MODAL COMBINATION IN RESPONSE SPECTRUM ANALYSIS: EXPLORING THE VARIATION OF GROUND MOTIONS IN TIME

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Abstract

Response Spectrum Analysis (RSA), one of the most popular methods to carry out the seismic design of multi-degree-of-freedom (MDOF) structures, is based on the concept of modal superposition, by which the uncoupled equations of motion that represent each mode of vibration of the system can be solved independently and the resulting responses superimposed by assuming linear elastic behaviour. Each mode is represented by a single-degree-of-freedom (SDOF) system, whose peak response is retrieved from response spectra deemed suitable for design. However, while modal superposition allows for the total response of a MDOF system to be determined by simple addition of the individual modal responses at each time step, combination of spectral values needs to take into account the fact that peak modal responses do not necessarily occur at the same time or along the same horizontal directions.

These considerations give rise to the use of modal and spatial combination rules that aim to calculate the likely peak response of a MDOF system instead of conservatively carrying out an algebraic sum of maxima. Current design codes prescribe methodologies that were defined in the 1970s and 1980s, such as the Complete Quadratic Combination (CQC) [1], its three-dimensional extension CQC3 [2], the Square Root of the Sum of the Squares (SRSS) [3], or the 30% rules [4], based mostly on random vibration theory. However, access to large numbers of ground motion records at the present time allow us to revisit these approaches from a data-driven perspective, and investigate the relationship across the peaks of SDOF responses to seismic excitation at different orientations and at different points in time, with the ultimate goal of characterising this relationship in a fully probabilistic way.

This paper presents results of a study of SDOF demands obtained considering 1,218 accelerograms from the RESORCE database [5], whose two horizontal perpendicular components were rotated around all non-redundant angles every 2° and applied to SDOF systems with periods of vibration of 0.2, 1.0 and 3.0 seconds, and sets of secondary systems with periods ranging from 0.5 through 0.95 times the three aforementioned periods. The concept of peak response was extended to include all peaks with amplitudes above two alternative thresholds of 80% and 95% of the maximum absolute response. Two main kinds of parameters were studied and are presented: (i) time differences between peaks of the same component and across perpendicular components, and (ii) ratios of instantaneous displacement demands between perpendicular components and the same component for different oscillator periods, as one of the components reaches a peak in the oscillator's response. While results for the latter resemble the idea of the 0.3 coefficient from the 30% rule in average terms, the dispersion associated with all these parameters is large and should not be neglected.

Keywords: response spectrum analysis, modal combination, spatial combination, modal superposition, 30% rule

1. Introduction

The assumption that structures can be designed for seismic action using elastic theory as long as the input pseudo-acceleration response spectra accounts for the expected relationship between elastic and inelastic behaviour (*e.g.*, through the concept of a ductility-dependent reduction factor) allows for the concept of modal superposition to be used in everyday engineering design. Response Spectrum Analysis (RSA), one of the most popular methods to carry out the seismic design of multi-degree-of-freedom (MDOF) structures, is based on this concept, by which the uncoupled equations of motion that represent each mode of vibration of the system can be solved independently and the resulting responses superimposed at the end. In RSA, each mode is represented by a single-degree-of-freedom (SDOF) system, whose peak demand is retrieved from the aforementioned response spectra, deemed suitable for design. However, while the step-by-step addition of individual modal responses is valid to determine the total response of a MDOF system when working in the time domain, the combination of spectral values (*i.e.*, the maxima of the time series) needs to take into account the fact that peak modal responses do not necessarily occur at the same time for all modes or along different horizontal directions of motion.

The need to carry out a summation of maxima in a way that does not over-conservatively assume that all peaks occur simultaneously gives rise to the use of modal and spatial combination rules that aim to calculate the likely peak response of a MDOF system as a whole. Some of the most popular such rules are the Complete Quadratic Combination (CQC) [1], its three-dimensional extension CQC3 [2], the Square Root of the Sum of the Squares (SRSS) [3], and the 30% rules [4], which were defined in the 1970s and 1980s, based mostly on random vibration theory. In terms of modal response, the SRSS rule can be considered to be a sub-case of the CQC rule when the different modes have well-separated natural frequencies and, thus, their modal cross-correlation tends to zero, while the CQC rule explicitly takes the modal cross-correlation into account. In terms of the combination of different components of ground motion, the CQC3 rule attempts to take into account both the modal cross-correlation and the cross-correlation between the components of the seismic excitation. Under certain conditions, such as a structure in which the closely-spaced frequencies correspond to modal vectors situated in perpendicular planes, the SRSS spatial combination rule gives results sufficiently similar to those of the CQC3 rule. The 30% spatial combination rule is an approximate procedure that assumes lack of correlation of the input motion along any pair of orthogonal directions, and indicates that any member of the structure needs to be designed to the simultaneous effect of 100% of the spectrum applied along direction X of the building and 30% applied along direction Y, and vice versa, whichever is largest.

While these rules have been of use for the earthquake engineering community for decades, the increasingly large number of ground motion records available nowadays allows us to revisit these approaches from a data-driven perspective, and investigate the relationship across the peaks of SDOF responses to seismic excitation at different orientations and at different points in time, with the ultimate goal of characterising this relationship in a fully probabilistic way. This paper represents a first step taken in this direction by presenting a study carried out using 1,218 accelerograms from the RESORCE database [5], according to the methodology described in the following section. Results regarding the time differences between peaks of the same component and across perpendicular components are presented in section 3, while section 4 discusses those pertaining the ratios of instantaneous displacement demands between SDOF oscillators of different periods when subject to the same and perpendicular components of ground motion, as one of the two reaches a peak. These results and their implications for current prescriptions in seismic codes are discussed in section 5, where conclusions are drawn.

2. Methodology

For the present study, records were selected from the RESORCE database [5] to satisfy the following criteria: no pulse-like characteristics (as determined by means of the algorithm of Shahi and Baker [6]), available value of V_{s30}, available value of moment magnitude, maximum usable period of at least 3.2 seconds, and sensor location other than the upper floors of a building. This search yielded 1,218 pairs of accelerograms, of which 89 (7.3%) and 770 (63.2%) are reported to be located in accelerometer shelters and either the basement or ground floor of buildings, respectively, while for the remaining 359 (29.5%) the relative location of the sensors with respect to structures is unknown.

