Comparative study of the wave and vibration method of seismic response of one-dimensional uniform shear straight bar media model

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Abstract

The observation data of building response under the actual earthquake and the test data of a large number of vibratory station tests show that as the height of buildings increases, the wave characteristics of seismic responses become increasingly obvious. For such high-rise buildings, it is clearly unreasonable to ignore the wave characteristics of their seismic response. Based on the one-dimensional uniform shear straight bar medium model of a closed system, this paper conducts theoretical and numerical comparisons of the displacement response of the two types of solutions under horizontal seismic action. The results show that the wave method based on continuous mass distribution can truly reflect the actual state of the medium model, and the accuracy of the vibration method based on discrete mass distribution depends on the degree of mass dispersion and the number of modal superpositions. Therefore, for super-high-rise buildings with high mass distribution of vertical rod spouts and significant seismic fluctuation effects, when using the vibration method to solve the earthquake response, the mass distribution characteristics should be considered and a reasonable physical model should be selected.

Keywords: wave characteristics, seismic response, high-rise buildings, closed system, wave method, vibration method

1. Introduction

Based on the one-dimensional (1D) uniform shear straight bar medium model of a closed system, this paper conducts theoretical and numerical comparative analysis of the displacement response of the two types of solutions under a horizontal seismic action. The results show that the wave method based on continuous mass distribution can truly reflect the actual state of the medium model, and the accuracy of the vibration method based on discrete mass distribution depends on the degree of mass dispersion and the number of modal superpositions. Therefore, for the super-high-rise buildings with high quality distribution of vertical rod spouts and significant seismic fluctuation effects, when using the vibration method to solve the earthquake response, the quality distribution characteristics should be considered and a rational physical model should be selected.

2. Calculation model

The 1D uniform shear straight bar medium model of the closed system is shown in Fig. 1. Based on the micro-elemnt analysis, the governing equation of the bar can be expressed as

$$c^2 \frac{\partial^2 u}{\partial z^2} - \frac{\partial^2 u}{\partial t^2} = 0 \tag{1}$$

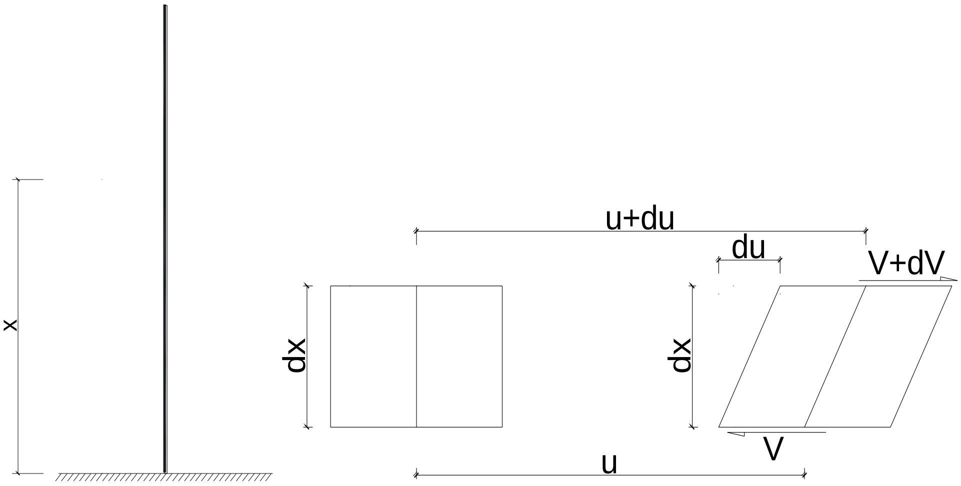


Fig. 1. The one-dimensional uniform shear straight bar medium model.

where u is the horizontal displacement, $c = \sqrt{\frac{GA}{m}}$ is the shear wave velocity, G , A and m are the shear modulus, cross-sectional area and density of the straight bar, respectively..

