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MAXIMUM RESPONSE OCCURRENCE TIME AND SPECTRA

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Abstract

Typically, in seismic response analysis, the focus is on absolute maximum response, without specific interest in its attainment time. The definition of response spectrum, for example, does not provide information about such a time and the issue is somewhat scarcely investigated in the literature, while it is the subject of the presented very simple study, where linear and non-linear single-degree-of-freedom systems are investigated when excited by a large set of ground motion records. For the linear systems, the occurrence time of maximum displacements is measured with respect to: (i) the initial instant defining the *significant duration* of the record; (ii) the initial instant defining the *bracketed duration* of the record; (iii) the time of occurrence of *peak ground acceleration* (PGA). For the non-linear systems, the maximum displacements are measured with respect to the time of occurrence of peak ground acceleration, for two different hysteretic behaviors, stable and evolutionary, considering several vibration periods and ductility levels. The occurrence time of inelastic response is compared between the two non-linear systems and to that of the corresponding elastic system. A characteristic behavior is found for the time of occurrence of the maximum displacement in linear systems that, as a trend, systematically occurs after the PGA, independently of the vibration period. The occurrence time tends to get closer to the PGA time as the damping increases. Finally, the comparison of the time of maximum response for the two non-linear systems and those linear, shows some relationships between the two.

Keywords: Earthquake record; non-linear models; response spectra; PGA; duration.

1. Introduction

The response spectrum does not provide information about the time of occurrence of the maximum response, an issue which, to the authors' knowledge, has received scarce attention in the existing literature (e.g., [1]). The aim of this work is to analyze this issue using both linear and non-linear single-degree-of-freedom (SDoF) systems.

Within the context of linear systems forced by ground motion records, the time of occurrence of the maximum displacement is assessed for a wide range of vibration periods and some damping factors. The work focuses on the relationship between the maximum response time with respect to: (i) the time of occurrence of the peak ground acceleration (PGA); (ii) the first instant defining the *significant duration* [2]; (iii) the first instant defining the *bracketed duration* [3].

For non-linear SDoF systems, the time of occurrence of maxima is investigated considering elastic-perfectly-plastic and peak-oriented [4] hysteric behaviors, in a fixed-ductility approach. Subsequently, the results, for linear and non-linear systems are compared. All the analyses are carried out through the use of ground motion records from NESS1 dataset [5], made of approximately fifteen-hundred ground motions.

The paper is structured such that the next section provides the main characteristics of the considered record dataset. Then, some details about the linear analyses are given followed by the related results. Finally, non-linear analyses are presented and discussed, also in comparison with the linear case.

2. Ground motion dataset

The set of accelerograms used in this study is a collection of near-source strong-motion waveforms named NESS1 [5]. This set is made of about fifteen-hundreds strong motion three component waveforms from about seven hundred accelerometric seismic stations around the world and caused by more than seventy events (occurred from 1933 to 2016), with moment magnitude larger than 5.5 and hypocentral depth smaller than 40 km (crustal events). Normal, strike-slip, and thrust focal mechanisms are included in the dataset. In Fig. 1, the pseudo-acceleration spectra for the whole set of records and their *Husid plots* [2] are shown.

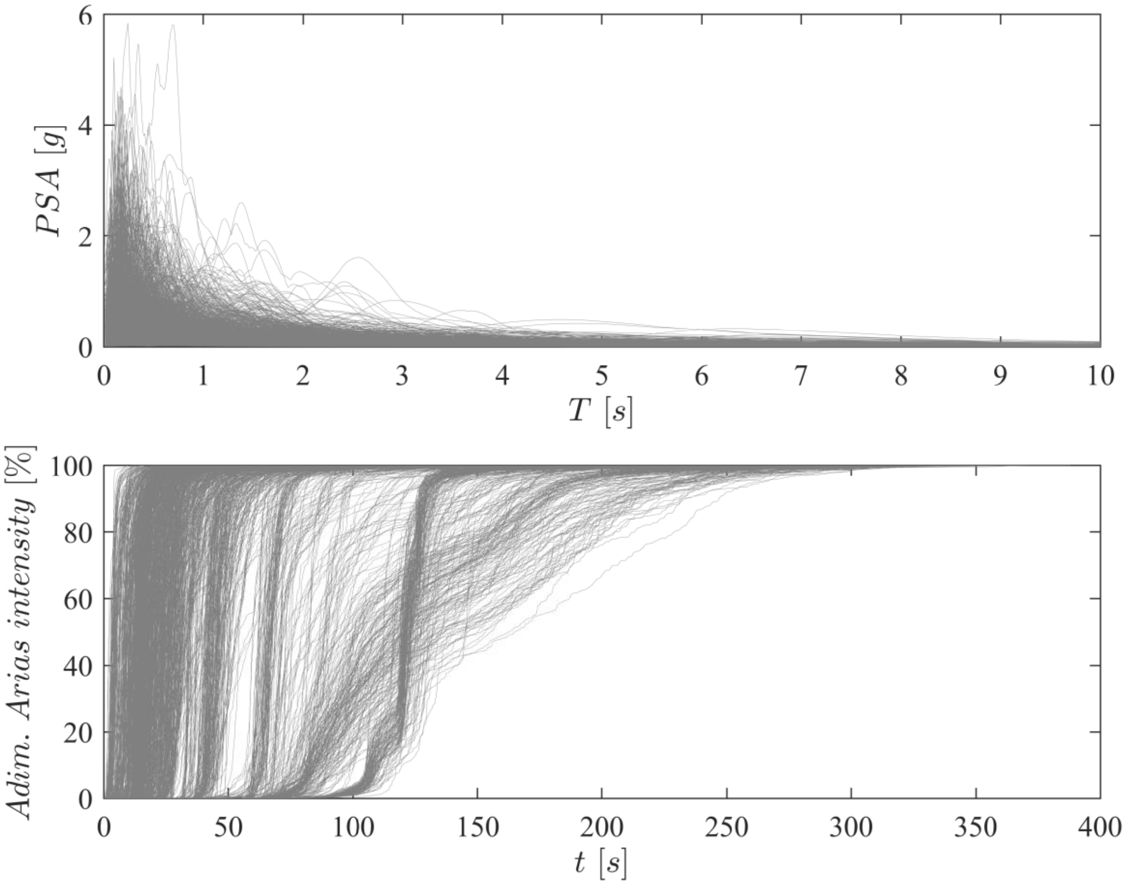

Fig. 1 – Pseudo-acceleration spectra (top) and Husid plots (bottom) of NESS1 dataset