The two perpendicular components of each of the 1,218 earthquake acceleration records were rotated around all non-redundant angles every 2° and used as excitation for a series of elastic single-degree-of-freedom (SDOF) oscillators with 5% damping ratio. At each orientation, the cross-correlation coefficient between the two components X, Y of ground motion was determined as a function of the ratio α_{AI} of the Arias intensities along the two principal directions of motion (i.e., the orientations at which the cross-correlation between perpendicular components is zero) and the angle between the X component and the main principal direction, θ, as per Eq. (1) [7].

$$\rho_{XY} = -(1-\alpha_{AI}) \cdot \sin(2\theta) / \sqrt{(1+\alpha_{AI})^2 - (1-\alpha_{AI})^2 \cdot [\cos(2\theta)]^2} \tag{1}$$

Fig.1 shows the distribution of the cross-correlation values obtained for all 1,218 records. The plot, which considers only the maximum of all possible angles of incidence for each record, shows that statistics for cross-correlation values larger than about 0.4 are likely to be irrelevant and, as will be shown, excessively noisy. The same tendency for larger values of maximum cross-correlation to be uncommon was observed for the whole of the RESORCE database [5], opening the question of whether the lack of such values is (or not) a world-wide phenomenon and reflects (or not) some kind of physical constraint [8]. Exploration of other ground motion databases in the future could shed some light on this matter.

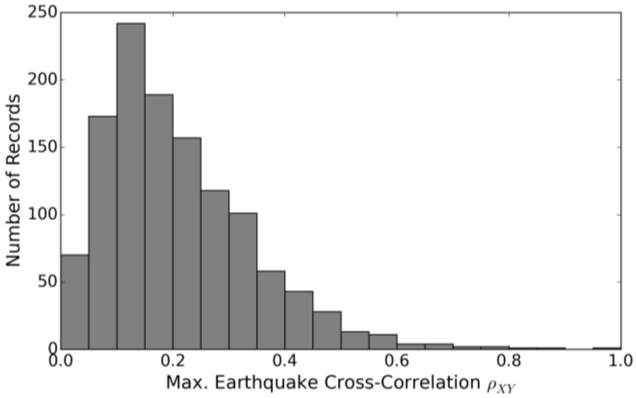

Fig. 1 – Cross-correlation coefficient between horizontal components ρ_{XY} of the 1,218 records. The maximum of all possible angles of incidence was selected for each record.

Three primary oscillator periods Ti were used, of 0.2, 1.0 and 3.0 seconds, and each of these in combination with a series of secondary periods Tj defined by seven Tj/Ti ratios: 0.95, 0.90, 0.85, 0.80, 0.70, 0.60 and 0.50. These are associated with modal cross-correlation coefficients ρ_{ij} of 0.791, 0.473, 0.273, 0.166, 0.071, 0.035 and 0.018 as per Eq. (2), where r = Tj/Ti [1]. Eq. (2) was developed by [9] using a white noise input.

$$\rho_{ij} = \left[8 \cdot \sqrt{\xi_i \cdot \xi_j} \cdot (\xi_i + r \cdot \xi_j) r^{3/2}\right] / \left[(1-r^2)^2 + 4 \cdot \xi_i \cdot \xi_j \cdot r \cdot (1+r^2) + 4 \cdot (\xi_i^2 + \xi_j^2) \cdot r^2\right] \tag{2}$$

The parameters used to assess the variation of the response of the SDOF oscillators in time, computed at each considered orientation, were:

- The minimum time difference between each of the peaks in the response of a SDOF system with period Ti to one component (X, Y) with respect to their closest peaks in the response of a SDOF system with each of the Tj periods to the same component.
- The same, but crossing components (i.e., comparing component X applied to a SDOF with period Ti against component Y applied to a SDOF with period Tj).

- The percentage/fraction of Sd(Tj) that the response to a component develops at the time of each of the peaks in the response to the same component for Ti (i.e., comparing component X applied to two SDOFs, one with period Ti and the other with period Tj).
- The percentage/fraction of Sd(Tj) that the response to a component develops at the time of each of the peaks in the response to the perpendicular component for Ti (i.e., comparing component X applied to a SDOF with period Ti against component Y applied to a SDOF with period Tj ; the same as above, but crossing components).
- The maximum percentages/fractions for the two above, per record and orientation.

The percentage of Sd(T) at the occurrence of the peaks in the other component/period is measured with respect to the same component. For example, if component X is applied to a SDOF with period Ti, component Y is applied to a SDOF with period Tj, and the peaks of the response $d_X(Ti, t)$ are identified, the output parameter is the displacement of the SDOF system with period Tj when subject to component Y divided by the spectral displacement of component Y at period Tj, i.e., $d_Y(Tj, t_{peak\ X,i})/Sd_Y(Tj)$, where $t_{peak\ X,i}$ is the time of the peak of SDOF with Ti subject to component X. The purpose of this is to be able to differentiate the relationship between the overall intensity of the two perpendicular components ($Sd_X(T)/Sd_Y(T)$) from the variation in time of the intensity within each of them, as the former can be addressed by means of the ratio of spectral demand with respect to the direction of maximum response, as suggested elsewhere [10,11].

Local peaks in the response of the SDOF systems were identified as described in Appendix E of Nievas (2016) [8], and then selected according to five different threshold levels, 50%, 70%, 80%, 90% and 95%. This was done, rather than only considering the absolute peak, because, normally, the latter is attained only once, but several peaks of slightly smaller but still significant intensities can occur. The overall procedure of detection of peaks and assessment of the response at the perpendicular component is illustrated in Fig.2 (the procedure is the same for acceleration time histories and the response of SDOF systems). Results for only two of the threshold levels—80% and 95%, aimed at illustrating the range—will be presented in what follows.

