)$$

$$i = \sqrt{-1}$$

Finally, the instantaneous frequency ω can be expressed as the time derivative of the phase function as is shown in Eq. (4).

$$\omega = \frac{d\theta(t)}{dt} \tag{4}$$

From the above, the Hilbert spectrum can be expressed as H(ω,t) which establishes the contribution of energy for a particular frequency at a certain time and then the marginal spectrum can be estimated as in Eq. (5) where the spectrum is summed over the time domain of 0 and *T*.

$$h(\omega) = \int_0^T H(\omega, t)dt \tag{5}$$

2.2 Empirical mode decomposition

The EMD considers that any signal is integrated by different simple intrinsic modes of oscillations and each intrinsic mode represents an oscillation that has the same number of extrema and zero-crossings. Thus the algorithm decomposes a specific signal into a set of functions called intrinsic mode functions (IMF). These IMF are defined as signals with zero mean and whose number of extrema and zero-crossings differ by at most one.

In fact, the method is an iterative process to subtract the local minima from the corresponding signal and is as follows:

1. Settle on the local extrema (maxima, minima) of the signal.
2. Link the maxima with an interpolation function, producing an upper envelope of the signal.
3. Link the minima with an interpolation function, producing a lower envelope of the signal.
4. Calculate the local mean as half the difference between the upper and lower envelopes.
5. Take from the local mean from the signal.
6. Iterate on the residual.

At the end of the iterative process and gotten, all the IMF´s along with the final residue (R_n), the decomposed original signal $D(t)$ can be expressed as follows Eq. (6):

$$D(t) = R_n(t) + \sum_{j=1}^{n} IMF_j\ (t) \tag{6}$$

Figure1 presents the overall algorithm layout to estimate Hilbert-Huang transform considering both processes the Hilbert spectral analysis and EMD.