3. Occurrence response time in linear systems

The key parameter of this study is the occurrence time of maximum response of a linear SDoF, $t_{S_a(T)}$; i.e., the time at which a linear SDoF system, forced by a ground motion record, reaches the maximum absolute displacement value, $t_{S_a(T)} = \max_t |x(t)|$ (Fig. 2 - top). This work analyzes, in particular, the relationship between $t_{S_a(T)}$ for a system characterized by vibration period T and damping factor ν, and: (i) the time of occurrence of PGA, t_{PGA} (Fig. 2 – center); (ii) the first time defining the significant duration, t_{IA}, that is the instant in which the normalized *Arias Intensity* is equal to 5% [2] (Fig. 2 - bottom); (iii) the initial instant defining the bracketed duration, t_{BK}, that is the first exceedance time of $\pm 0.05g$ in ground motion record [3] (Fig. 2 – center).

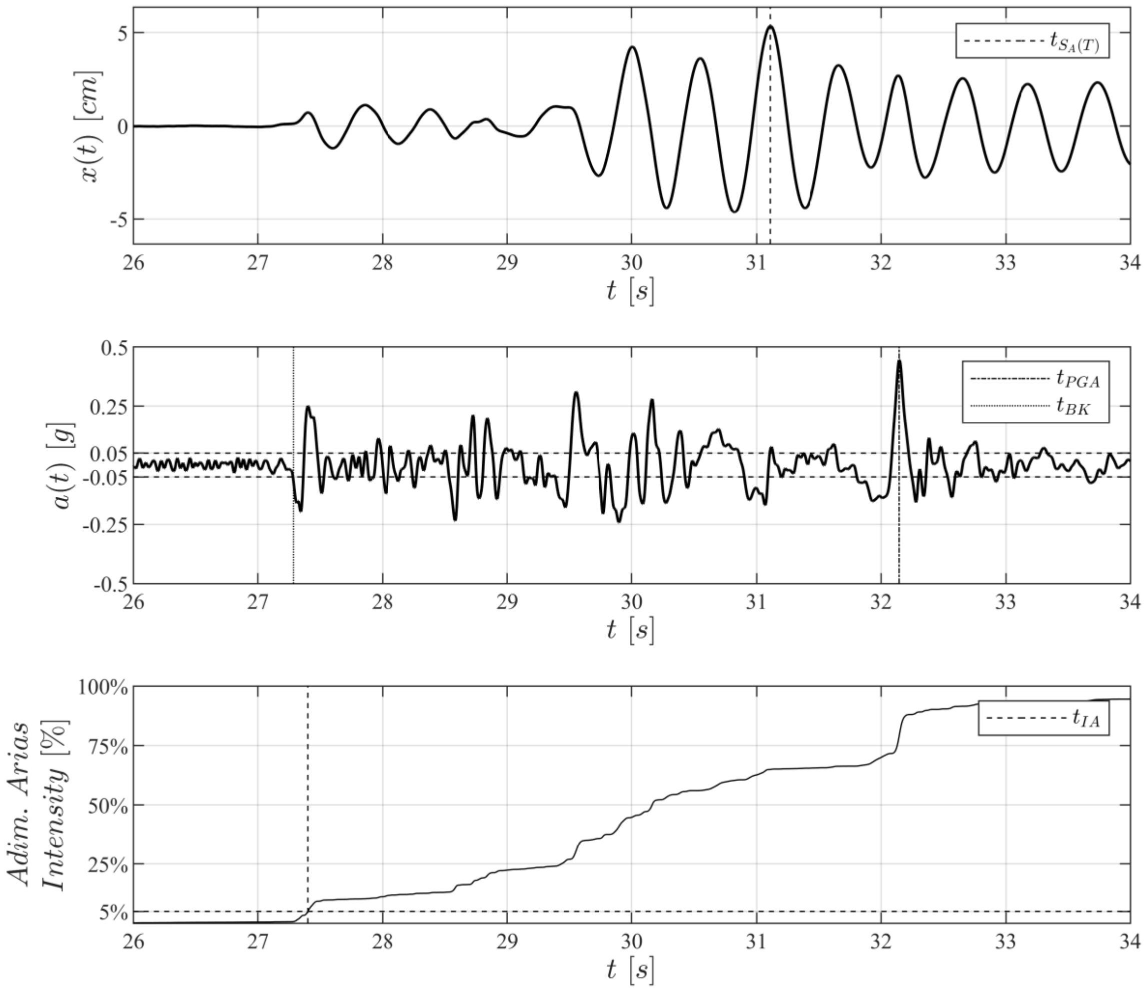

Fig. 2 – Example of identification of $t_{S_a(T)}$, t_{PGA}, t_{IA} and t_{BK} on a ground motion record; displacement response (top) of a linear SDoF ($T = 0.55$ s; $\nu = 0.05$) forced by the accelerogram $a(t)$ (center); Husid plot of $a(t)$ (bottom)

Because $t_{S_a(T)}$ changes for any T, and ν, it is possible to plot a sort of *spectrum*, where $t_{S_a(T)}$ is a function of T given ν. In particular, this work focuses on the spectra concerning the differences between $t_{S_a(T)}$ and the characteristic times of the ground motion records: $\left(t_{S_a(T)} - t_{PGA}\right)/t_{PGA}$, $\left(t_{S_a(T)} - t_{IA}\right)/t_{IA}$ and $\left(t_{S_a(T)} - t_{BK}\right)/t_{BK}$, spectra. Such spectra have been calculated, in a range of periods from 0.01 s to 10 s (0.005 s step) and damping factor values equal to 0.00 (undamped systems), 0.01, 0.02 and 0.05.