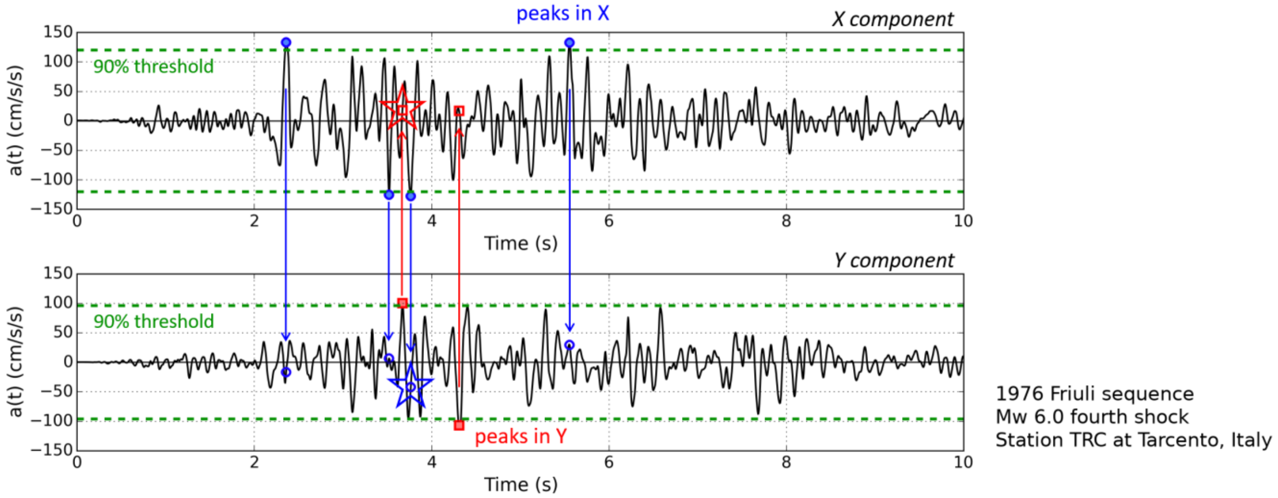

Fig. 2 – Example of the procedure followed to identify peaks and relevant parameters. Filled blue circles and red squares indicate the peaks above the 0.90 threshold in the X and Y components, respectively. Empty blue circles and red squares indicate the points in the Y and X components that correspond to the peaks in the X and Y components, respectively. The stars show the largest of these latter values.

3. Time Difference Between Peaks

3.1 Same Component

Fig.3 shows the relationship observed between the minimum time difference between peaks of the responses of oscillators with different periods (Ti vs Tj) to the same record component and the modal cross-correlation coefficients ρ_{ij} between the Ti and Tj periods. Each point shown corresponds to one of the Tj/Ti ratios considered, while each colour makes reference to the primary period Ti only. A separate analysis of the data did not allow to reach a conclusion regarding whether the minimum time differences calculated between peaks of responses of SDOF system oscillators with different periods to the same component follow a specific theoretical distribution. For this reason, Fig.3 shows the results obtained both in linear (left) and logarithmic (right) spaces. As can be observed, the means decrease with increasing modal cross-correlation coefficients ρ_{ij}, as would be expected. The standard deviations are also observed to decrease with increasing ρ_{ij}. The trivial case of Ti=Tj (the same component and the same period, i.e., the same displacement history) for which $\rho_{ij}=1$ would lead to a zero-time difference between peaks and a null standard deviation. The plot in Fig.3 thus confirms the relevance of the modal cross-correlation coefficient for representing the synchronicity of the several modes of vibration of a structure subject to seismic excitation, a fundamental premise of the CQC method. It is noted that the use of thresholds to define the peaks to consider leads to the calculated time differences reflecting not only the syncing between the responses (*i.e.*, the alignment of the peaks) but also on the variation in the amplitudes of the motion for various consecutive cycles.

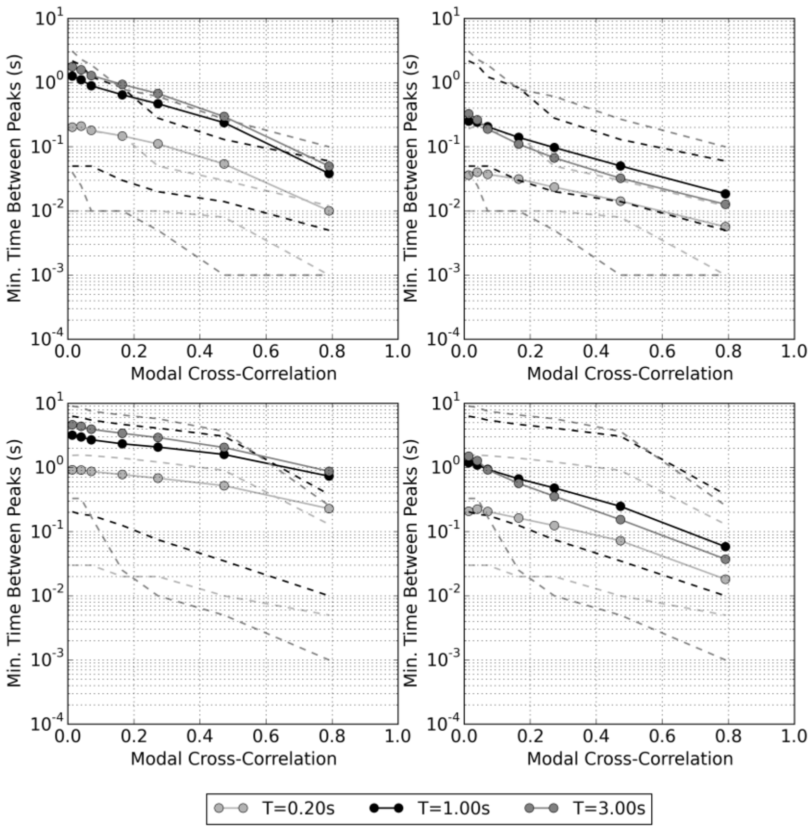

Fig. 3 – Minimum time difference between peaks of the responses of oscillators with different periods to the same component, against modal cross-correlation coefficients ρ_{ij} for two threshold levels: 80% (top) and 95% (bottom). Indicated periods are the primary Ti values. Solid lines indicate mean values, dashed lines indicate the 16th and 84th percentiles. Plots on the left refer to the data in linear space (as-is), plots on the right refer to the natural logarithm of the data[1].