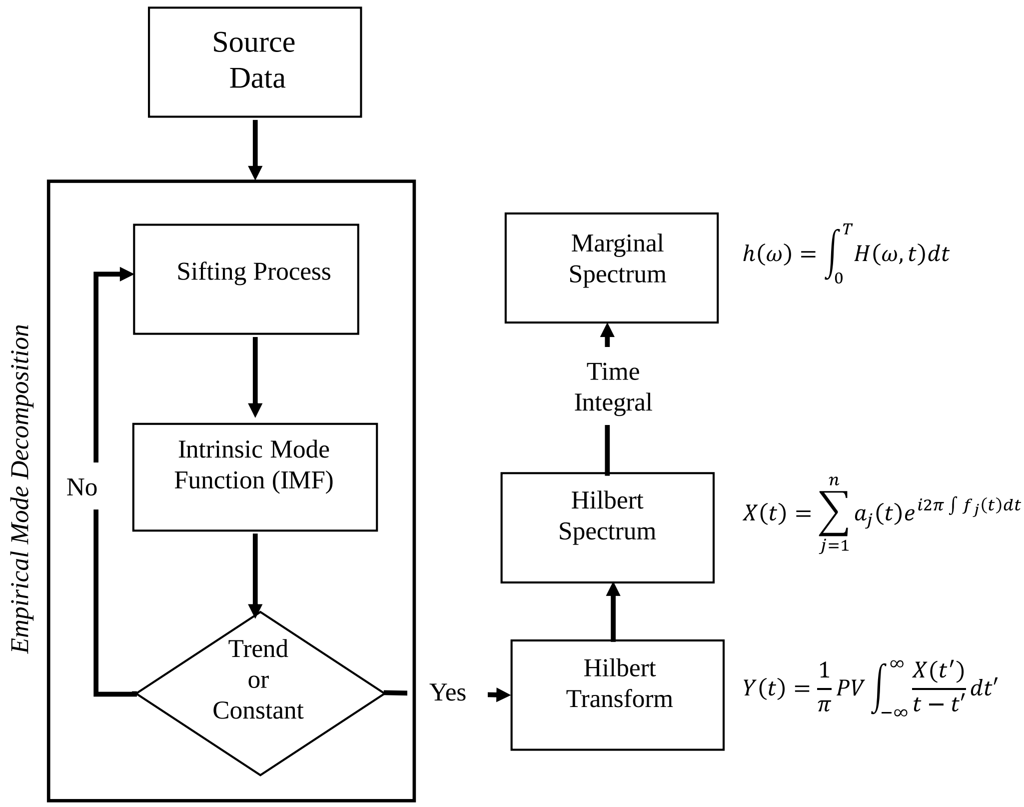

Fig. 1 Hilbert-Huang transformation flow diagram. The first stage (left) is the EMD process to get IMF´s and the second stage (right) is the Hilbert spectral analysis, (modified from [12]).

3. Hilbert-Huang transformation of seismic response

3.1 Oaxaca earthquake of September 08, 2017 (M8.2)

The Mexican Seismological Service (SSN reported an earthquake on September 7, 2017 (Mw=8.2). The epicenter was located at 14.761° N 94.103° W and an estimated depth of 45.9 km in the Gulf of Tehuantepec, Mexico (see Fig. 2) [13]. In the other hand, the Global Centroid-Moment-Tensor (CMT) website catalog [14, 15] reported that the shock had a normal fault mechanism, the epicenter was placed at 15.38°N and -94.6°W, estimated depth = 44.8 km and occurred at 4:49:46.7 (GMT). The Tehuantepec Gulf produced over 9369 aftershocks because of this large earthquake. The bigger ones were those of September 8 (M5.8) and September 23 (M6.1). Figure 2 shows location of the three main events and the distribution of the seismic swarm.

The earthquake was located in the Mexican subduction zone along the Pacific Ocean where Cocos plate subducts the North-American plate. In the period, that goes from 1900 to 2017 four more earthquakes $M \geq 8$ have occurred in that region: January 15, 1931 (Ms=8), October 9, 1995 (Mw=8), September 19, 1985 (Mw=8) and June 3, 1932 (M8.2).

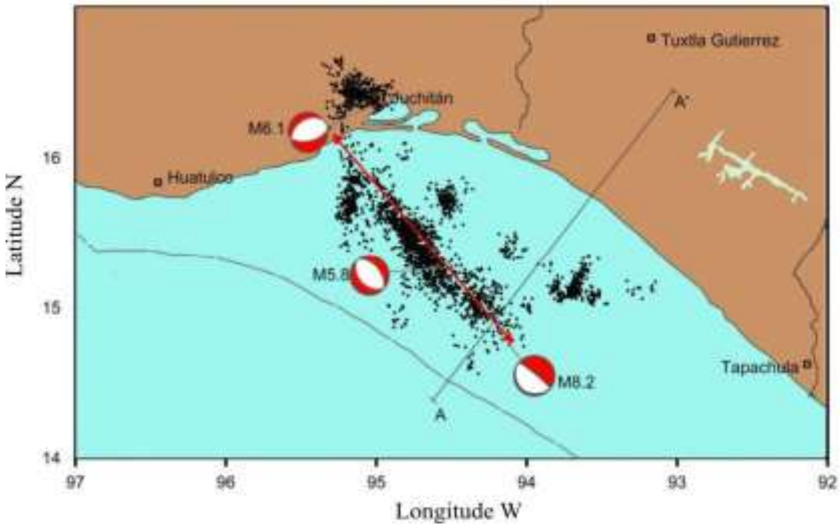

Fig. 2 Earthquake of September 8, 2017 (M8.2) and the aftershock distribution (modified from [13])

3.2 Accelerogram recordings in Oaxaca City

The state of Oaxaca is located in the Southwest region of Mexico and along the Pacific Ocean where the Cocos plate subducts that of North American. The Engineering Institute of UNAM operates a small seismic network in the capital city of the state also called Oaxaca [16]. The arrangement is composed of nine accelerographic stations and the instrumented sites were selected based on the existing preliminary zoning, the geotechnical information as well as on environmental vibration studies. This accelerographic network recorded the earthquake of September 8, 2017 (M8.2).