3.1 Median trends

Fig. 3 shows the median values of response-time spectra for the whole NESS1 dataset, which significantly decreases with increasing damping. The occurrence of the maximum, on average, follows the time of occurrence of the PGA (Fig. 3 - top). For undamped systems, this difference reaches its maximum value of about 70% of t_{PGA}, in a range of periods from 2.5 s to 4.0 s. For systems with damping factor of 0.05, the difference reaches a maximum value of about 15% in a range of periods from 4.0 s to 6.0 s. In particular, for periods up to 1 s, the median values of $\left(t_{S_a(T)} - t_{PGA}\right)/t_{PGA}$ fluctuate between 0% and 5%. It is possible to also observe, regardless of the damping factor, always a non-monotonic behavior, that is, an increasing and then a decreasing trend.

The spectra related to $\left(t_{S_a(T)} - t_{IA}\right)/t_{IA}$ and $\left(t_{S_a(T)} - t_{BK}\right)/t_{BK}$ show a similar behavior. For undamped systems, both spectra show the maximum value of about 100% in a range of periods from 2.5 s to 4.5 s. In the range of periods from 3.5 s to 5.5 s, the ordinates for damped systems ($\nu = 0.05$) reach a maximum value of about 35%.

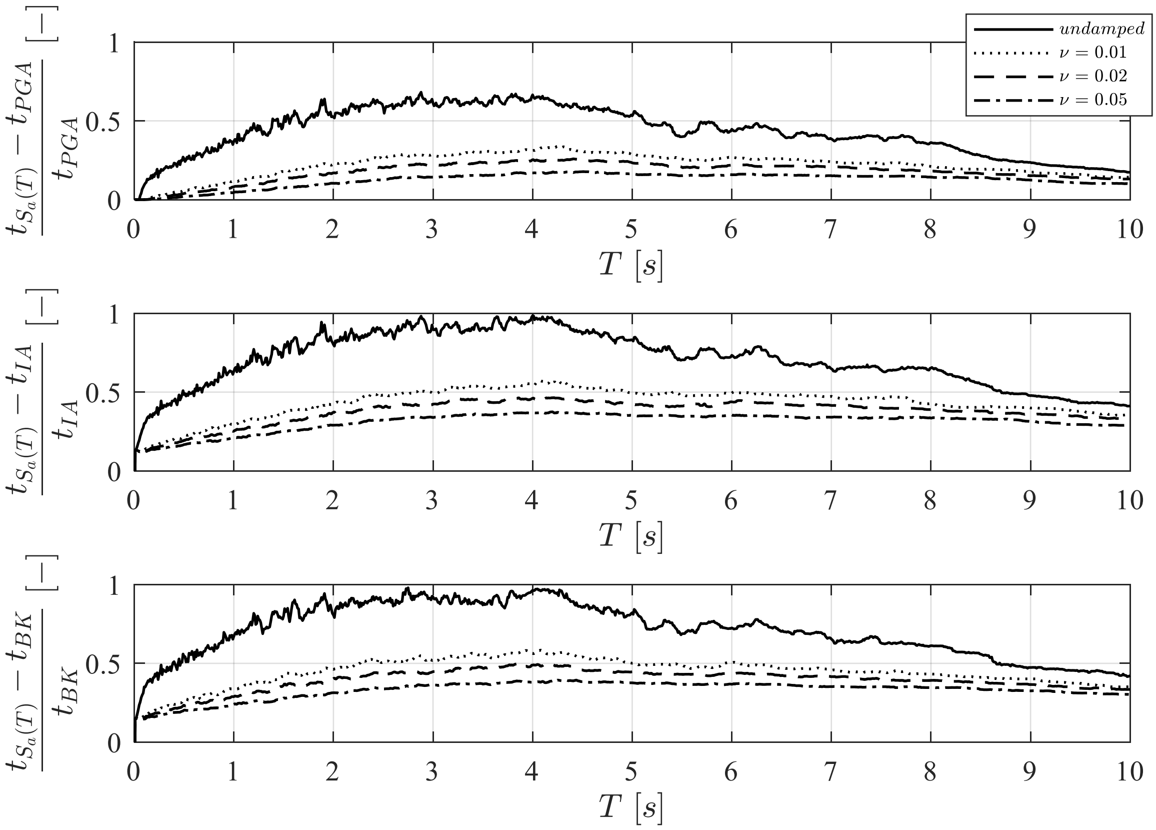

Fig. 3 – Median *response-time spectra* for the NESS1 set of ground motion records

3.2 Individual record behavior

It is to note that the median trend is not quite that of a single record. As an example, in Fig. 4 (top) illustrates the spectral ordinates $t_{S_a(T)} - t_{PGA}$, which fluctuate between positive and negative values. This means that the occurrence time of maximum oscillates around t_{PGA}. It can also be seen that $t_{S_a(T)}$ is a piece-wise constant function, due to significant changes in response time, even for small period fluctuations. To appreciate that, in Fig. 4 (center and bottom) the significant variations in the response time to changes in vibration period are shown. For $T = 2$ s, $t_{S_a(T)}$ matches t_{PGA}, while increasing the period, as it possible to observe that the maximum responses move to a different range; i.e., between 55 s and 65 s. This leads to a value of $t_{S_a(T)} - t_{IA}$ equal to 26 s. A positive value, lower than 25 s, is observed for $T = 4\,\text{s}$, because the response changes, and the range of maximum values moves between 40 s and 60 s. Finally, for $T = 5\,\text{s}$, the maximum response times move backwards around t_{PGA}. In particular, the maximum occurs about 1 s before the PGA and causes a negative $t_{S_a(T)} - t_{IA}$ value. However, these negative values are less recurrent and lower than positive ones, in absolute terms. This characteristic is highlighted in Fig. 5, in which the spectra for all the records of NESS1 and their median value, for $\nu = 0.05$ are represented.