[1] The means correspond to means in linear space, i.e. $\bar{x}$, and means in logarithmic space, i.e. $\overline{\ln x}$, transformed back into linear space, i.e. $e^{\overline{\ln x}}$.

3.2 Perpendicular Components

When the peaks in one component are confronted against those in the perpendicular component, the cross-correlation between the earthquake acceleration components ρ_{XY} needs to be accounted for. Fig.4 shows the means and standard deviations obtained against ρ_{XY} for the data binned according to this parameter. Statistics are shown in logarithmic space, as it was observed that data binned according to ρ_{XY} seem to tend to a log-normal distribution, even though the fit is not as good as could be observed when the ground motions themselves were considered (instead of the response of oscillators) in such an analysis [8]. As can be observed in Fig.4, the obtained means decrease with increasing cross-correlation between the earthquake components, while the standard deviations appear as relatively constant for cross-correlation values of up to around 0.4, the limit of what can be considered to represent meaningful statistics due to the lack of records associated with larger values (see Fig.1). In linear (*i.e.*, non-logarithmic) space, the means present a similar trend to that shown in Fig.4, while the standard deviations appear to be more dependent on the cross-correlation between earthquake components and the period of the primary oscillator as well.

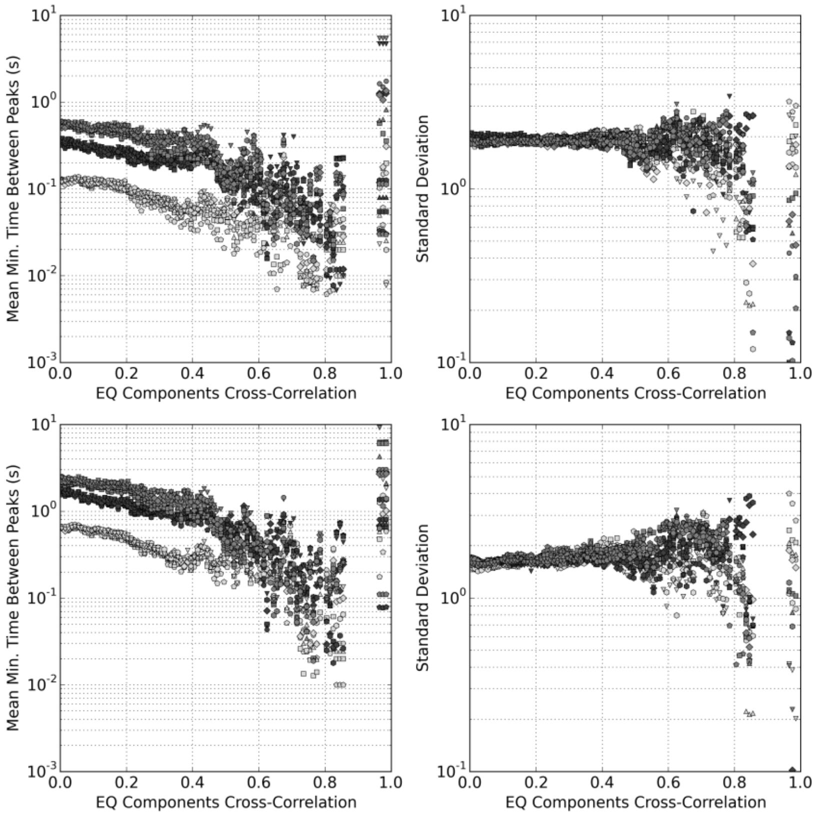

Fig. 4 – Minimum time difference (left: means, right: standard deviations; computed in terms of the natural logarithm of the data) between peaks of the responses of oscillators with different periods to perpendicular components, against cross-correlation coefficients of the earthquake components ρ_{XY} for two threshold levels (80%, top, 95%, bottom), three primary Ti values (0.2 s, extremely light grey, 1.0 s, black, 3.0 s, light grey), and seven Tj/Ti ratios (0.95, pentagons, 0.90, squares, 0.85, triangle up, 0.80, circle, 0.70, rhombus, 0.60, hexagon, 0.50, triangle down).

The influence of the modal cross-correlation values ρ_{ij} seems somewhat unclear. A close-up look at Fig.4 reveals that the means for the smallest Tj/Ti ratio considered (0.5, $\rho_{ij} = 0.018$, marked with downward-pointing triangular markers) appear to be consistently higher than those for the remaining ratios, while the latter seem to alternate more and not always be ordered by modal correlation coefficient. While Fig.3 indicates that smaller means should be associated with larger values of modal correlation coefficients, Fig.4 suggests that the influence of the modal correlation between periods of vibration might be overshadowed by the correlation between earthquake components when the time difference is measured between two perpendicular components instead of the same one. It is nevertheless noted that the case of the same component applied to different oscillators would be the ultimate extreme case of perfectly correlated perpendicular components (ρ_{XY} = 1.0) and that, as shown in Fig.1, only a reduced number of records with maximum ρ_{XY} larger than 0.4 are available.

