



A NEW TYPE ROTATIONAL ACCELEROMETER AND A DUAL MAGNETIC CIRCUIT SERVO TYPE ULTRA-LOW FREQUENCY ANGLE SHAKING TABLE

Feng GAO⁽¹⁾, Xueshan YANG⁽²⁾, Lei WANG⁽³⁾

⁽¹⁾ Associate Professor, Institute of Engineering Mechanics, CEA, e-mail address: gaofenggao@126.com

⁽²⁾ Professor, Institute of Engineering Mechanics, CEA, e-mail address: yangxs_iem@126.com

⁽³⁾ Associate Professor, Institute of Engineering Mechanics, CEA, e-mail address: wanglei_hlj@hotmail.com

Abstract

The structure principle of a new type of strong motion rotational accelerometer and a dual magnetic circuit servo type ultra-low frequency angle shaking table is given. The differential equations, frequency characteristics and sensitivity calculation formulas of the two instruments are derived. In the national calibration department, the sensitivity, linearity and amplitude-frequency characteristics of the two are calibrated. The lateral rotation sensitivity, horizontal vibration and vertical vibration of the rotary accelerometer are more comprehensive in the laboratory. The technical indicators were tested, and the technical indexes such as linearity and stability of the angle vibration table were tested. The test results show that the strong vibration rotary accelerometer and the angular vibration table have good performance, and the calculation results are in good agreement with the test results. The strong vibration rotary accelerometer can be used for the measurement of the rotational acceleration of the strong earthquake and the rotational acceleration of the building structure. The angular vibration table can be used for the calibration and detection of the strong vibration rotary accelerometer and other rotating sensors. These two new instruments have obtained national patents.

Keywords: rotational accelerometer; angle vibration table; strong earthquake; capacitive sensing; low frequency

1. Introduction

It is well known that in strong earthquake observations the most widely used instrument is the triaxial accelerometer. The acceleration record is usually considered as east-and-west, south-and-north, and vertical vibrations. In fact, during the earthquake the complete motion characteristics of a rigid body at a certain point also include rotations, which requires the measurement of six degrees of freedom, i.e. three translational and three rotational. The rotational component becomes more obvious for near-field earthquake have been records in history about rotational damages in seve.

Japan (Takeshi, 1998) and Taiwan (Huang et al. 2006, Liu et al. 2009) studied the near-field earthquake measurements and showed that the rotational earthquake is several times larger than that predicted by classical elasticity theory.

Based on the importance of the rotational component, Lee et al. (2009) of the US Geological Survey set up the International Working Group on Rotational Seismology (IWGoRS) in Munich, Germany and have done a lot of research and analysis on rotary motions. Europe has incorporated the rotational earthquake into design specifications. The specification is to consider the structure of rotating loads tower building, for structure or steel for reinforced concrete chimney, there are special requirements specification. However, due to the lack of rotational component records and other reasons, many countries do not consider rotation role in seismic design structure.

The rotational component of a strong earthquake can be measured by an angular velocimeter or angular accelerometer. There are many manufacturing solutions in the development of angular accelerometers; however, few of them reached the practical stage, as exemplified by the high cost of the mechanical gyroscopic angular accelerometer sensor and the short measurement range of $1 \sim 0.001 \text{ rad/s}^2$. The inertial angular



accelerometer made by FanceBanie machinery factory uses the supported inertial wheel as the sensing element, but it also uses the inductive converter. Published by the U.S. Patent 1986, an anti-vibration angular acceleration sensor uses the piezoelectric principle. Another angular acceleration sensor uses the eddy current induction principle, but its sensitivity is only $16.5\mu\text{V}/(\text{rad}/\text{s}^2)$. The resolution of an angular acceleration piezoelectric gyroscope is only $\pm 0.1^\circ/\text{s}^2$. With the frequent occurrence of large earthquakes, countries around the world have strengthened the observations of the three translational records of earthquakes, and have carried out research and development of instruments for seismic rotation components, and have proposed a variety of methods for measuring the seismic rotation components [1,2,3] And developed related measuring instruments [4,5,6,7]. How to calibrate these instruments and obtain valuable rotation components is a concern of seismic science and technology workers, so it is urgent to develop a practical calibration device for seismic rotation sensors.