According to the characteristics of the model, we assume the following boundary conditions and initial conditions:

(1) The boundary conditions:

$$\frac{\partial u}{\partial z}\Big|_{z=H} = 0 \tag{2}$$

$$\frac{\partial^2 u}{\partial t^2}\Big|_{z=0} = \ddot{u}_g(t) \tag{3}$$

(2) The initial conditions:

$$u\big|_{t=0} = 0 \tag{4}$$

$$\frac{\partial u}{\partial t}\Big|_{t=0} = 0 \tag{5}$$

where H is the height of the straight bar, $\ddot{u}_g(t)$ is the acceleration of the ground.

In the following calculations, we assume that the ground acceleration is an instantaneous sinusoidal load and can be expressed as

$$\ddot{u}_g(t)=\begin{cases}\dfrac{\omega_0^2}{2\pi}\sin\omega_0 t, & 0\le t\le T_0\\ 0, & T_0<t\end{cases} \tag{6}$$

where $\omega_0=\dfrac{2\pi}{T_0}$, T_0 is the period of the instantaneous sinusoidal load.

3. Calculation method

3.1 Wave method[1] [2]

We input a unit sinusoidal acceleration pulse load, $\delta(t)$, at the bottom of the straight bar when t is zero. Then the pulse load travels in the straight bar with a velocity, c .. When t is H/c, the pulse reaches the top of the straight bar, and is an upward travel wave in the time of $\left[0,\ \dfrac{H}{c}\right]$. Due to the free top boundary of the straight bar, a reflected wave generates at the top and propagates downward in the time of $\left[\dfrac{H}{c},\ 2\dfrac{H}{c}\right]$. Because the straight bar is fixed at the bottom, when the reflected wave reaches the bottom, a reversed reflected wavegenerates and propagates upward In the time of $\left[0,\ 2\dfrac{H}{c}\right]$, the pulse wave completes a cyclic prioagation in the bar.

In the first cycle, i.e., in the time of $\left[0,\ 2\dfrac{H}{c}\right]$,

$$\dot{u}_1^+=\delta(t-\frac{z}{c}),0\le t\le\frac{H}{c} \tag{7}$$

$$\dot{u}_1^- = \delta(t - \frac{H}{c} - \frac{H-z}{c})$$
$$= \delta(t - \frac{2H}{c} + \frac{z}{c}), \frac{H}{c} \le t \le \frac{2H}{c} \qquad (8)$$

In the N-th cycle, i.e., when $\frac{2(N-1)H}{c} \le t \le \frac{2NH}{c}$, the sum of the absolute speeds of the upward and downward waves can be expressed as

$$\dot{u}_a(z,t) = \sum_{n=1}^{N} (-1)^{n-1} \left[\delta\left(t - \frac{2(n-1)H}{c} - \frac{z}{c}\right) + \delta\left(t - \frac{2nH}{c} + \frac{z}{c}\right) \right] \qquad (9)$$

When the pulse shown in Eq. 6 is input at the bottom of thebar, the absolute acceleration of a point in the bar can be derived as

$$a(z,t) = \sum_{n=1}^{N} (-1)^{n-1} \left[\ddot{u}_g\left(t - \frac{2(n-1)H}{c} - \frac{z}{c}\right) + \ddot{u}_g\left(t - \frac{2nH}{c} + \frac{z}{c}\right) \right] \qquad (10)$$

3.2 Vibration method[3]

The natural angular frequency of the 1D uniform shear straight bar with infinite degrees of freedom can be expressed as

$$\omega_n = \frac{(2n-1)\pi c}{2H} \qquad (11)$$

where ω_n is the n-th natural angular frequency, and n =1,2,3,…….

The *n*-th mode function can be expressed as

$$\Psi_n(z) = (\frac{2}{H})^{\frac{1}{2}} \sin \frac{(2n-1)\pi}{2H} z \qquad (12)$$

where $\sqrt{\frac{2}{H}}$ is used for normalized.