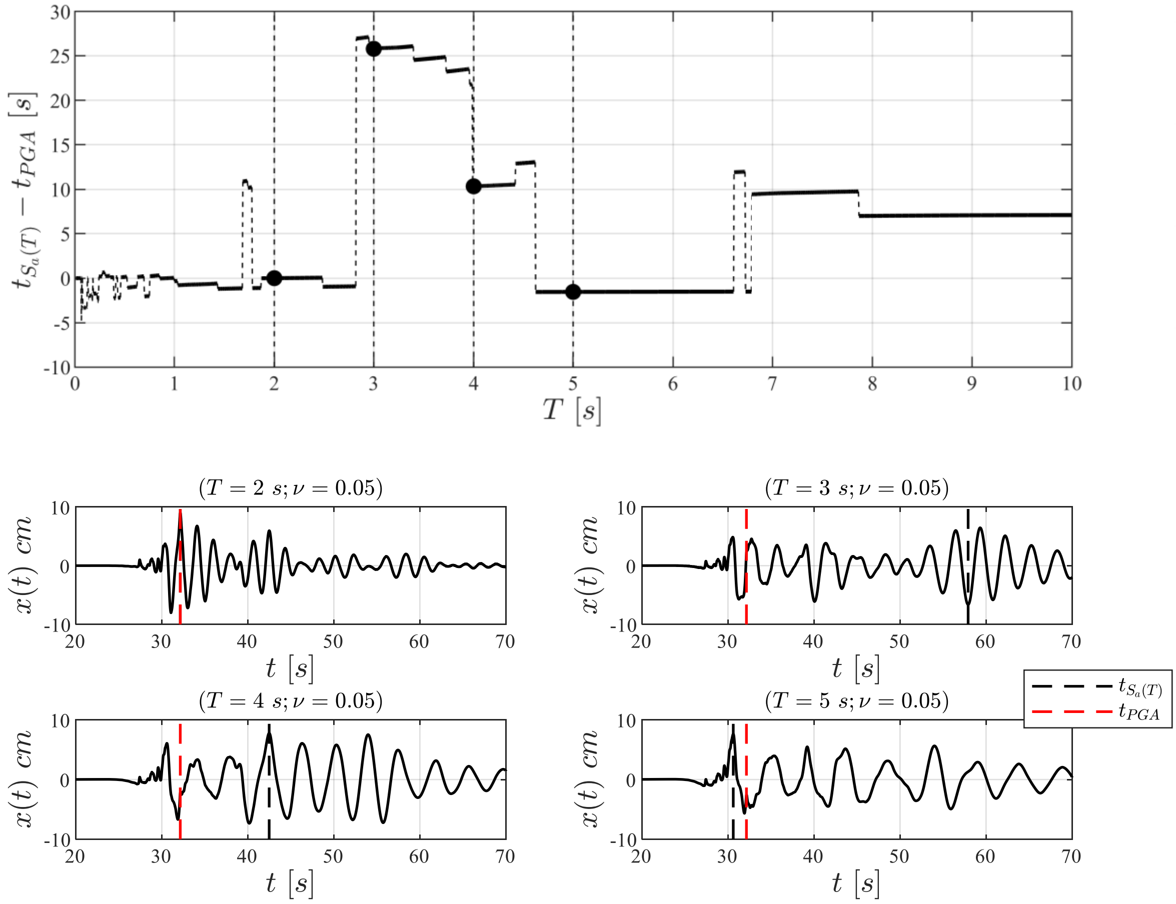

Fig. 4 – $t_{S_a(T)} - t_{PGA}$ response-time spectrum (top) for record of Fig. 1 $(\nu = 0.05)$ and displacement response (center and bottom) for the four different systems marked

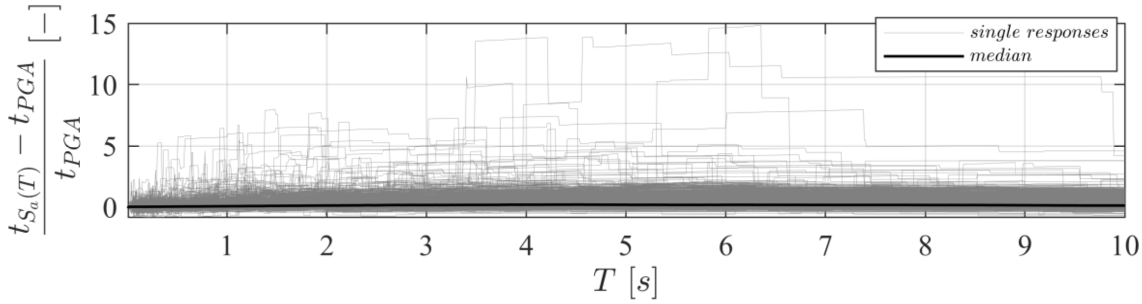

Fig. 5 – Single and Median response-time spectra for the NESS1 set of ground motion records ($\nu = 0.05$)

3.3 Effect of damping

Fig. 6 clarifies the behaviour of linear systems as the damping increases. The same system ($T = 0.5$ s) responds differently to the record, for the various levels of damping and, in accordance with Fig. 3, $t_{S_a(T)}$ is getting closer to t_{PGA}, to confirm the ordinates reduction as the damping increases.