2. Structure and theoretical derivation of strong earthquake rotational accelerometer

The structure diagram of the strong earthquake rotational accelerometer is shown in Fig.1, and its schematic block diagram is shown in Fig.2. It consists of a spoke type mass-spring system, a symmetrically reverse-mounted same parameter double pendulum capacitive sensing transducer, a moving coil damper, a conditioning circuit, a synthesis circuit, and a housing base and a shell.

On both sides of the mass ring are anti-symmetrically installed two tilting differential capacitance transducers of the same parameters. Each transducer is composed of two movable plates and a fixed plate. The change in the capacitor plate gap leads to the change of in the differential capacitance. The output voltage can be obtained through voltage conversion circuit and the conditioning circuit. The output voltage of the anti-symmetrically installed two same-parameter tilting differential capacitance transducers is input to the synthesis circuit, which forms the strong earthquake rotational accelerometer. Synchronized with the movement of each differential capacitor plate, the bobbin is wound with two coils, one of which is called the damping coil and the other is called self-calibration coil. The two coils move in the gap between two anti-symmetrically installed magnetic systems. The damping coil adjusts the circuit and provides the desired damping ratio required by the rotational accelerometer. Self-calibration coil inputs the sinusoidal signal, which can be used to measure the technical indicators of the rotational accelerometer.

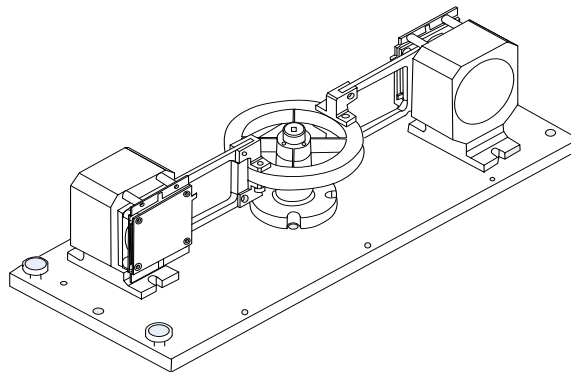


Fig. 1 – Schematic representation of the structure of the strong earthquake rotation accelerometer.

In Fig.2, K_1 is the moment of inertia of the double pendulum swinging about the rotational axis. b is the damping coefficient. k is the angular stiffness. θ is the displacement. L_k is the length of the indicator pendulum. \ddot{y} is the ground translational acceleration. $\ddot{\varphi}$ is the ground rotational acceleration. G_1 is the electric constant of the damping coil. i_1 is the current flowing into the damping coil. G_2 is the electric constant of the self-calibration coil. i_2 is the current flowing into the self-calibration coil. K_c is the



sensitivity of the capacitive transducer. x is the displacement of the capacitor plates. K_2 is the conditioning factor of the conditioning circuit. u_0 is the output voltage of the each pendulum capacitive transducer rotational accelerometer. u_{0z} is the combined voltage of the two pendulum capacitive transducer rotational accelerometers, which is output voltage of the strong earthquake rotational accelerometer.

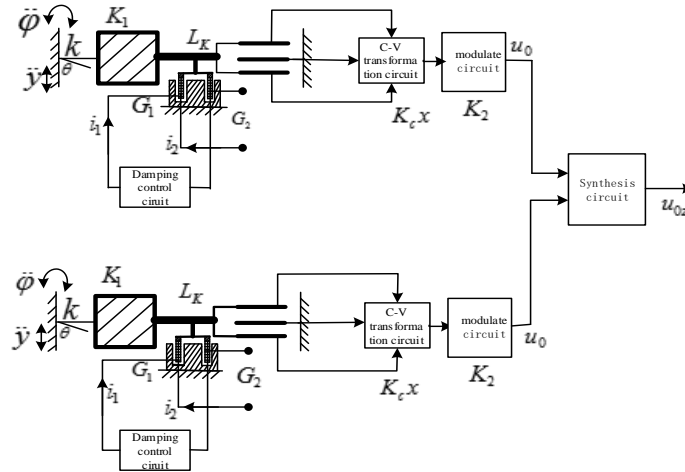


Fig. 2 –Schematic diagram of the principle of the strong earthquake rotation accelerometer