4. Percentage of Sd(T) at Peaks

4.1 Same Component

The distributions of the percentages (or fractions) of Sd(Tj) of the response of the secondary oscillators at all peaks of the primary one when both are subject to the same component of ground motion appear to vary for different values of the modal cross-correlation coefficient ρ_{ij} and the threshold values used to select the peaks. As a separate analysis (reported in [8]) aimed at analysing the potential use of different theoretical distributions to describe this parameter was not conclusive (though it did indicate the potential relevance of the Weibull distribution to describe its sign-inverted logarithm), results are presented in what follows in both logarithmic and non-logarithmic terms. Fig.5a depicts the resulting mean values and 16th and 84th percentiles against the modal cross-correlation values ρ_{ij} (as in Fig.3). As would be expected, the mean percentage of Sd(T) attained by the oscillator with period Tj at the peaks of the primary oscillator (with period Ti) increases with ρ_{ij}. For the trivial case of Ti=Tj (the same component and the same period, i.e., the same displacement history) for which ρ_{ij}=1 would lead to peaks in Ti to fully coincide with peaks in Tj and thus the minimum percentage of Sd(Tj) possible to be the threshold considered to define the peaks (80% and 95% in Fig.5a). Consequently, the mean values would then always be above the corresponding thresholds for ρ_{ij}=1. This is reflected, firstly, in the narrowing of the 16th-84th percentile band for increasing values of ρ_{ij} and, secondly, in larger values of the modal cross-correlation coefficient ρ_{ij} being associated with a more accentuated tendency for the means to increase with increasing values of the thresholds.

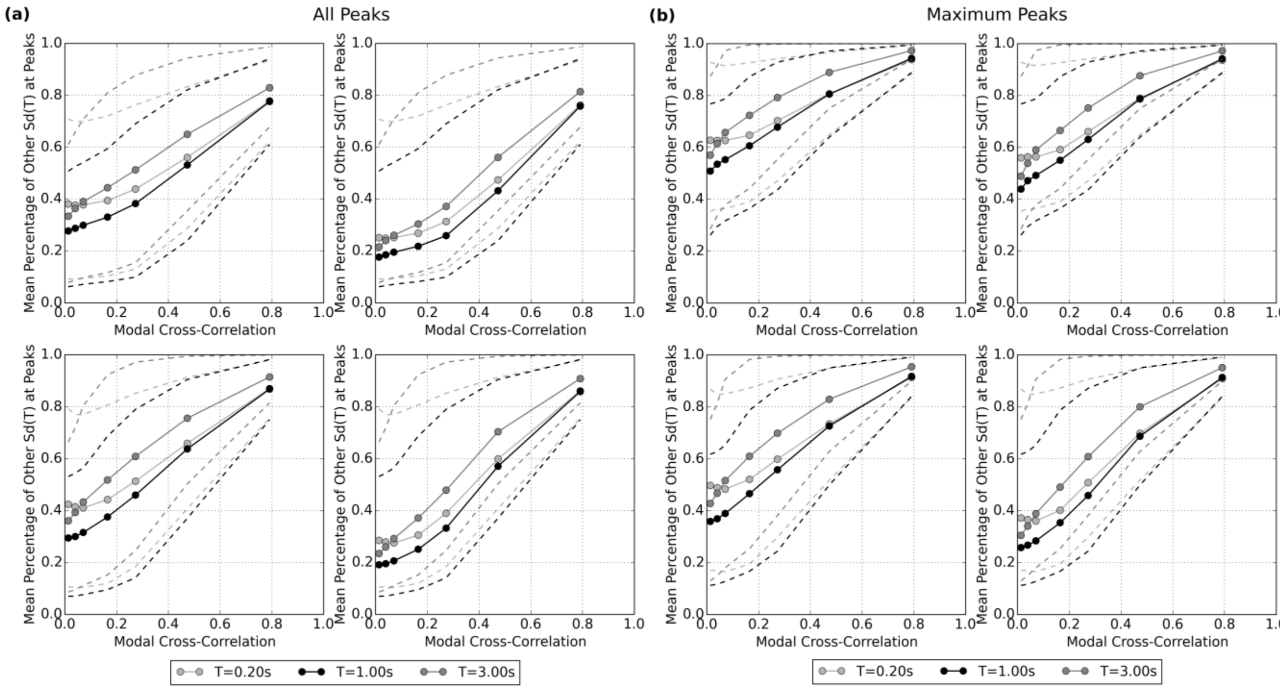

Fig. 5 – (a) Means (solid lines) and 16th – 84th percentiles (dashed lines) of the fraction (0-1=0-100%) of Sd(Tj) of the response of the secondary SDOF systems at all peaks of the response for Ti (0.2, 1.0 and 3.0 s, as per indicated greyscale) when subject to the same component, against modal cross-correlation coefficients ρ_{ij} for two threshold levels: 80% (top) and 95% (bottom). Plots on the left refer to the data in linear space (as-is), plots on the right refer to the natural logarithm of the data. (b) Same as (a), but considering only the peaks at which the maximum fraction of Sd(Tj) occurs.

An interesting feature of Fig.5a is that the means are not in order (either ascending or descending) according to the primary oscillator periods Ti. While the cause of this phenomenon has not been identified in the present study and could be due to chance (*i.e.*, a peculiarity of the ground motions used), it is possible that it may be related to an observation by Baker and Cornell [12] who, developing correlation models for the residuals of spectral acceleration values at different oscillator periods (for the same component), observed that the correlation does not always increase with increasing separation of periods and identified (from their data) T=0.189 s as an inflection point for their model, but were unable to provide a reason for this phenomenon.

Considering only the peaks at which the maximum fraction of Sd(Tj) occurs, the means tend to be smaller for increasing values of the thresholds used to define the peaks, as shown in Fig.5b. This is the opposite of what occurs in Fig.5a when all peaks are considered and is probably due to a big-number effect: the smaller the threshold, the more peaks that get considered and, consequently, the higher the chances that one of these will be associated with a larger fraction of Sd(Tj). For the trivial case of Ti=Tj (ρ_{ij}=1), the only possible value of the fraction of Sd(Tj) in Fig.5b would be 1.0, as peaks are aligned with peaks and the largest of all, that is, Sd(Tj) itself, is selected.