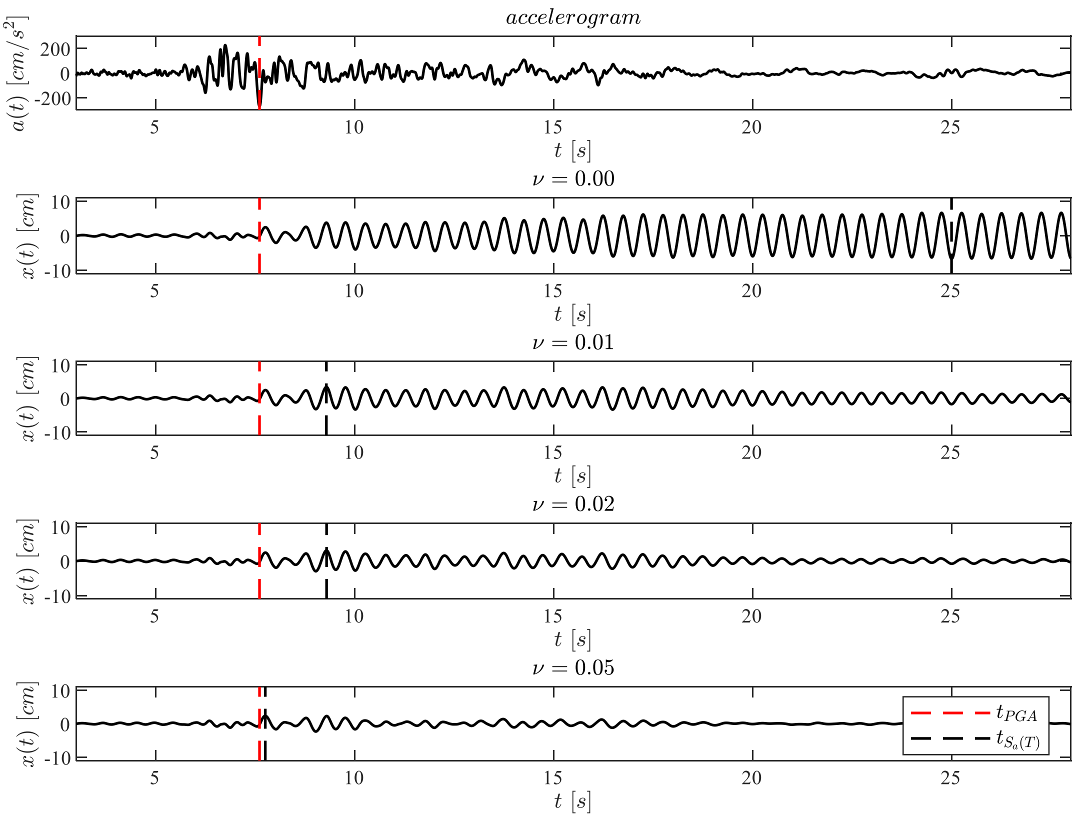

Fig. 6 – Response-time history for the same system ($T = 0.5$ s), forced by one record, $a(t)$, for four increasing damping values: 0.00, 0.01, 0.02 and 0.05

4. Occurrence response time in non-linear systems

The non-linear systems investigated first are characterized by an *elastic perfectly plastic* (EPP) stable hysteretic model. For these systems, characterized by vibration periods equal to 0.4 s, 0.8 s, 1.0 s, 1.2 s, 1.6 s, 2 s and damping factor equal to 0.05, the time of occurrence of maximum response is recorded in a controlled-ductility (μ) approach where the latter equals 2, 4 and 8.

The second set of investigated systems refers to the modified *Ibarra-Medina-Krawinkler* (IMK) hysteric model, characterized by a peak-oriented behavior. Again, the maximum response time is recorded in a fixed ductility approach. The systems' parameters explored are the same as the EPP systems.

After having recorded the response-time of non-linear systems, it is possible to compare the time of occurrence of the non-linear systems with the linear counterpart having the same initial vibration period, T_{in}. The comparison has been extended to about eighty-five randomly selected records from NESS1 dataset. In Fig. 7 it is possible to observe this comparison for a system with $T = 2\,\mathrm{s}$, forced by the same accelerogram, in order to return $\mu = 2$ in both non-linear cases, while Fig. 8, shows the same cases when ductility equals to four and eight.