In the study carried out here, we present the results for two stations: Las Canteras (OXLC) and Faculty of Medicine (OXFM). The first is located on an andesitic tuff with shear wave velocity of Vs> 700 m/s and the second one on a clastic deposit with surface rigid soils, with shear wave velocity of 250 <Vs <350 m/s. The 20170908 earthquake was recorded in three orthogonal components (two horizontals and one vertical). Peak ground accelerations (in cm/s^2) for OXLC were 59 (N-S), 68(V), 103 (E-W) and for OXFM were 269 (N-S), 83 (V), 147 (E-W), figure 3 presents the corresponding recordings.

3.3 Decomposition of accelerograms

According to the procedure described in 2.2, the accelerograms were decomposed into their IMF pieces. Figure 4 presents the results only for PGA components, E-W for OXLC and N-S for OXFM. As is shown, for both cases 11 IMFs were necessary for decomposing the original signals and they can be represented according to equation 4.

The IMFs are signals with zero mean and whose number of extrema and zero-crossings differs by at most one, as it was required in section 2.2. IMF11 is the residue and represents the final of the process because has become a function from which no more IMFs can be extracted. IMF1 contains the highest frequency content which diminishes for higher IMF components. In fact, the summation of IMF1 to IMF5 represents the EMD-based high-frequency component and the summation of IMF6 to IMF11 the EMD-based low-frequency component.

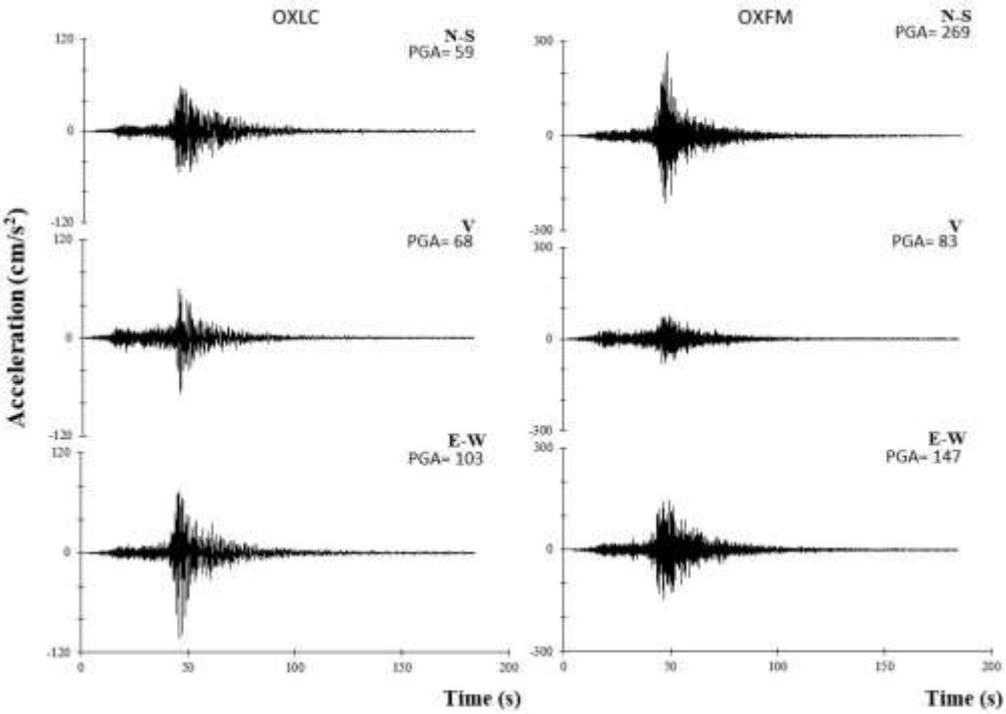

Fig. 3 Accelerogram recordings in stations OXLC and OXFM [17]

The EMD process showed in figure 4 reveals that the first few components are the more significant and represent the true seismic waves, in contrast, the rest are considered noise and numerical error associated to the process.

According to the previous results, we can reproduce the original accelerograms registered in the stations OXCL and OXFM from Oaxaca City during the September 8, 2017 earthquake (M8.2). For that, we only considered the first 5 IMF as is depicted in figure 5. On the upper side are the original recordings and below them, the signal reproduced which are almost identical both in time duration and in the PGA values.