4.2 Perpendicular Components

The assessment of results obtained for perpendicular components of ground motion benefits from taking a first look at the ground motions themselves. As a Weibull distribution appeared to fit well the sign-inverted logarithm of the data, the statistics presented in Fig.6 were calculated in logarithmic space. As can be observed, the fraction (0.0-1.0 of the total) of the peak ground acceleration (PGA) developed at the moment of the peaks of the perpendicular component tends to increase with an increase in the cross-correlation between the earthquake acceleration components ρ_{XY}. It is interesting to note that at null cross-correlation, the mean values are almost the same for all values of the threshold when all peaks are considered (plot on the left), but then different thresholds attain different means for increasing correlation values. The opposite is true when only the maximum peaks are considered (plot on the right), possibly due to the big-number effect described earlier. While statistics for cross-correlation values larger than 0.4 are not considered reliable (see Fig.1), it is interesting to note that the tendency for all means to converge to 1.0 when the maximum peaks are considered (and the same to occur for the means stemming from the 95% threshold when all peaks are considered) is reasonable from the theoretical point of view, as a cross-correlation ρ_{XY} of 1 would indicate that the two components are completely in sync and their peaks always aligned. It would be beneficial to verify this observation with additional records with highly correlated components, if available. The overall tendencies of the mean values in non-logarithmic space is the same as those shown in Fig.6, albeit with slightly larger values (*e.g.*, around 0.25-0.3 for ρ_{XY} =0). Standard deviations are very large in all cases, and density plots depicting all data show that the whole range of ratios (0-1) occur, at least where a sufficiently large number of records is available.

Returning to the response of the SDOF oscillators, Fig.7 shows the means and standard deviations obtained against the cross-correlation of earthquake components ρ_{XY}. As can be observed, both statistics are relatively stable for cross-correlation values below around 0.4, though there seems to be a tendency for the mean to increase slightly in a behaviour similar to that in the top left of Fig.6. In order to be able to better assess the influence of the modal cross-correlation, “slices” of correlation of earthquake components ρ_{XY} were examined in Fig.8 (left) for the 95% threshold level. Although tending to slightly increase with increasing values of the modal cross-correlation coefficients, the means appear to be relatively stable with respect to this parameter, especially for modal correlation coefficients above 0.1 and the first three ρ_{XY} bins shown, which correspond to the range where data is more abundant. Just like for the case of the ground motions themselves, standard deviations are very large. The 1.1 logarithmic standard deviations depicted in Fig.8 imply, for example, that ratios up to 0.6 are only one standard deviation away from the 0.2 shown as the expectation in logarithmic space.

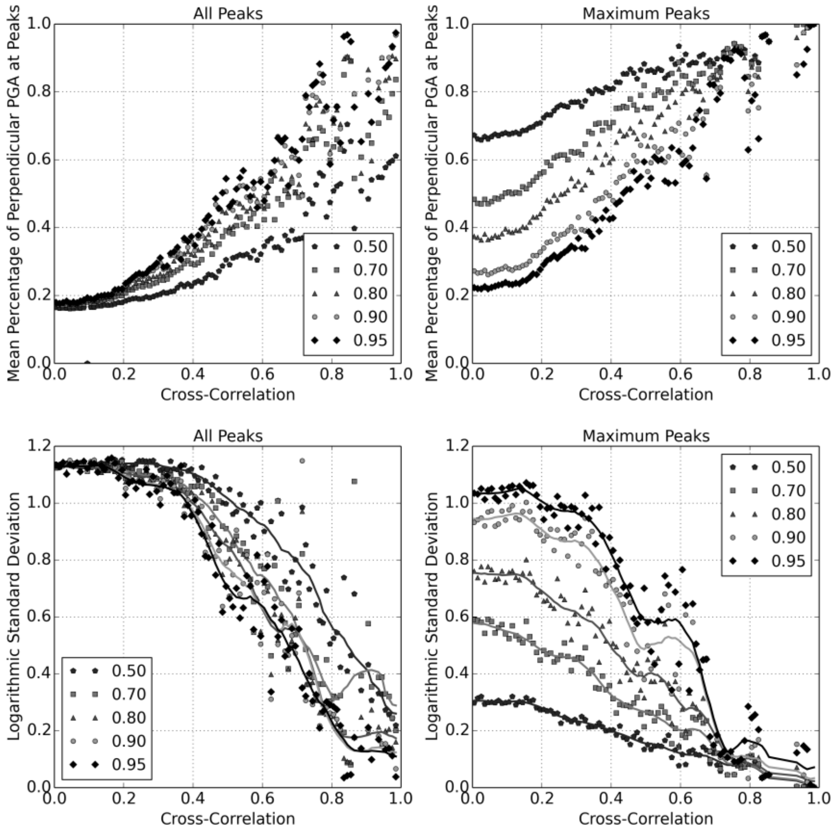

Fig. 6 – Mean (top) and standard deviation (bottom) of the fraction (0.0-1.0) of PGA of the perpendicular components at peaks of the other component, for five thresholds levels (from 50% through 95%, see legend), for all peaks (left) and for the peaks at which the maximum fraction occurs (right), against the cross-correlation of the earthquake components (ρ_{XY}). Statistics calculated from the natural logarithm of the data.

The similarities and differences in the behaviour of the means in the top left plot of Fig.6 and the plots on the left of Fig.7 are worth highlighting, and may indicate the influence of the modal responses and their different degrees of cross-correlation on the syncing of demands of the two perpendicular components. The fact that the means tend to be more independent of ρ_{XY} in Fig.7 than Fig.6 for ρ_{XY}<0.4 suggests that the oscillators have a stabilising effect on the variable under study. Moreover, it is noted that this behaviour is observed at all threshold levels, and that the latter have only a small influence in the actual mean values observed. More records with ρ_{XY}>0.4 are needed to understand what occurs at this other range.

Values of Fig.7 and Fig.8 were calculated in logarithmic space to be comparable against those of Fig.6, though the appropriateness of such decision needs to be corroborated in further studies. The slight tendency for the mean to increase with increasing cross-correlation values is more pronounced in the linear space, and the values of the means themselves are larger (they lie in the range ~0.3-0.4 for ρ_{XY}=0 instead of ~0.2-0.3, when all peaks are considered, and in the range ~0.3-0.45 for ρ_{XY}=0 instead of ~0.25-0.3, for only those of the maxima, 95%).

