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# IMPROVING THE SEISMIC PERFORMANCE OF STRUCTURES BY ASSIGNING CHANGES IN MASS, STIFFNESS AND ADDED DAMPING

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## Abstract

This paper presents a two-stage seismic design methodology for improving the seismic performance of multi-story frame structures by assigning changes in floor-mass, changes in horizontal-stiffness, and added linear viscous dampers. In the first stage, use is made of the general system interconnection paradigm that represents the linearity between the ground acceleration and absolute accelerations, together with the modified $H_\infty$ synthesis, to attain optimal stiffness and mass changes. The modified $H_\infty$ synthesis is based on the analogy between the $H_\infty$ norm and the square sum of the squares (SRSS) of the peak absolute acceleration gains in the frequency domain. In the second stage, the analysis-redesign approach for the fully stressed design of linear viscous dampers is introduced to limit the maximum interstory drifts to their prescribed allowable limits. An example of a 5-story shear-type building is studied. Optimal changes are obtained to show significant improvement in seismic performance. The results clearly indicate the efficiency of the proposed methodology that possesses the capability of attaining optimal changes in all the structure's physical characteristics, while reducing absolute accelerations and limiting the maximum interstory drifts.

*Keywords: Seismic Design, Acceleration Reduction, Changes in Mass and Stiffness, Optimal Damper Placement*

## 1. Introduction

The field of optimal control offers innovative approaches for designing closed-loop systems subjected to disturbances/measurement-noises while maintaining their dynamic stability. In Earthquake Engineering, numerous optimal control solution procedures are employed for designing structures to withstand strong earthquakes. Optimal control procedures differ from other kinds of seismic design procedures, such as the analysis-redesign (e.g., Levy and Lavan [1]), genetic-algorithms (e.g., Wongprasert and Symans [2]; Liu et al. [3]), sequential search methods (e.g., Zhang and Soong 1992[4]; Wu et al. [5]; Lopez-Garcia [6]; Shmerling [7]), due to their ability of regulate multiple types of structural response, simultaneously, while attaining optimal structural changes in mass and/or stiffness and/or damping.

The Linear Quadratic Regulator (LQR) solution procedure is quite popular. The procedure provides the optimal gain matrix of the closed-loop system, which correlates with the optimal stiffness changes and optimal added linear damping. Although the gain matrix is unsymmetric and its form does not resemble the structural matrices, various researches proposed equivalence approaches for deriving the stiffness changes and added linear viscous dampers (e.g., Gluck et al. [8]; Loh et al. [9]; Cimellaro [10]). There is a drawback in utilizing the LQR because each approach results with different structural changes and, consequently, different "optimal" seismic performance. Another drawback is that the LQR aims at improving the entropy of the closed-loop system, and not minimizing the maximum response. Shmerling et al. [11] cater to both drawbacks by combining the LQR solution procedure with the analysis-redesign approach of Levy and Lavan [1] for the fully stressed design of linear viscous dampers. The optimal stiffness changes are derived from the LQR gain matrix, while the optimal damping distribution and sizing are determined according to the analysis-redesign approach so that the maximum interstory drifts are limited to prescribed quantities. Shmerling et al. [11] methodology demonstrates that combing optimal control solutions with the analysis-redesign approach is very successful. This concept is adopted and examined in this paper as well but utilizes an optimal control problem that has a clear relation to regulating the peak response of the structure while adhering to structural matrix constraints. These types of problems of are the $H_\infty$ synthesis, which deals with minimizing the $H_\infty$ norm.