Fig. 3 –Photograph of the strong earthquake rotation accelerometer

The mathematical model of the strong earthquake rotation accelerometer is shown in Fig.4. The equation of the motion is given by:

$$K_1\ddot{\theta} + b\dot{\theta} + k\theta + G_1i_1 = -\sum m_i H_i \times \ddot{y} - K_1\ddot{\phi} \tag{1}$$

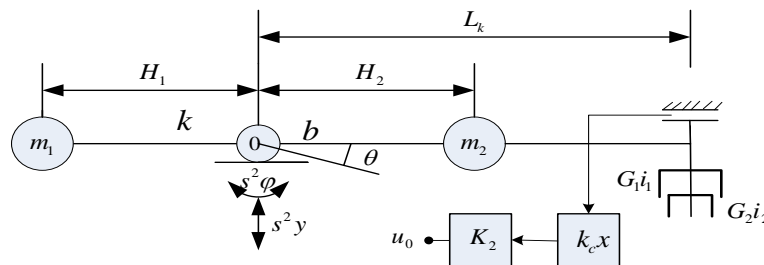


Fig. 4 – Schematic representation of the mathematical model of the strong earthquake rotation accelerometer

In this equation, m_1, m_2 are the effective masses of the double pendulum. H_1, H_2 are the distances between the two centers of mass and the rotational axis. l_0 is the equivalent pendulum length. s is the



operator. n is the circular frequency of self-vibration. D is the damping ratio, which is adjusted by the damping circuit.

$$\sum m_i H_i = m_1 H_1 - m_2 H_2, \quad K_1 = m_1 H_1^2 + m_2 H_2^2 + K_{10} + K_{20} \quad (2)$$

K_{10} is the moment of inertia of the reduced mass m_1 about the rotational axis. K_{20} is the moment of inertia of the reduced mass m_2 about the rotational axis.

Solution of Eq. (1) is:

$$\theta(s) = \frac{-s^2}{(s^2 + 2Dns + n^2)} \left(\frac{1}{l_0} y - \varphi \right) \quad (3)$$

$$\text{where, } n^2 = \frac{k}{K_1}, \quad 2Dn = \frac{b}{K_1}, \quad l_0 = \frac{K_1}{\sum m_i H_i} = \frac{K_1}{m_1 H_1 - m_2 H_2}$$

If we ignore the air damping, the damping force of the pendulum is $F = Gi$, and the damping ratio is:

$$D = \frac{G^2}{2K_1 n R} \quad (4)$$

where R is the loop resistance.

When $m_1 H_1 = m_2 H_2$, $l_0 = \infty$, the instrument will only reflect the rotational angle, and not the translation.

The solution of the rotation accelerometer equation is:

$$\theta(s) = - \frac{s^2 \varphi}{(s^2 + 2Dns + n^2)} \quad (5)$$

Under the effect of the ground rotational acceleration, the output voltage of the capacitive transducer is:

$$u_0(s) = K_c x K_2 = K_c L_K \theta K_2 = -K_c L_K K_2 \frac{s^2 \varphi}{(s^2 + 2Dns + n^2)} \quad (6)$$

where K_c is the sensitivity of the capacitive transducer. $x = L_K \theta$ is the displacement of the movable plate relative to the fixed plate. L_K is the length of the indicator pendulum. K_2 is the conditioning factor, which is used to adjust the sensitivity of the rotational accelerometer.