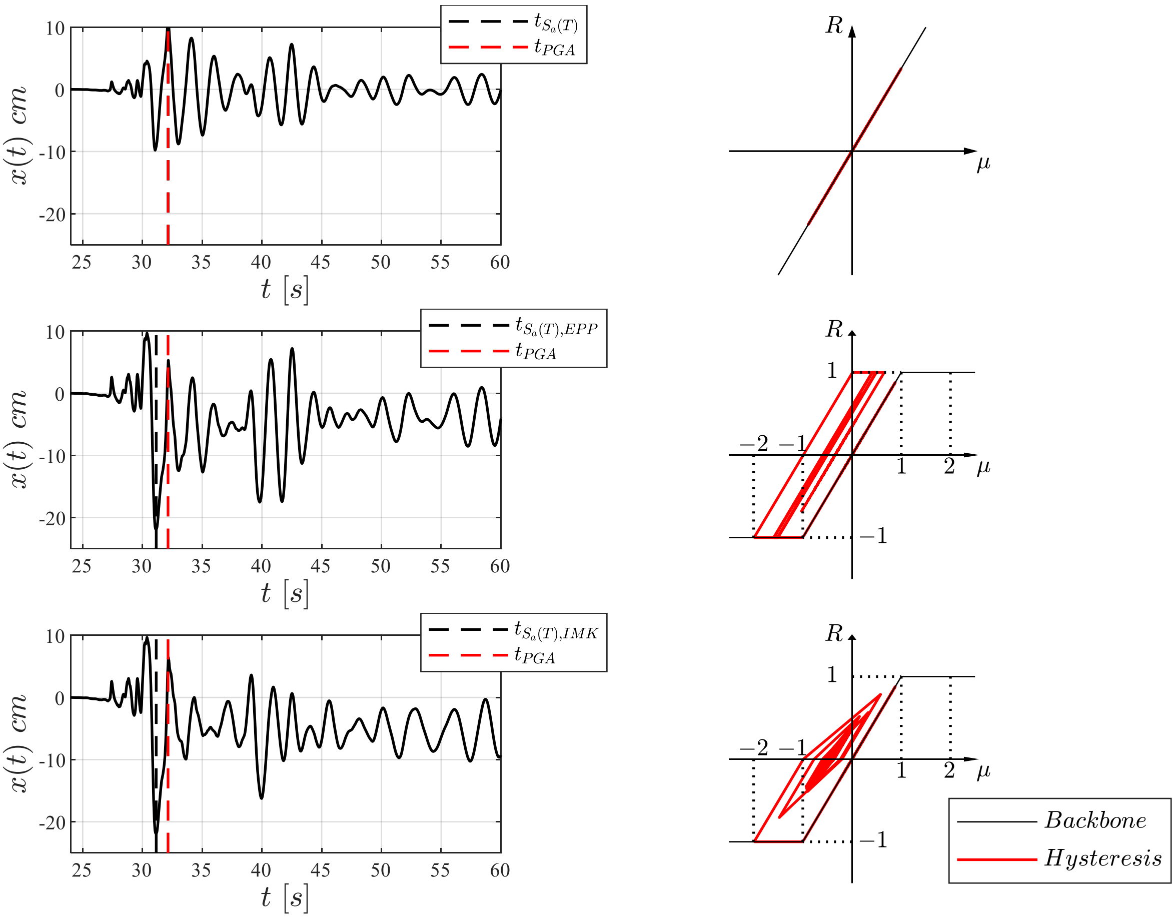

Fig. 7 – On the left, the identification of $t_{S_a(T)}$ and t_{PGA} on displacement responses of linear (top) and non-linear SDoF (EPP - center; IMK - bottom), forced by the record of Fig. 1 when $\mu = 2$. On the right, the corresponding hysteretic responses

From Fig. 7 and Fig. 8 it is possible to observe how the occurrence time of non-linear systems, $t_{S_a(T),EPP}$ and $t_{S_a(T),IMK}$, with the same initial period, changes for different target ductility values. However, this susceptibility is different for the two different non-linear models. Fig. 9 show the comparison of linear and non-linear responses, for EPP and IMK models separately. In particular, $\left(t_{S_a(T),EPP} - t_{PGA}\right)/t_{PGA}$ and $\left(t_{S_a(T),IMK} - t_{PGA}\right)/t_{PGA}$ are shown for each record and their median values versus T_{in}, for each ductility value investigated. As in the linear case, in the non-linear systems the occurrence times of maxima follow, as a trend, the t_{PGA}, regardless of the period investigated and the level of ductility imposed, although only slightly.