The $H_\infty$ norm equals to the supreme eigenvalue of the closed-loop transfer function matrix overall excitation frequencies, and the closed-loop system is presented by the general system interconnection paradigm of disturbances/measurement-noises inputs, regulated outputs, control feedbacks, and control forces. A common approach for solving $H_\infty$ control problems is the “Standard 2-Riccati solution” (Glover and Doyle [12]), which consists of imposing a desired $H_\infty$ quantity followed by solving the two algebraic Riccati equations. If both solutions exist, the controlled system is stable and the solutions are substituted into the term of the optimal controller. Spencer et al. [13] discuss the seismic analysis of multi-story structures in the frequency domain, where the external input and the regulated outputs are the ground acceleration and floor displacements respectively. They studied an 8-story shear-type building with a tuned-mass damper placed on the rooftop and demonstrated the clear resemblance between the $H_\infty$ norm quantity and the square sum of the squares (SRSS) of maximum displacements for the case of varying excitation frequencies. In Shmerling and Levy [14], it is mathematically explained how the $H_\infty$ norm is similar to the SRSS of the regulated dynamic response (e.g., displacements, interstory drifts, absolute accelerations) when the general system interconnection is of SIMO. Bai et al. [15] followed suit and proposed their seismic design methodology for optimally assigning linear viscous dampers to frame structures so that the $H_\infty$ norm is minimized. Shmerling and Levy [14] utilized the modified $H_\infty$ synthesis of Apkarian and Nool [16] and proposed a general system interconnection which represents the linearity between the ground acceleration (input) and the absolute accelerations (regulated outputs). Optimal changes in mass and stiffness are assigned for minimizing the SRSS of the absolute acceleration gains.

In this paper, the seismic methodology of Shmerling and Levy [14] is combined with the analysis-redesign procedure of Levy and Lavan [1] into a unified two-stage procedure that is able to mitigate the peak absolute acceleration while limiting the peak interstory drifts according to prescribed allowable quantities.



 

## 2. Objective and constraints

The current seismic design methodology aims at minimizing the SRSS of peak absolute acceleration gains, and minimizing the sum of added damping, simultaneously, while subjected to dynamic equilibrium and prescribed allowable interstory drifts. The formal optimization problem formulation of two objective functions and dynamic equilibrium constraints is:

$$
\begin{aligned}
&Obj.1 \quad \underset{\Delta m_1,\ldots,\Delta m_N,\Delta k_1,\ldots,\Delta k_N}{\text{minimize}} \left\{ J = \sup_{\omega_g \in [0 \quad \infty]} \sqrt{\sum_{n=1}^{N} \left( \left| a^{abs}{}_n(\omega_g) \right| / PGA \right)^2} \right\} \\
&Obj.2 \quad \text{minimize}\{\Delta c_1 + \Delta c_2 + \cdots \Delta c_N\} \\
&\text{S.T.} \\
&(\mathbf{M} + \Delta\mathbf{M})\,\mathbf{a}^{abs}(t) + (\mathbf{C} + \Delta\mathbf{C})\dot{\mathbf{x}}(t) + (\mathbf{T}_x)^T \mathbf{f}^{\mathbf{R}}\left(\dot{\boldsymbol{\delta}}(t), \dot{\mathbf{f}}^{\mathbf{R}}\right) + \Delta\mathbf{K}\,\mathbf{x}(t) = \mathbf{0} \\
&\boldsymbol{\delta}(t) = \mathbf{T}_x\,\mathbf{x}(t) \quad \leftrightarrow \quad \mathbf{T}_x = \begin{bmatrix} 1 & & & \\ -1 & 1 & & \\ & \ddots & \ddots & \\ & & -1 & 1 \end{bmatrix} \\
&\mathbf{a}^{abs}(t) = \ddot{\mathbf{x}}(t) + \mathbf{1} a_g(t) \\
&\max_{n=1,\ldots,N} (|\delta_n(t)|) / \delta_n^{all} \le 1.0
\end{aligned}
\tag{1}
$$

Here, the term "$\left| a^{abs}{}_n(\omega_g) \right| / PGA$" is the absolute acceleration gain of the $n^{th}$ story for the ground excitation frequency $\omega_g$, and PGA stands for peak ground acceleration. The term $\delta_n^{all}$ is the prescribed allowable interstory drift of the $n^{th}$ story, and $\mathbf{1}$ is vectors of ones. The interstory resisting force, $\mathbf{f}^{\mathbf{R}}$, is considered as bilinear resisting forces whose inelastic behavior for the $n^{th}$ story is depicted by Fig. 1.