According to Eq. (6), the output voltage u_0 of the accelerometer is proportional to the rotational acceleration $s^2 \varphi$. Because the differential capacitive accelerometers are reversely installed on the two sides of the mass ring, after the synthesis circuit the output voltage of the two accelerometers is proportional to the output voltage of the vertical rotational acceleration. The output voltage of the synthesis circuit is:

$$u_{0z} = 2u_0 \quad (7)$$

The sensitivity of the rotational accelerometer is:

$$S_{\varphi}(s) = \frac{2u_0}{s^2 \varphi} = 2 \frac{K_c L_K K_2}{n^2} \frac{1}{\left(\frac{s^2}{n^2} + \frac{2D}{n} s + 1 \right)} = 2 \frac{K_c L_K K_2}{n^2} H \quad (8)$$



where $H = \frac{1}{\left(\frac{s^2}{n^2} + \frac{2D}{n}s + 1\right)}$ is the frequency characteristics of the rotational accelerometer.

This rotational accelerometer can be used for the observation of the earthquake rotational acceleration and measurement of civil engineering structural rotations.

3. Main technical indicators

Pass band: 0-20 Hz

Sensitivity: $38.5\text{mV}/(\text{rad}/\text{s}^2)$ - $2.5\text{V}/(\text{rad}/\text{s}^2)$ adjustable

Lateral rotation sensitivity ratio: $\leq 5\%$

Effects of horizontal vibration on rotational accelerometer: $\leq 0.06\text{rad}/(\text{m}/\text{s}^2)$

Dynamic range: larger than 100dB

Maximum range: larger than $30\text{rad}/\text{s}^2$

4. Calculation Results

4.1. Calculated results for the frequency response characteristics

The calculated results for the frequency response characteristics are shown in Fig.5: the top figure shows the frequency dependence of the amplitude, and the bottom figure displays the frequency dependence of the phase.

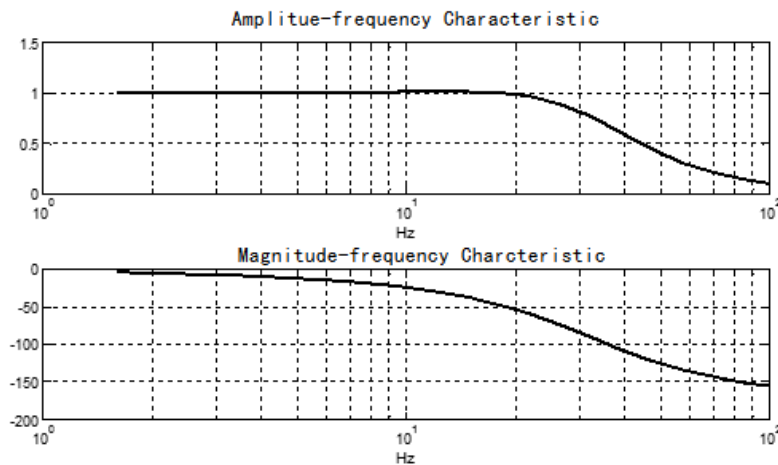


Fig. 5 – Frequency response curves of the strong earthquake rotational accelerometer

4.2 Calculated results for the sensitivity

In the straight section of the amplitude-frequency characteristics, the sensitivity of the rotational accelerometer is

$$S_{\dot{\varphi}} = \frac{u_0}{s^2 \varphi} = 2 \frac{K_c L_K K_2}{n^2} = 2 \frac{6600 \times 0.12 \times 1}{(2\pi \times 32)^2} = 39.2\text{mV}/(\text{rad}/\text{s}^2)$$

5. Experimental results



5.1 Measured results of the amplitude-frequency characteristics and sensitivity of the rotational accelerometer

Calibration results on the National Standards angular vibration table :Table 1 shows the calibration results of the amplitude-frequency characteristics of the rotational accelerometer, obtained using the National Standards angular-vibration table of the China Aviation Industry Corporation Beijing Changcheng Institute of Measurement and Testing Technology[8]. (See the Calibration Certificate of China Aviation Industry Corporation Beijing Changcheng Institute of Measurement and Testing Technology, Certificate Number: Li D02 No. 2014030053).