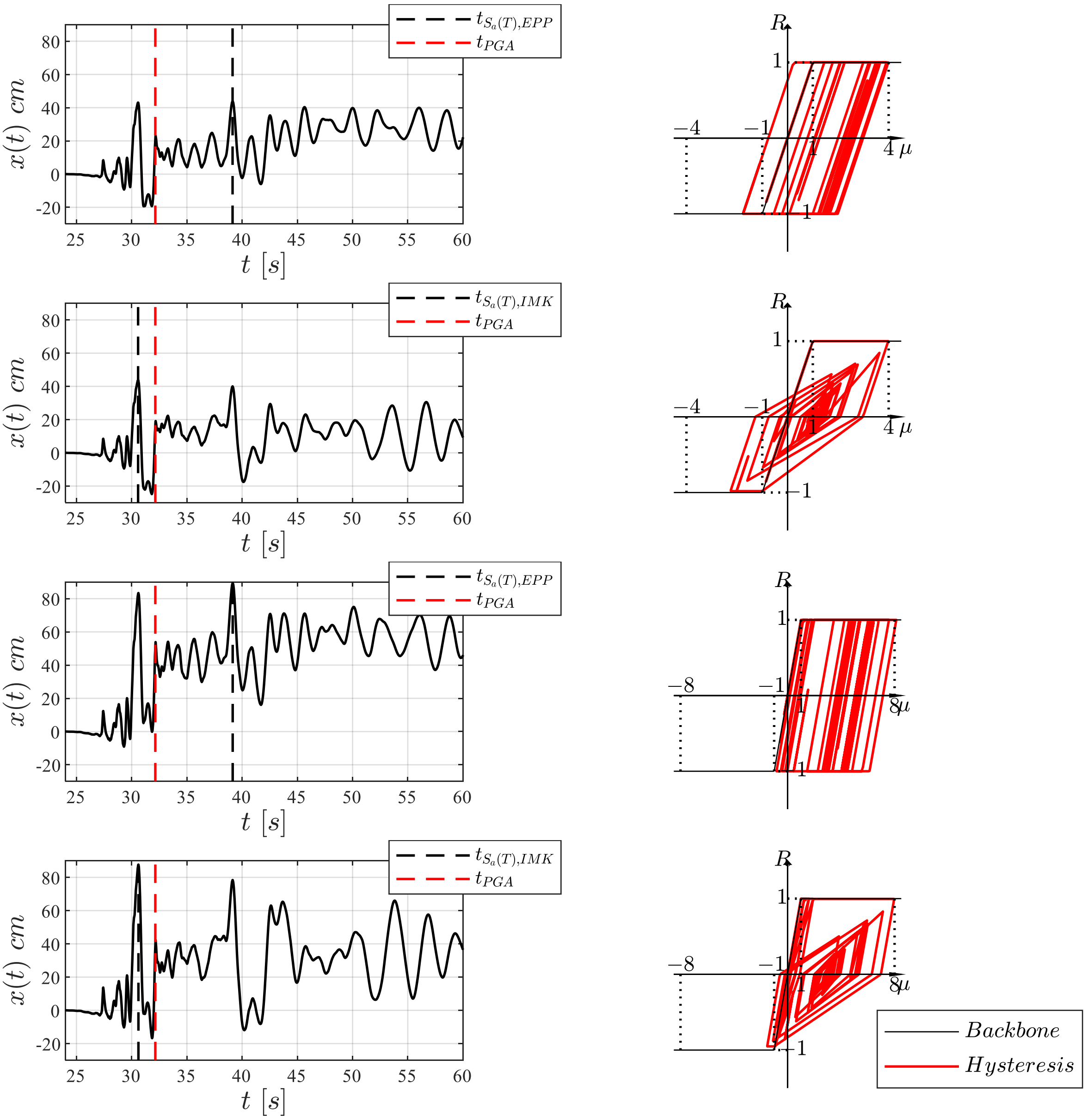

Fig. 8 – Displacement (left) and hysteretic (right) responses of same SDoF system of Fig. 7, when $\mu = 4$ (top) and $\mu = 8$ (bottom)

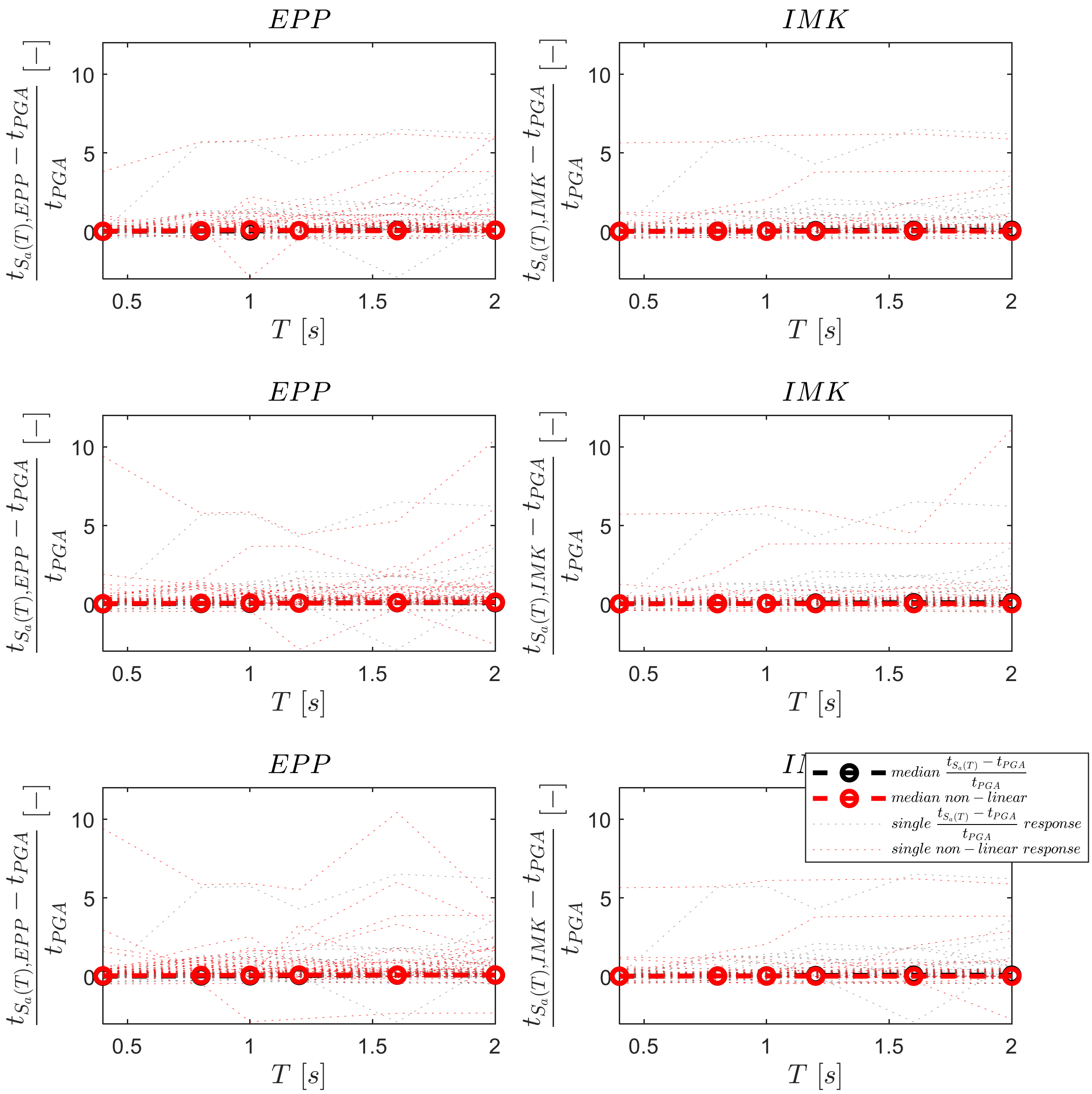

Fig. 9 – Linear and non-linear single differences normalized with respect to t_{PGA} ($\nu = 0.05$) for 82 record and their median values, for ductility 2 (top), 4 (center) and 8 (bottom)

In order to study, more deeply, the relationship between the occurrence times of linear and non-linear maxima, Fig. 10 provides the ratios of non-linear to linear or non-linear to non-linear response occurrence times, for the same ground motion record: $t_{S_a(T),EPP} / t_{S_a(T)}$ (top), $t_{S_a(T),IMK} / t_{S_a(T)}$ (center) and $t_{S_a(T),IMK} / t_{S_a(T),EPP}$ (bottom). It is possible to observe that all the median values of these ratios are approximately equal to one, regardless of target ductility and periods.