The aforementioned tendencies of the means are similar when only the peaks of the response for Ti where the maximum fraction of Sd(Tj) occurs are considered, though in this case the mean values significantly decrease for increasing thresholds, as was observed for the same component (Fig.5b). In all cases they are larger than those shown in Fig.7. The plots on the right of Fig.8 show the behaviour of the means and standard deviations for the 95% threshold level in term of "slices" of correlation of earthquake components ρ_{XY}. As can be observed, the tendencies are similar to the plots on the left of Fig.8 for the case of all peaks being considered, but hover around 0.3 rather than 0.2.

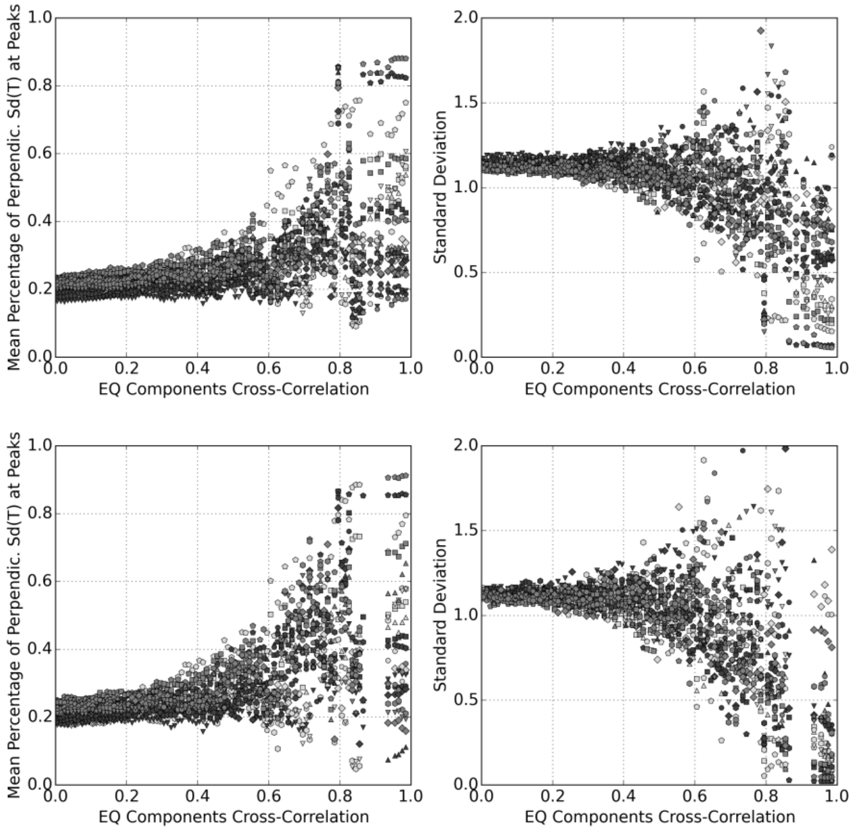

Fig. 7 – Means (left) and standard deviations (right) (computed in terms of the natural logarithm of the data) of the fraction (0-1=0-100%) of Sd(Tj) of the response of the secondary SDOF systems at all peaks of the response for Ti when subject to the perpendicular component, against cross-correlation coefficients of the earthquake components ρ_{XY}, for two threshold levels (80%, top, 95%, bottom), three primary Ti values, and seven Tj/Ti ratios. Colours by Ti and markers by Tj/Ti as in Fig.4.

5. Discussion and Conclusions

A strong link appears to exist between the modal cross-correlation coefficient ρ_{ij} (as defined by [1]) and the response of SDOF oscillators with different fundamental periods of vibration when subject to the same component of ground motion, both in terms of the minimum time difference between peaks (Fig.3) and the percentage of Sd(T) attained at different periods (Fig.5). This observation is of relevance, as it confirms the importance of accounting for modal cross-correlation in modal superposition of planar (2D) systems, as is highlighted by the CQC rule [1]. The relationship between response peaks and modal correlation ρ_{ij} is not as strong when the different SDOF oscillators are instead subject to perpendicular components of excitation (Fig.4, Fig.8). In the case of the minimum time differences shown in Fig.4, one can observe instead a strong link with the cross-correlation coefficient between the two ground motion components ρ_{XY} (as defined by [7]).

A strong link was also seen to exist between ρ_{XY} and the percentage of the PGA of a component that occurs at the instant of the peaks of the perpendicular component, as shown in Fig.6. Interestingly, the mean values appear to lie in the range 0.2 (logarithmic space) to 0.3 (linear space) for null cross-correlation between components, which is one of the main underlying assumptions of the 30% rule [4]. However, this observation cannot be deemed to represent a validation of the 30% rule for a series of reasons. Firstly, real earthquake ground motions do present correlation between their components, as shown in Fig.1, and Fig.6 suggests that the mean values of percentage of PGA at the perpendicular component increase with this correlation. Secondly, the dispersions depicted in Fig.6 and in plots of the data in [8] are very large and indicate that the whole range of ratios between 0 and 1 are indeed possible, posing the question of the relevance of adopting a “one size fits all" approach by looking at a simple average. Thirdly, while the 0.3 factor of the 30% rule accounts for both (i) the ratio between the PGAs of the two perpendicular components and (ii) the correlation (or lack of) in the response of SDOF systems to the simultaneous action of the two components, the percentages of PGA considered herein only represent the latter. This last point implies that, in order to rigorously evaluate the 30% rule, the values obtained in the present work should still be multiplied by the ratio between the demands at the two perpendicular components. However, for accurate estimates of seismic risk we argue that the ratio in demands should be treated as a variable in its own right, across the whole process of defining seismic hazard at a site (the interested reader is referred to [10,11] for details). Finally, modal superposition and spatial combination of ground motions for design imply the combination of modal responses and not of ground motions themselves, hence the focus of the present paper on the response of SDOF systems.