Table 1 – calibration results of the amplitude-frequency characteristics of the rotational accelerometer on the National Standards angular-vibration table

Frequency (Hz)	Standard angular acceleration (rad/s²)	Rotational accelerometer output (mV)	Sensitivity mV/(rad/s²)
0.05	0.04118	15.17	368.3
0.5	0.4796	185.04	385.8
1	1.1149	429.91	385.6
3	1.5455	597.73	386.8
5	2.0385	788.61	386.9
15	1.8331	698.86	381.2
25	2.3117	677.51	293.1

5.2 Linearity calibration of the rotational accelerometer

Calibration results of the linearity on a national standards angular vibration table, Linearity calibration results of the rotational accelerometer are shown in Table 2.

Table 2 – Calibration results of the linearity of the accelerometer on a national standards angular vibration table

Frequency (Hz)	Standard angular acceleration (rad/s²)	Rotational accelerometer output (mV)
2	0.1189	45.04
2	0.4789	184.30
2	1.0196	393.61
2	1.4371	555.38
2	2.0396	787.25
2	2.5229	974.71
2	2.9946	1158.2
2	3.5741	1381.6
2	4.0557	1568.8
2	4.9966	1933.3



Linearity : 0.05%, Sensitivity : 387.1 mV/(rad/s²)

As can be seen from Table 2, when the conditioning factor $K_2=10$, and the angular acceleration is within the range of 0.1189–4.9966(rad/s²), the linearity is 0.05%.

5.3 Effects of horizontal vibration to the rotational accelerometer

In this experiment, the rotational accelerometer and horizontal 941B vibration pickup were installed on a horizontal vibration table. A sinusoidal signal was given to the vibration table to drive the horizontal vibrations. The output voltages of both the 941B vibration pickup and the rotational accelerometer were measured and converted into countertop acceleration and the angular acceleration. The results are shown in Table 3.

Table 3 –Test results of the effects of horizontal vibrations to the rotational accelerometer

Frequency (Hz)	Table horizontal acceleration (m/s ²)	Converted angular acceleration (rad)	effects of horizontal vibrations to the rotational accelerometer (rad/(m/s ²))
5	1.55	0.089	0.057
10	1.63	0.059	0.036

The results shown in Table 3 are the results on a small portable horizontal vibration table. Since the small vibration table uses a four-spring suspension structure, there are inevitably rotational components arising from factors such as unevenness of the countertops, and transverse sensitivity. If the experiment is carried out on a low-frequency vibration standard device, the output voltage of the rotational accelerometer will be even smaller. As seen in Table 3, the effects of the horizontal vibration on the rotational accelerometer are smaller than 0.06rad/(m/s²).

5.4 Test of effects of vertical vibrations on the rotational accelerometer

In this test, the rotational accelerometer and the vertical 941B pickup are installed on the vertical vibration table. A sinusoidal signal was given to the vibration table to drive the vertical vibrations. The output voltages of both the 941B vibration pickup and the rotational accelerometer were measured and converted into countertop acceleration and the angular acceleration. The results are shown in Table 4.

Table 4 – Test results of the effects of vertical vibrations on the rotational accelerometer

Frequency (Hz)	Vertical acceleration (m/s ²)	Converted angular acceleration (rad)	effects of vertical vibrations to the rotational accelerometer (rad/(m/s ²))
3	0.7615	0.0061	0.0080
5	0.9576	0.0107	0.0110
10	0.8450	0.0152	0.0180

As can be seen in Table 4, since the vertical stiffness of the spoke spring structure is much larger than the horizontal one, the effects of vertical vibrations on the rotational accelerometer are smaller.

5.5 Test of the rotational transverse sensitivity



In this test, the rotational accelerometer was installed on the vibration table according to the sensitive axis. Under the condition of constant voltage or constant current, the rotational vibration table was excited by a sinusoidal signal to make angular vibrations and the output voltage was measured. Then the rotational accelerometer was rotated by 90 degrees (perpendicular to the sensitive axis) and installed on the rotational table. Under the same condition of constant voltage or constant current, the rotational vibration table was again excited by a sinusoidal signal to make angular vibrations and the output voltage was measured. Then we change the input frequency and performed the measurements using the same method, and we obtained the transverse sensitivity ratio at different frequencies. The results are shown in Table 5.