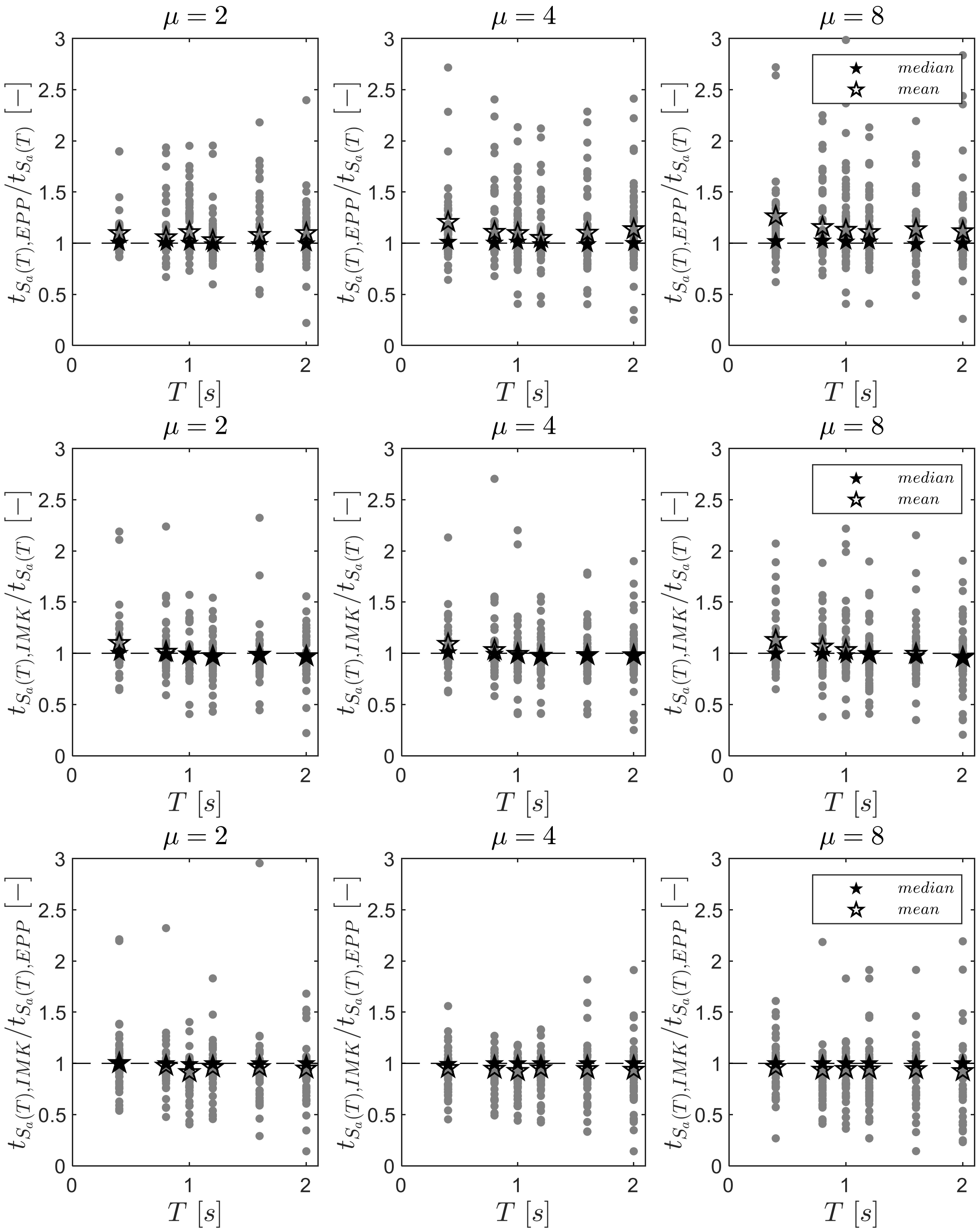

Fig. 10 – $t_{S_a(T),EPP}$ / $t_{S_a(T)}$ (top), $t_{S_a(T),IMK}$ / $t_{S_a(T)}$ (center) and $t_{S_a(T),IMK}$ / $t_{S_a(T),EPP}$ (bottom) over T for 82 records; the results are separately with regard to the required level of ductility: 2 (left), 4 (center) and 8 (right)

5. Final remarks

The lack of attention in the literature on the timing of maximum displacement motivated this very simple investigation through the dynamic analysis of linear and non-linear SDoF systems, forced by the records of near-source strong-motion dataset NESS1. The work has shown the following.

- The time of occurrence of the maximum displacement for linear systems results, on average, later than the time of occurrence of PGA. It is also possible to observe a non-monotonic trend of the spectra representing the time of occurrence of the maximum as a function of the elastic period. A similar behavior is found for the other occurrence-time spectra investigated.

- As the damping increases, the ordinates of the $\left(t_{S_a(T)} - t_{PGA}\right)/t_{PGA}$ spectrum decrease. In particular, for a damping factor equal to 0.05, and periods up to 1 s, the median values of $\left(t_{S_a(T)} - t_{PGA}\right)/t_{PGA}$ fluctuate between 0% and 5%. Similar behavior is found for the other occurrence-time spectra investigated.

- In the range of periods investigated, also the time of occurrence of the maximum displacement for non-linear systems results, as a trend, always successive to the time of occurrence of PGA. Moreover, an equivalence of median occurrence times of the maxima for linear and non-linear systems has been found.

All these results, as well as the study as a whole, must be considered very preliminary.

6. References

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