At the bottom of the figure as a way to estimate the capability of the method, we calculate the Fourier spectrums for the original signals and for the ones reproduced using IMF 1 to 5. Fourier spectrums are almost identical for the full range of frequencies. From there we can see that these IMF1 to IMF5 capture the most important features of the seismic signals.

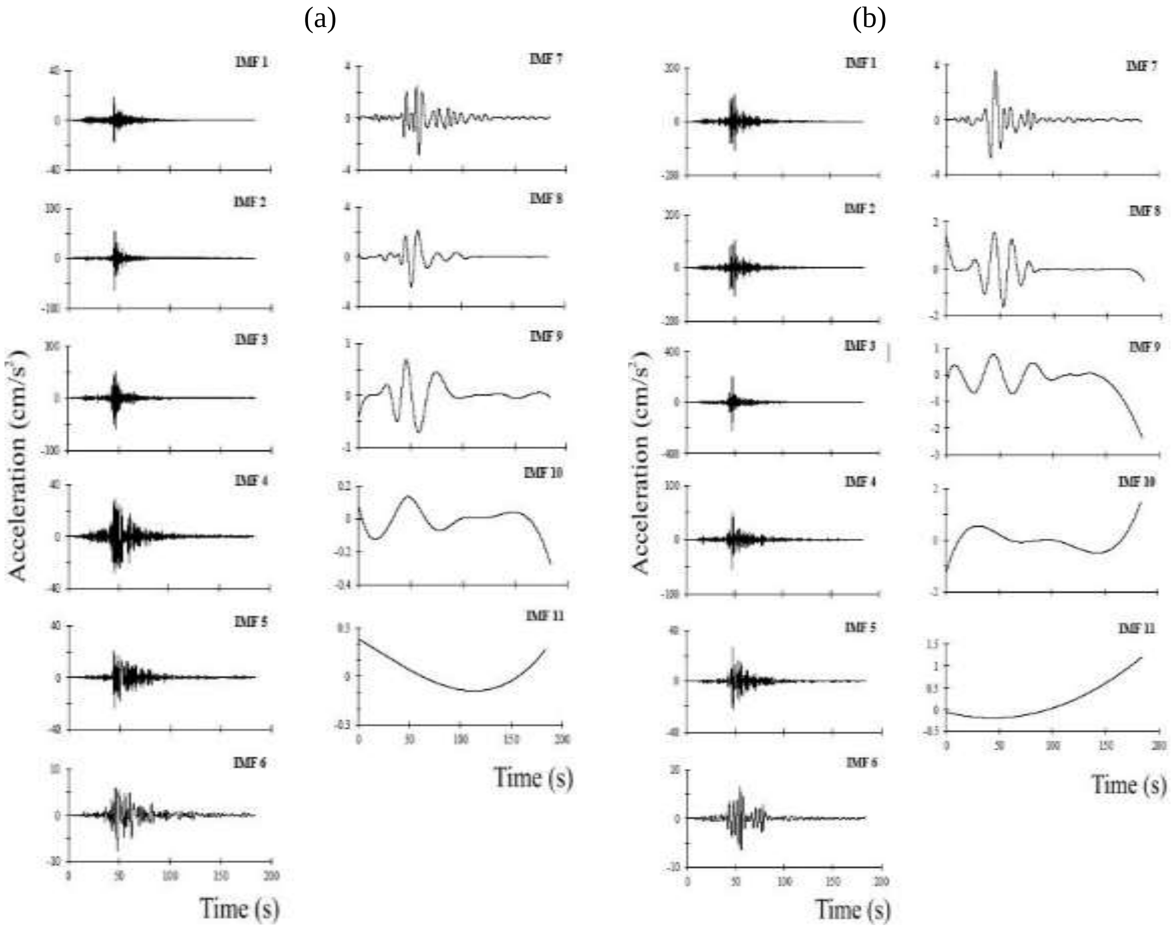

Fig. 4 Intrinsic Mode Functions (IMF) obtained from accelerogram records of Oaxaca City during September 8, 2017, earthquake (M8.2). a) OXLC station component E-W and b) OXFM station component N-S.