Table 5 – Test results of the rotational transverse sensitivity of the rotational accelerometer

Frequency (Hz)	Sensitive axis output voltage (mV)	Transverse output voltage (mV)	Transverse sensitivity ratio (%)
1	96.9	4.20	4.33
2	375	18.0	4.80
5	183	8.73	4.77

As seen in Table 5, the rotational accelerometer has good resistance to effects of transverse rotational vibration.

6. Structural principle of dual magnetic circuit servo type ultra-low frequency angular vibration table

The structural principle of angular vibration table is shown in Fig. 6. The main magnetic circuit of angular vibration table is compound of two E-type magnetic steel as Fig.7. The driving model is showed in Fig.8, and the ultra-low frequency angle vibration table was designed in Fig. 9.

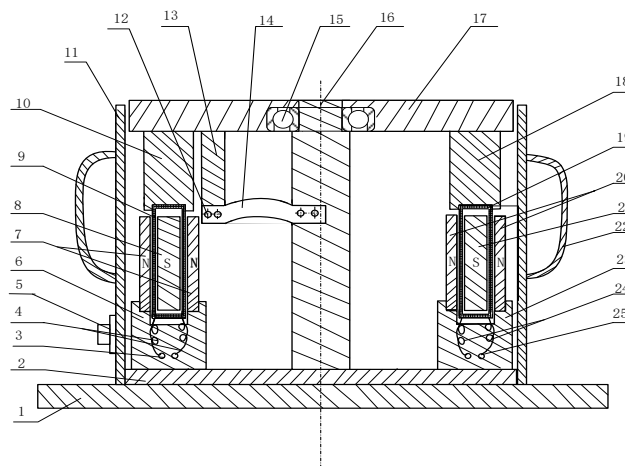


Fig. 6 – double moving coil servo type super low frequency angular vibration table section

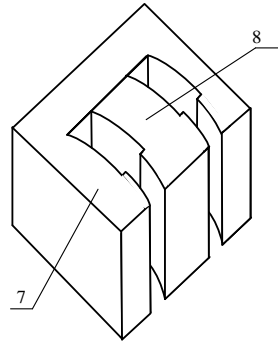


Fig. 7 – driving magnetic circuit and magnetic circuit structure diagram of feedback

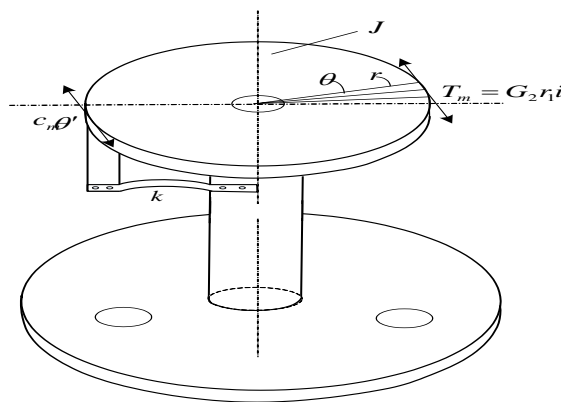


Fig. 8 – double moving coil servo type super low frequency angular vibration table mechanics model diagram



ZD-013D Large Range Ultra-Low Frequency Rotating Shaking-table

Fig.9 – Photograph of the ultra-low frequency angle vibration table

7. Technical index of dual magnetic circuit servo type ultra-low frequency angular vibration table

Angular vibration frequency range: 0.1 ~ 40Hz;

Angular displacement: 0 ~ 50°;

Angular speed: 30° / s;

Angular acceleration: 3rad / s²;

Turning torque: 2N m;



Maximum load: 17kg

8. Linear measurement of angular vibration of an angular vibration table in a closed loop

In the laboratory, according to the standard test procedures [9,10,11], the frequency of the signal source is fixed, the output voltage of the signal source is changed, and the output of the rotary table is measured to obtain the linearity of the angular vibration table. The measurement results are shown in Table 6.