According to Ref. 1, the relative displacement of the 1D uniform shear straight bar with infinite degrees of freedom can be expressed asa

$$u(z,t)=\begin{cases}\displaystyle\sum_{n=1}^{\infty}\frac{-2\omega_0^2}{(2n-1)\pi^2\omega_n}\left(\frac{\omega_n\sin\omega_n t-\omega_0\sin\omega_0 t}{\omega_0^2-\omega_n^2}\right)\sin\left(\frac{\omega_n}{c}z\right), & 0\le t\le T_0\\ \displaystyle\sum_{n=1}^{\infty}\frac{-2\omega_0^3}{(2n-1)\pi^2\omega_n}\left(\frac{\sin\omega_n t-\sin\omega_n(t-T_0)}{\omega_0^2-\omega_n^2}\right)\sin\left(\frac{\omega_n}{c}z\right), & T_0<t\end{cases} \tag{13}$$

By derivating Eq. 13 twice, the relative acceleration of the 1D uniform shear straight bar can be expressed as

$$\ddot{u}(z,t)=\begin{cases}\displaystyle\sum_{n=1}^{\infty}\frac{2\omega_0^3}{(2n-1)\pi^2}\left(\frac{\omega_n\sin\omega_n t-\omega_0\sin\omega_0 t}{\omega_0^2-\omega_n^2}\right)\sin\left(\frac{\omega_n}{c}z\right), & 0\le t\le T_0\\ \displaystyle\sum_{n=1}^{\infty}\frac{2\omega_0^3\omega_n}{(2n-1)\pi^2}\left(\frac{\sin\omega_n t-\sin\omega_n(t-T_0)}{\omega_0^2-\omega_n^2}\right)\sin\left(\frac{\omega_n}{c}z\right), & T_0<t\end{cases} \tag{14}$$

Adding the relative acceleration to the ground acceleration, the acceleration of the bar can be expressed as

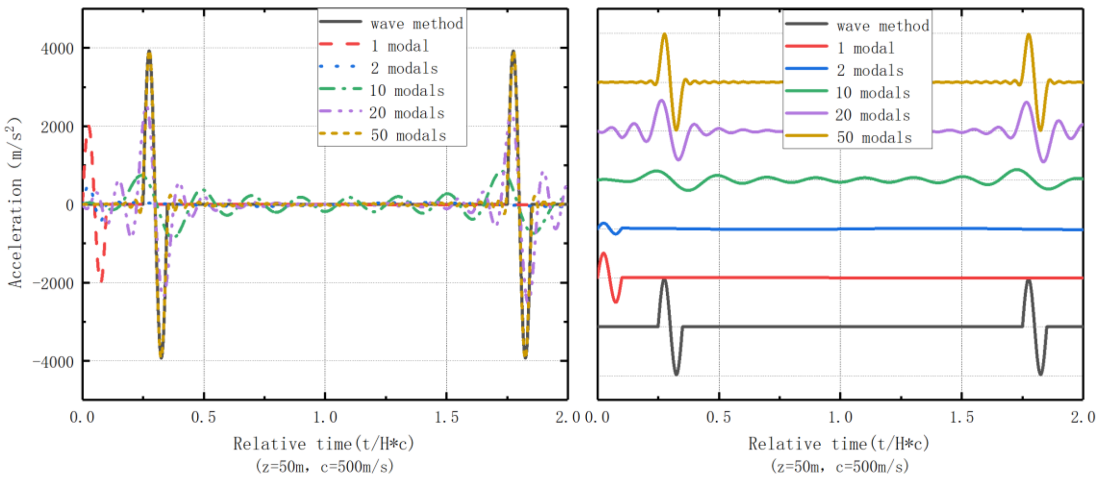
$$a(z,t)=\ddot{u}(z,t)+\ddot{u}_g(t) \tag{15}$$