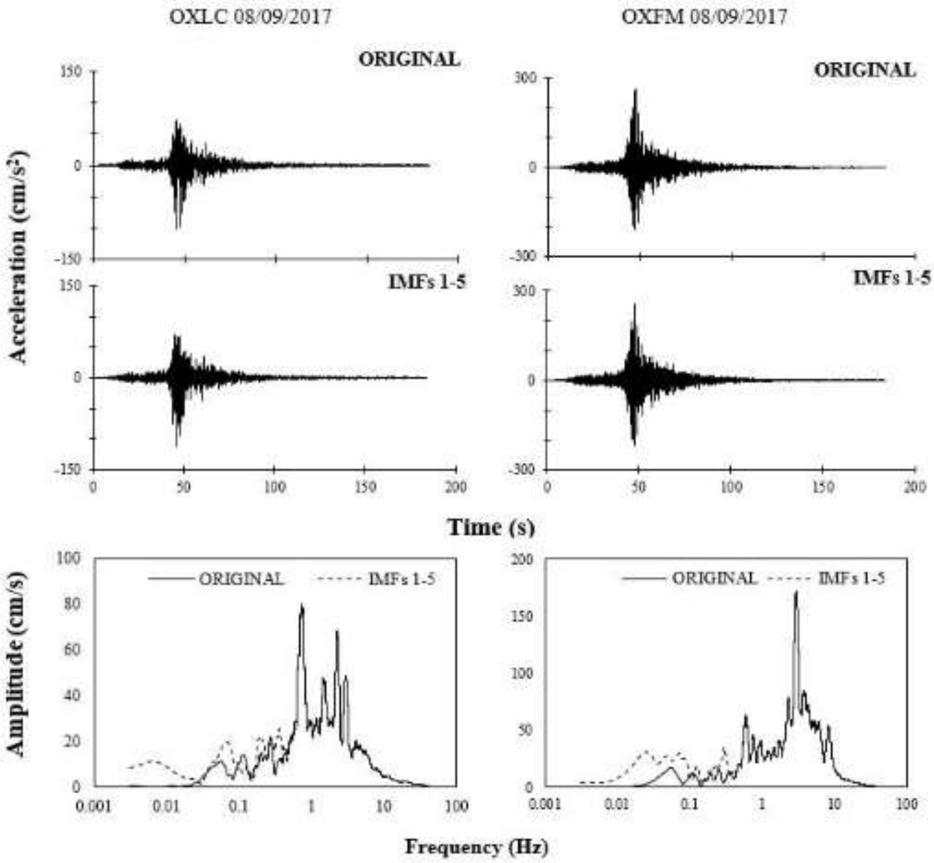

Fig. 5 Representation of the original accelerograms considering their IMFs. The component E-W of station OXLC is presented on the left side and the component N-S of station OXFM is on the right side.

3.4 Hilbert spectra of ground motions.

Once the IMFs have been obtained, the Hilbert transform can be applied to each IMF to estimate the instantaneous frequencies using Eq. (2) and (4) and after performing that we can represent the real part R of the original data using Eq. 11. Because of the residue (r_n) is either a constant or a monotonous signal has been left out. In the equation, both the amplitude and frequency are expressed as functions of time. The same data in a Fourier representation would be as is shown in Eq. (8). It is clear that Eq. (7) in a generalized version of Eq. (8) that enables to consider nonlinear and nonstationary data

In the following, we present the Hilbert spectra analysis for the accelerograms registered in two seismic stations of Oaxaca City during the September 8, 2017 earthquake (M8.2). Only the components where maximum acceleration or PGA was presented are included, but similar findings were obtained in the other components.

$$x(t) = R\left\{\sum_{j=1}^{n} a_j(t)\exp\left[i\int \omega_j(t)dt\right]\right\} \qquad (7)$$

$$x(t) = R\left[\sum_{j=1}^{n} a_j e^{i\omega_j(t)}\right] \qquad (8)$$

Given de Fourier Spectra in figure 5, it can be observed that the frequency content of the signals is spread out having the maximum spectral amplitudes at 0.7, 3 Hz for OXLC and at 0.7, 4 Hz for OXFM. In contrast, HS shows quantitatively the temporal-frequency distribution of vibration characteristics in the ground motion in figures 6 and 7. Although the recording time in both stations was 180 s, the most significant amplitudes are presented in a short interval that goes from 40 to 60 s and in a frequency span from one to four Hz (Figs. 6a and 7a).