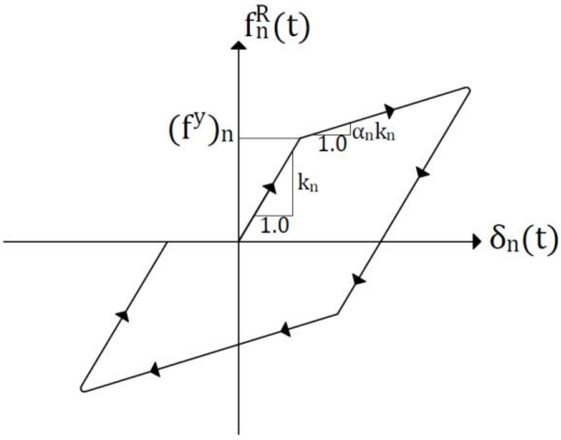


Fig. 1 – The $n^{th}$ interstory bilinear resisting force

Accordingly, the ratio $\dot{f}^R{}_n / \dot{\delta}_n$ (i.e., tangent stiffness) contains the following three cases:

$$
\dot{f}_n^R(t)/\dot{\delta}_n(t) = \begin{cases} k_n + \Delta k_n & \leftrightarrow \quad \text{sgn}\left(f_n^H \dot{\delta}_n(t)\right) > 0 \quad \& \quad \left|f_n^H(t)\right| \le |(1-\alpha_n)(f^y)_n| \\ \alpha_n(k_n + \Delta k_n) & \leftrightarrow \quad \text{sgn}\left(f_n^H \dot{\delta}_n(t)\right) > 0 \quad \& \quad \left|f_n^H(t)\right| > |(1-\alpha_n)(f^y)_n| \\ k_n + \Delta k_n & \leftrightarrow \quad \text{sgn}\left(f_n^H \dot{\delta}_n(t)\right) < 0 \end{cases}
\tag{2}
$$

where $f_n^H(t)$ is the hysteretic part of the resisting force:

$$
f_n^H(t) = f_n^R(t) - \alpha_n(k_n + \Delta k_n)\delta_n(t)
\tag{3}
$$



 

The matrices **M**, Δ**M,** Δ**K**, **C**, and Δ**C** are the initial mass, mass changes, initial stiffness, stiffness changes, inherent damping and added damping respectively:

$$\begin{aligned}
&\Delta\mathbf{M} = \mathrm{diag}\{\Delta m_1, \dots, \Delta m_N\} \\
&\Delta\mathbf{K} = \mathbf{T_x}^T \mathrm{diag}\{\Delta k_1, \dots, \Delta k_N\}\mathbf{T_x} \\
&\Delta\mathbf{C} = \mathbf{T_x}^T \mathrm{diag}\{\Delta c_1, \dots, \Delta c_N\}\mathbf{T_x} \\
&\mathbf{C} = a_0(\mathbf{M} + \Delta\mathbf{M}) + a_1(\mathbf{K} + \Delta\mathbf{K}) \\
&a_0 = 2.0 \cdot 10^{-5} \\
&a_1 = 2.0 \cdot 10^{-8} \\
&\Delta m^{max}{}_n \geq \Delta m_n \geq \Delta m^{min}{}_n \quad \forall\, n = 1, \dots, N \\
&\Delta k^{max}{}_n \geq \Delta k_n \geq \Delta k^{min}{}_n \quad \forall\, n = 1, \dots, N \\
&\Delta c^{max}{}_n \geq \Delta c_n \geq \Delta c^{min}{}_n \quad \forall\, n = 1, \dots, N
\end{aligned} \tag{4}$$

The inherent damping matrix, **C**, is defined according to Rayleigh classical damping and its' coefficients, $a_0$ and $a_1$, are set according to the technique presented in Shmerling and Levy [14] for having a close to unified modal damping ratio as depicted by Fig. 2 for the $i^{th}$ modal frequency "$\omega_i$".