4. Calculation results

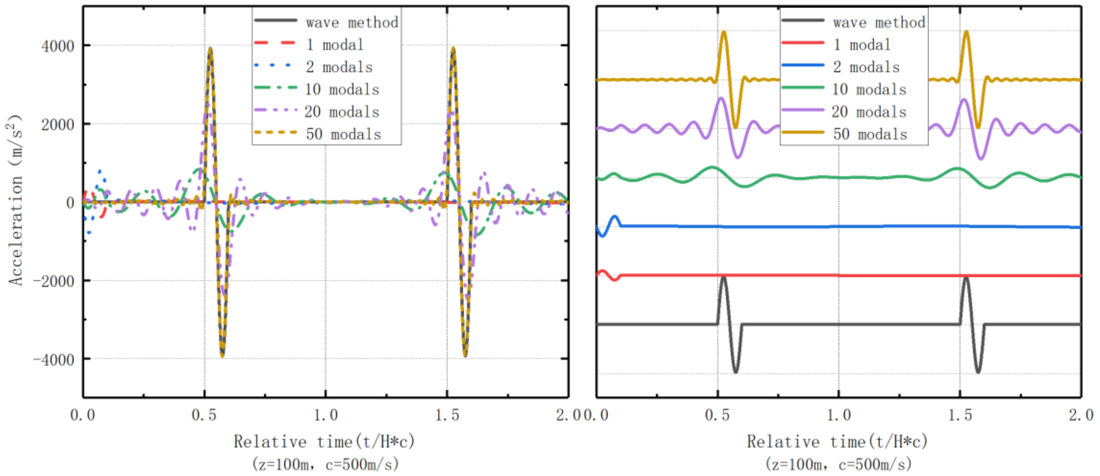
The input ground acceleration is shown in Eq. 6 and T_0 =0.04 s:

The model parameters are as follows:

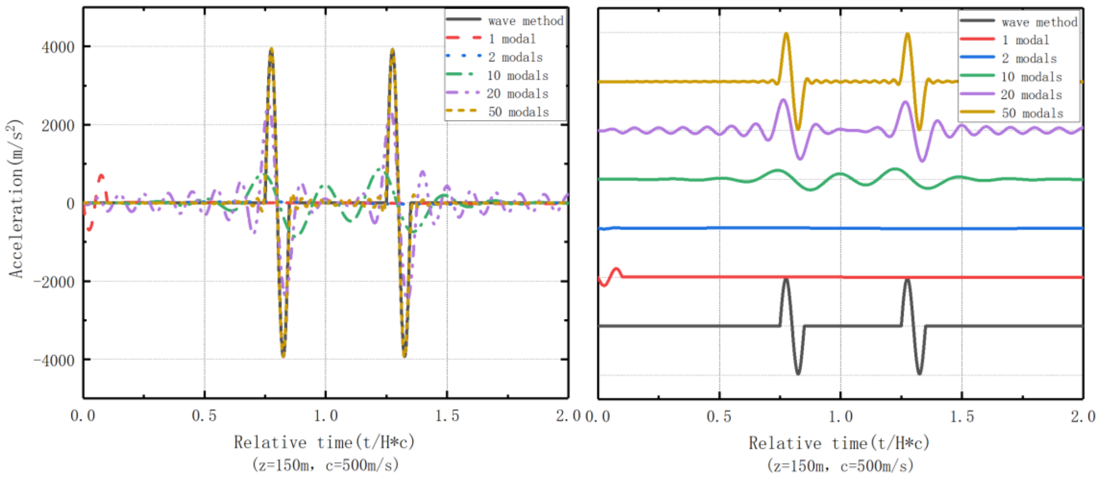
The height, cross-sectional area, mass per unit length, shear modulus and shear wave velocity of the straight bar are200 m, 1.0m^2, 1.0 kg,250000N/$\text{m}^2$and 500 m/s, respectively.