The expanded regions of HS presented in figures 6b and 7b show that significant energy varies in terms of both time and frequency. In the case of station OXLC (rock deposit), these intensities range from 10 to 100 cm/s^2 and from 1 to 4.5 Hz and for OXFM (hard soil) range from 20 to 269 cm/s^2 and from 2 to 6 Hz. Although the major intensities are developed in a brief period from 44 to 47.5 s (OXLC) and 46 to 49 s (OXFM), in both cases their HS show the arrivals of energy in three packets with increasing frequencies. These observations are unclearly shown in the Fourier spectra for both stations (Fig. 5).

Strong ground motion duration is another relevant aspect when site effect is being studied because this parameter is frequently used to characterize the soil response and the seismic damage accumulated in the structures. Again, Fourier analysis is limited in this aspect and therefore is better to use HS because it exposes the signal duration where most meaningful amplitudes and their associated frequencies are presented.

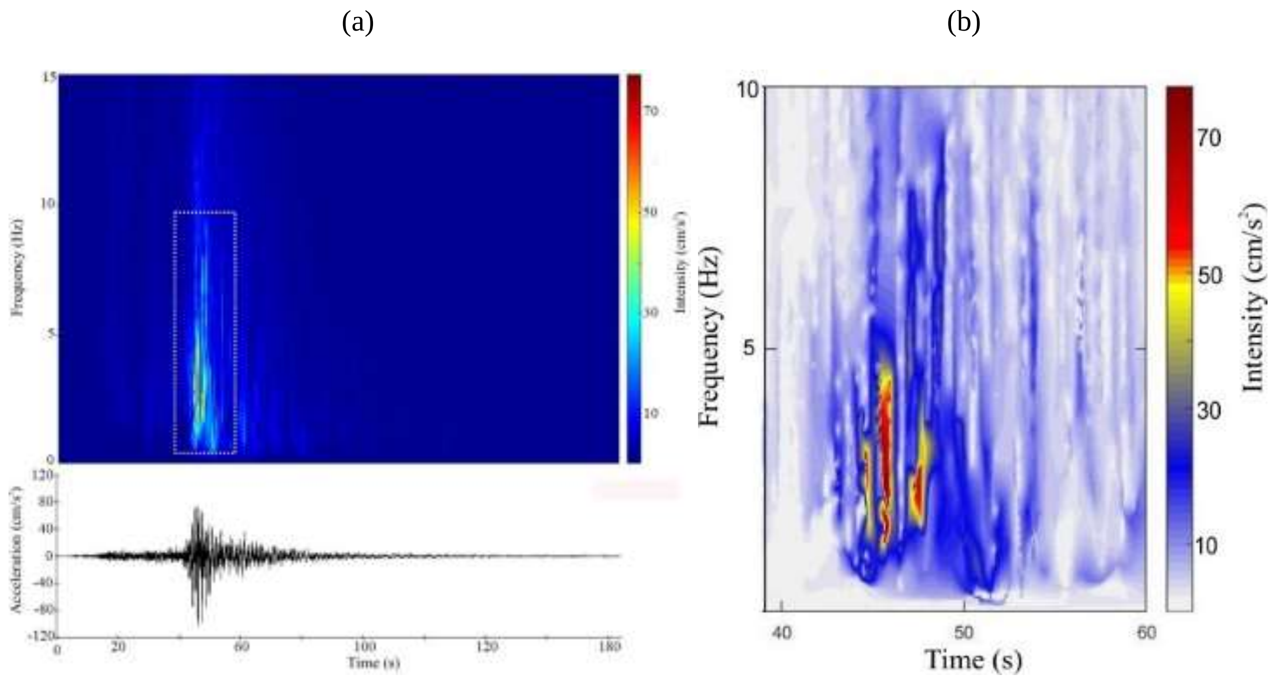

Fig. 6 Hilbert spectrum of the accelerogram recorded at station OXLC component E-W

a) HS at the top and below the original seismic signal with its entire duration,
b) an expanded region of HS that goes from 40 to 60 seconds and from 0 to 10 Hz.