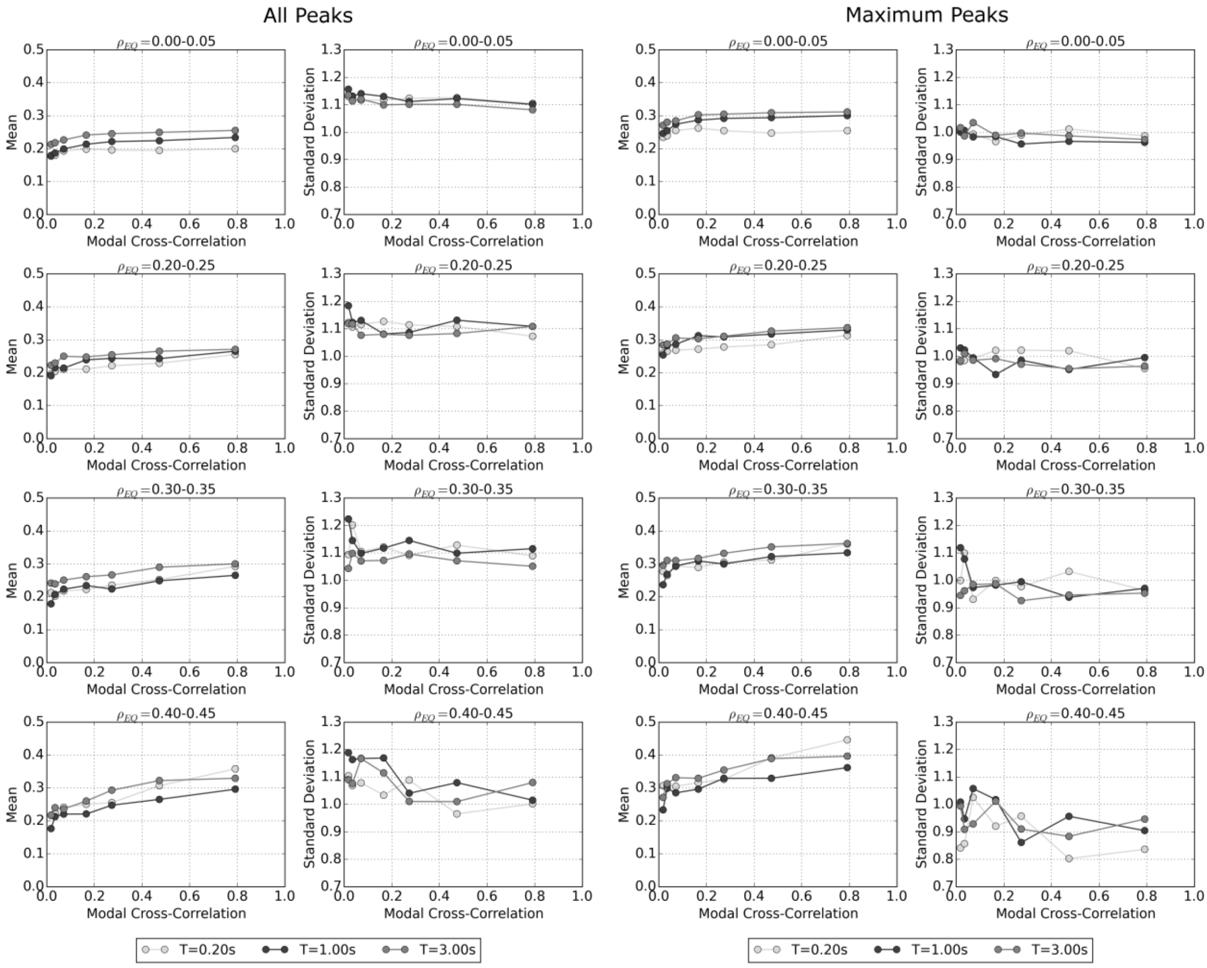

Fig. 8 – Means and standard deviations (computed in terms of the natural logarithm of the data) of the fraction (0-1=0-100%) of Sd(Tj) of the response of the secondary SDOF systems at all peaks of the response for Ti when subject to the perpendicular component (left, "All Peaks") and at peaks of the response for Ti where the maximum fraction of Sd(Tj) occurs (right, "Maximum Peaks"), against the modal cross-correlation coefficients ρ_{ij}, for a threshold value of 95% and data binned according to the cross-correlation coefficients of the earthquake components ρ_{XY} (labelled ρ_{EQ} in the plots).

In relation to the response of SDOF systems it is interesting to see that the relationship between ρ_{XY} and the percentage of the peak response Sd(T) that occurs in the instant of the peaks of the perpendicular direction appears to become weaker when analysing the response of SDOF oscillators with different periods of vibration (Fig.7). For ρ_{XY}<0.4, where most of the data is available, the mean percentages of the response of one component at the peaks of the other component seem to only increase slightly with both ρ_{XY} and ρ_{ij}, as illustrated by the plots in Fig.8. According to these, mean percentages of the response of one component at the peaks of the other component appear to hover around 0.20-0.45 and be relatively stable with respect to both ρ_{XY} and ρ_{ij}. These observations support the idea of the 30% rule being reasonable, but only in an average way and without account for the ratio of maximum demands between components, as discussed above.

The results presented herein are characterised by large standard deviations and this should serve as motivation to keep exploring ways to account for the uncertainty in seismic demands for the design and assessment of engineering structures. A study such as this should not be considered in isolation and is part of a larger ongoing effort of the community to characterise the variation of demands with angle of incidence in a more holistic way [*e.g.*, 8,10,11,12]. The current practice of applying the same design spectrum in two perpendicular directions and combining the responses in a manner that might only be able to reflect reality in an average sense obscures the influence of each of the variables involved. Explicit consideration of these variables would allow instead for a refined definition of seismic demands and quantification of the associated epistemic and aleatory uncertainties. Future steps in the context of the present study should thus be directed towards a fully probabilistic characterisation of the ratio of demands imposed to different oscillator periods by perpendicular components, in conjunction with the examination of additional ground motion databases that could shed some light over the frequency with which ground motions present correlations ρ_{XY} between components of more than ~0.4 and the behaviour of the variables explored in the present work in such cases.

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