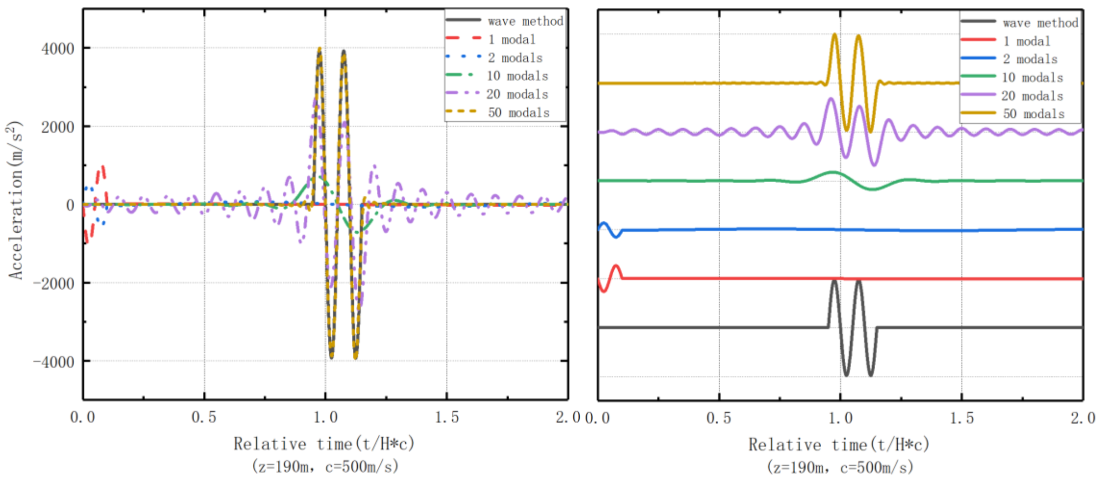
(a)



(b)



(c)



(d)

Fig. 2. Comparison of acceleration time history curves of wave method and vibration method at different building heights.

It can be seen from Fig. 2 that theoscillation peak value and waveform of the wave solution remain unchanged at different floor height positions. This is because the wave solution method is not only suitable for linear and non-linear structures, but also for closed and open systems. For the vibration solution, theoscillation peaks and waveform variation of the curves depend on the number of modes. When the number of modes is 1, 2, the waveforms of the vibration solution and the wave solution do not coincide at all, which indicates that the vibration solution isnot accuracy for a low mode number. This is because the input ground motion period is greatly different from the basic natural period of the structure. From a physical point of view, the principle of the vibration solution based on the overall vibration of the structure lies in the superposition of the internal standing waves of the linear elastic structure, which lacks the linear superposition condition of the internal standing waves. When the number of modes is 10 or 20, the waveforms of the vibration solutions tend to be closer to the wave solution. When the number of modes is 50, these two solutions are almost identical. This shows that the larger the number of modes, the closer the waveform of the vibration solution to the wave solution.

In summary, when the number of modes is sufficiently large, the calculation results of the vibration method and the wave method are almost indentical. The acceleration values and the change laws are basically the same, and the vibration method meets the accuracy requirements. The wave method can reflect the real situation of the dynamic response of the structure and the time effect of the tarvel wave propagation, but the calculation procedure is more complicated. The vibration method cannot reflect the time effect of the tarvel wave, and can only reflect the change of relative time, but the calculation procedure is relatively simple. The appropriate modal number can be seleceted according to the actual situation when using vibration method, and the calculation results is basically consistent with the wave method, which is more practical.

5. Conclusion

(1) For highly flexible structures, such as super high-rise buildings, when the value of the modal number is not reasonable， the calculation results of the modal superposition method have large deviation from the wave method and thus cannot be used for design guidence.

(2) The wave solution is the real dynamic response process of the structure that truly reflects the dynamic characteristics of the wave propagation in the structure, and the wave solution is an accurate solution.

(3) By comparing and analyzing the results of the two calculation methods, we can find that when the number of modes is sufficiently large, the results of the continuum vibration modal superposition method

and wave method are almost identical, and the acceleration values and the change laws are basically the same. The calculation accuracy of the vibration modal superposition method meets the requirements.

6. References

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