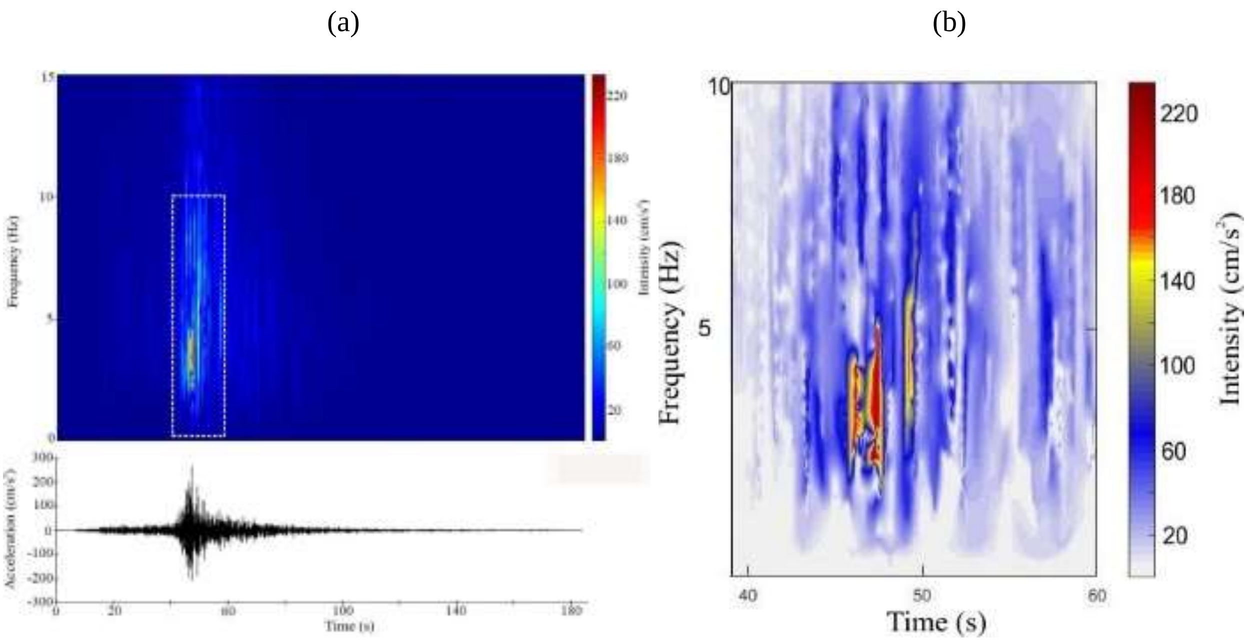

Fig. 7 Hilbert spectrum of the accelerogram recorded at station OXFM component N-S
a) HS at the top and below the original seismic signal with its entire duration,
b) an expanded region of HS that goes from 40 to 60 seconds and from 0 to 10 Hz.

4. Conclusions

The paper presents a method of analysis through the Hilbert-Huang spectrum, which is based on the empirical decomposition method and Hilbert analysis. The study was carried on some accelerogram records recorded in the city of Oaxaca in Mexico during the occurrence of the large earthquake of September 8, 2017(M8.2).

The estimation of HS was performed in two sites, one in rock and the other in hard soil. Ten IMFs plus a residue were obtained using the EMD and from them was observed that IMFs 1-5 can reproduce with minimal differences in the original accelerogram. From the above, it can be established that the EMD adopted has shown its usefulness for separating oscillation modes and to permit discriminate between undesired signals to those more correlated to ones analyzed.