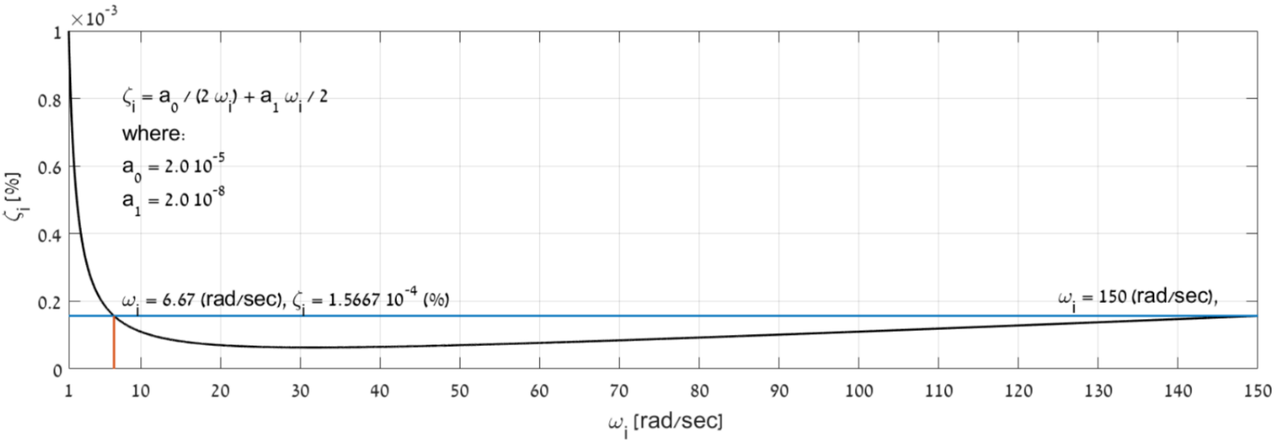


Fig. 2 – Modal damping ratio relationship

The proposed solution procedure of Eq. (1) is divided into two stages. In the first stage, $H_\infty$ synthesis is applied for optimally adjusting Δ**M** and Δ**K** in minimizing the SRSS of peak absolute acceleration gains, while adhering to the minimum/maximum quantity constraints. In the second stage, the analysis-redesign procedure for the fully stressed design of linear viscous dampers (i.e., Δ**C**) is applied, while referring to prescribed ground motion records.

## 3. Two-stage solution

### 3.1. First stage: $H_\infty$ synthesis

The optimal Δ**M** and Δ**K** are the optimal solution of the following modified $H_\infty$ synthesis problem:



 

$$
\begin{aligned}
&\underset{\mathbf{Q}_\Delta}{\text{minimize}}\{J = \left\|\mathbf{CL}_{w_1\to y_1}(\mathbf{Q}_\Delta, s)\right\|_\infty\} \\
&\text{s.t.} \\
&\mathbf{CL}_{w_1\to y_1}(\mathbf{Q}_\Delta, s) - \text{ stable} \\
&\mathbf{Q}_\Delta = [\Delta\mathbf{M} \quad \mathbf{C} \quad \Delta\mathbf{K}] \\
&\mathbf{C} = a_0(\mathbf{M}+\Delta\mathbf{M}) + a_1(\mathbf{K}+\Delta\mathbf{K}) \\
&\Delta\mathbf{K} = \mathbf{T}_x{}^T \text{diag}\{\Delta k_1, \dots, \Delta k_N\}\mathbf{T}_x \\
&\mathbf{M} = \text{diag}\{m_1, \dots, m_N\} \\
&\Delta\mathbf{M} = \text{diag}\{\Delta m_1, \dots, \Delta m_N\} \\
&\Delta m^{max}{}_n \geq \Delta m_n \geq \Delta m^{min}{}_n \quad \forall\, n = 1, \dots, N \\
&\Delta k^{max}{}_n \geq \Delta k_n \geq \Delta k^{min}{}_n \quad \forall\, n = 1, \dots, N
\end{aligned} \tag{5}
$$