Table 6 – the angular vibration table closed-loop linear measurements (1Hz)

Signal source output voltage(V)	20	18	16	14	12	10	8	6	4	2
Speed output voltage of angular shaker(mV)	536.8	480.6	429.1	372.9	318.4	265.1	211.3	155.2	102.1	51.5
Ratio of angular vibration table speed output voltage to signal source output voltage (x10⁻³)	26.84	26.70	26.82	26.60	26.50	26.50	26.40	25.86	25.53	25.77
Linear (%)	1.85	1.32	1.78	0.94	0.563	0.563	0.18	-1.87	-3.14	-2.20

9. Conclusions

Theoretical analysis and experimental results show that the strong-earthquake rotational accelerometer has good amplitude-frequency characteristics, linearity and good resistance of effects of other translational and rotational components. Results obtained at the national rotational table calibration device show that (see Tables 1 and 3), the calculated sensitivity results obtained from the rotational accelerometer agree with the national calibration results. Within the range of 0.1189-4.9966 (rad/s²) angular acceleration, the linearity is 0.05%. Self-calibration method can be used for lower-frequency amplitude-frequency characteristic measurements and linearity measurements of larger measurement range. Calibration results of the amplitude-frequency characteristics and linearity using the laboratory self-calibration are shown in Table 2 and Table 4, the amplitude-frequency characteristics are measured up to 0.01Hz. Comparing Table 1 and Table 2, within the frequency range of 0.05-2Hz, the national calibration results and the self-calibration results agree very well. Within the angular acceleration range of 0.0332 rad/s²~33.22 rad/s², the linearity of the rotational accelerometer is less than 0.09%. Within the angular acceleration range of 0.00976 rad/s²~33.22 rad/s², the linearity of the rotational accelerometer is less than 1.599%. Tables 5 to 6 show that the rotational accelerometer has the ability to resist transverse rotations and translations (horizontal and vertical vibrations). The strong-earthquake rotational accelerometer described in this article has obtained national patent, with the patent number: ZL201320066963.2. The angular vibration table has good low frequency characteristics, linearity and stability. The dual magnetic circuit servo-type ultra-low frequency angular vibration table can be used as the detection of seismic rotation sensors and other angular vibration sensors.

References

- [1] He Chao, Luo Qifeng, Hong Zhong(2014): A study of seismological rotational components, Journal of Seismological Research, 2011 (1).
- [2] Zhang Rongxiang, MengJunyi, Wei Gong. Overview of angular acceleration sensor studies.



- [3] Robert L. Nigbor. Six-Degree-of-Freedom Ground-Motion Measurement[J]. *Bulletin of the Seismological Society of America* (1994) 84 (5): 1665–1669.
- [4] Yang Zhenyu, Lu Haiyan. Six-degree freedom ground motion measurement method and its errors analysis[J]. *World Earthquake Engineering*. 2007,23(4):84-87.
- [5] Cai Naicheng, Fu Zizhong. Rotational seismograph developments.
- [6] Huang Zhenping, Wang Shaorong. Discussions of measuring rotational components using gyroscopic pendulum in strong earthquake measurements, *Earthquake engineering and engineering dynamics*, 24 (5) 2004.
- [7] Institute of Engineering Mechanics, China Earthquake Administration, Yang Xueshan, A seismological rotational accelerometer, China Utility Model Patent, CN 203054236 U.
- [8] China Aviation Industry Corporation Beijing Changcheng Institute of Measurement and Testing Technology, Certificate Number: Li D02 No. 2014030053
- [9] ISO 16063-15: 2006. Methods for the calibration of vibration and shock transducers —Part 15: Primary angular vibration calibration by laser interferometry.
- [10] GB/T 20485.15-2010: Methods for the calibration of vibration and shock transducers – Part 15: Primary angular vibration calibration by laser interferometry.
- [11] GJB/J6205-2008: Calibration specification for angular vibration transducers.