By detecting the oscillations that are directly linked to the real ground motion, the EMD is a useful tool to find the frequencies related to the highest energy concentrations and its variation as a function of the time and most important the EMD permits to deal with data from nonstationary and nonlinear processes.

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6. References

[1] Mexican Society for Earthquake Engineering SMIS (2000): Mexican strong motion database, CD-ROM Accelerograms v2, Mexico.

[2] Tong H (1990): Nonlinear Time Series Analysis: a dynamical system approach. Oxford University Press.

[3] Kantz H, Schreiber T (1997): Nonlinear Time Series Analysis. Cambridge University Press.

[4] Diks C (1999): Nonlinear Time Series Analysis: Methods and Applications. World Scientific Press. https://doi.org/10.1142/3823.

[5] Huang N, Long S, Shen Z (1996): The mechanism for frequency downshift in nonlinear wave evolution. *Adv. Appl. Mech.* 32: 59–111.

[6] Huang N, Shen Z, Long S, Wu M, Shih H, Zheng Q, Yen N, Tung C, Liu H (1998): The empirical mode decomposition and the Hilbert spectrum for nonlinear and non-stationary time series analysis. *Proc. R. Soc. London*. Ser. A 454: 903–995.

[7] Huang N, Shen Z, Long S (1999): A new view of nonlinear water waves: The Hilbert spectrum. *Annu. Rev. Fluid Mech.* 31: 417–457. http://dx.doi.org/10.1146/annurev.fluid.31.1.417.

[8] Huang N, Shen S (2014): Hilbert-Huang transform and its applications. 2nd ed. World Scientific Press. ISBN-13: 9789814508230. https://doi.org/10.1142/5862.

[9] Garcia SR, Romo MP, Alcántara L (2019): Analysis of non-linear and non-stationary seismic recordings of Mexico city. *Earthquake Engineering & Structural Dynamics*, **127** (2019) 105859. https://doi.org/10.1016/j.soildyn.2019.105859.

[10] Huang NE, Shen Z (2015): Interdisciplinary mathematical sciences. In: Huang NE, editor. Hilbert-Huang Transform and its applications. London: World Scientific 2015, 400 p. ISBN-13: 978-9814508230, ISBN-10: 9814508233.

[11] Huang NE, Attoh-Okine N (2005): The Hilbert-Huang Transform in Engineering. 1st ed. Boca Raton: CRC/Taylor & Francis; p328 ISBN: 978-0849334221.

[12] Lee MH, Shyu KK, Lee PL (2011): Hardware implementation of EMD using DSP and FPGA for online signal processing. IEEE Transactions on Industrial Electronics. **58**(6): 2473-2481.

[13] Mexican Seismological Service, SSN-UNAM (2017): Sismo de Tehuantepec (2017-09-07 23:49 Mw 8.2), http://http://www.ssn.unam.mx/sismicidad/reportes-especiales/2017/SSNMX_rep_esp_20170907_Tehuantepec_M82.pdf (*Last accessed 20200128*).

[14] Dziewonski AM, Chou T-A, Woodhouse JH (1981): Determination of earthquake source parameters from waveform data for studies of global and regional seismicity, *J. Geophys. Res*., 86, 2825-2852 doi:10.1029/JB086iB04p02825.

[15] Ekström G, Nettles M, Dziewonski AM (2012): The global CMT project 2004-2010: Centroid-moment tensors for 13,017 earthquakes, *Phys. Earth Planet. Inter*. 200-201, 1-9, 2012. doi:10.1016/j.pepi.2012.04.002.

[16] Alcantara L (2013): Análisis de movimientos sísmicos en las ciudades de Oaxaca y Puebla con redes neuronales Thesis (PhD), DEPFI Universidad Nacional Autónoma de México. (*in Spanish*)

[17] Unidad de Instrumentación sísmica IIUNAM (2020): Base de datos de registros acelerográficos de la RAII-UNAM, http://aplicaciones.iingen.unam.mx/AcelerogramasRSM (*Last accessed 20200128*).