where $\mathbf{Q}_\Delta$ is the closed-loop controller, and $\mathbf{CL}_{w_1\to y_1}(\mathbf{Q}_\Delta, s)$ is the closed-loop transfer function:

$$
\begin{aligned}
&\mathbf{CL}_{w_1\to y_1}(\mathbf{Q}_\Delta, s) = \mathbf{G}_{11}(s) + \mathbf{G}_{12}(s)\mathbf{Q}_\Delta[\mathbf{I} - \mathbf{G}_{22}(s)\,\mathbf{Q}_\Delta]^{-1}\,\mathbf{G}_{21}(s) \\
&\text{and:} \\
&\mathbf{G}_{11}(s) = -s^2\mathbf{P}(s)\mathbf{M}\boldsymbol{\iota} + \boldsymbol{\iota} \\
&\mathbf{G}_{12}(s) = s^2\mathbf{P}(s) \\
&\mathbf{G}_{21}(s) = -\begin{bmatrix} -s^2\mathbf{P}(s)\mathbf{M}\boldsymbol{\iota} + \boldsymbol{\iota} \\ -s\mathbf{P}(s)\mathbf{M}\boldsymbol{\iota} \\ -\mathbf{P}(s)\mathbf{M}\boldsymbol{\iota} \end{bmatrix} \\
&\mathbf{G}_{22}(s) = -\begin{bmatrix} s^2\mathbf{P}(s) \\ s\mathbf{P}(s) \\ \mathbf{P}(s) \end{bmatrix} \\
&\mathbf{P}(s) = [\mathbf{I} \quad \mathbf{0}]\left(\begin{bmatrix} s\mathbf{I} & -\mathbf{I} \\ \mathbf{M}^{-1}\mathbf{K} & s\mathbf{I} \end{bmatrix}\right)^{-1} \begin{bmatrix} \mathbf{0} \\ \mathbf{M}^{-1} \end{bmatrix}
\end{aligned} \tag{6}
$$

Our modified H∞ synthesis problem is solved using the first-order steepest descent algorithm of Apkarian and Noll [16], which can be described by the six fundamental steps below.

Define the set $\mathbf{x}_\Delta = \Delta k^{el}{}_1, \dots, \Delta k^{el}{}_N, \Delta m_1, \dots, \Delta m_N$ and the term $f(\mathbf{x}_\Delta{}^k) = \left\|\mathbf{CL}(\mathbf{Q}_\Delta{}^k, s)\right\|_\infty$.

Step 1. Fix 0<τ<1 and choose initial $\mathbf{x}_\Delta{}^0 = \Delta k^{el}{}_1, \dots, \Delta k^{el}{}_N, \Delta m_1, \dots, \Delta m_N$. set k=0.

Step 2. Step 2. Given $\mathbf{x}_\Delta{}^k$ choose a finite the set of $\boldsymbol{\omega}^k$ so that $\boldsymbol{\omega}^k \in [0, \bar{\omega}^k]$.

Step 3. Step 3. Compute the quantity $\mathcal{M}(\mathbf{Q}_\Delta{}^k) = f(\mathbf{x}_\Delta{}^{k-1}) - f(\mathbf{x}_\Delta{}^k)$. If $|\mathcal{M}(\mathbf{x}_\Delta)| < \text{TOL}$, then stop because of $\partial f(\mathbf{x}_\Delta{}^k) \cong 0$ and $f(\mathbf{x}_\Delta{}^k)$ is optimal.

Step 4. Use a line search to find a step vector "$\mathbf{t}^k$" such that:

$$\pi^k = f(\mathbf{x}_\Delta{}^k + \mathbf{t}^k\tau) - f(\mathbf{x}_\Delta{}^k) \leq -0.25\mathcal{M}(\mathbf{x}_\Delta{}^k) < 0$$

Put $\mathbf{x}_\Delta{}^{k+1} = \mathbf{x}_\Delta{}^k + t^k\tau$ and go to Step 6. If the inequality is not satisfied, proceed to Step 5.

Step 5. Obtain a finer mesh for $\boldsymbol{\omega}^k$, increase counter k by one, and go to Step 3.

Step 6. Increase counter k by one and go back to Step 2.

### 3.2. Second stage: analysis-redesign

Levy and Lavan [1] proposed an iterative analysis-redesign procedure for regulating the earthquake damage in terms of limiting the interstory drift and/or limiting the quantity of absorbed energy within the structure. Their procedure resolves in "fully stressed design" of linear viscous dampers and begins with identifying the "active" ground motion from a given ensemble of ground motion records. Then, an iterative analysis-redesign approach with the following recurrence relationship for the redesign of supplemental damping is used:

$$\Delta c_n{}^{j+1} = \Delta c_n{}^{j}\left(DI_n{}^{j}/DI_n^{ult}\right)^q \tag{7}$$

Where q is the convergence parameter, $\Delta c_n{}^j$ is the $n^{th}$ story damper at the $j^{th}$ iteration, $DI_n{}^j$ is the $n^{th}$ story damage index at the $j^{th}$ iteration that is defined herein as the maximum interstory drift, and $DI_n^{all}$ is the $n^{th}$ story allowable damage index that is defined herein as the allowable interstory drift:

$$\begin{aligned} DI_n{}^j &= \max_{n=1,\dots,N}(|\delta_n(t)|) \\ DI_n^{ult} &= \delta_n^{ult} \end{aligned} \quad \forall \quad n = 1, \dots, N \tag{8}$$

The time-history analysis is utilized to calculate the current interstory drift (analysis). Lastly, the maximum response of the damped structure for each of the remaining ground motions in the ensemble is separately evaluated using time-history analysis. If the design achieved in stage 2 violates the constraints of other records in the ensemble, i.e., $DI_n{}^j/DI_n^{all} > 1.0$, the ground motion for which $DI_n{}^j/DI_n^{all}$ achieves the largest value is added to the active set. Limiting interstory drifts will satisfy the constraints of Eq. (1).

## 4. Five-story building example

The study of this paper employs the five-story yielding shear-type building which has been optimized in Shmerling and Levy [14] using $H_\infty$ synthesis only. In this study, it will be optimized using $H_\infty$ synthesis as well as the analysis-redesign approach. The scheme of the employed yielding shear-type building is depicted in Fig. 3. The properties of the building are provided in Table 1. The building is designed to withstand the April 2011 Miyagi earthquake, which occurred off the coast of Miyagi Prefecture, approximately 70 kilometers east of Sendai, Japan. Accordingly, we address the six ground motion recordings indicated in Table 2. The ratios of interstory drift at which the story columns first yield to the story height are:

$$\begin{aligned} \delta^{yld}{}_1/h_1 &= 0.31(\%) \\ \delta^{yld}{}_2/h_2 &= 0.32(\%) \\ \delta^{yld}{}_3/h_3 &= 0.32(\%) \\ \delta^{yld}{}_4/h_4 &= 0.32(\%) \\ \delta^{yld}{}_5/h_5 &= 0.31(\%) \end{aligned}$$

These ratios remain stationary and, thus, when stiffness changes are introduced to the story, the story yielding forces is changed accordingly. In this study, we consider allowable ductility level of 3.0, and, thus, the ultimate/allowable interstory drift to story height ratios are:

$$\begin{aligned} \delta^{ult}{}_1/h_1 &= 0.94(\%) \\ \delta^{ult}{}_2/h_2 &= 0.96(\%) \\ \delta^{ult}{}_3/h_3 &= 0.96(\%) \\ \delta^{ult}{}_4/h_4 &= 0.95(\%) \\ \delta^{ult}{}_5/h_5 &= 0.92(\%) \end{aligned}$$

The maximum earthquake response of the initial building, in terms of absolute acceleration and ratio of interstory drift to allowable interstory drift, is shown in Table 3.



 

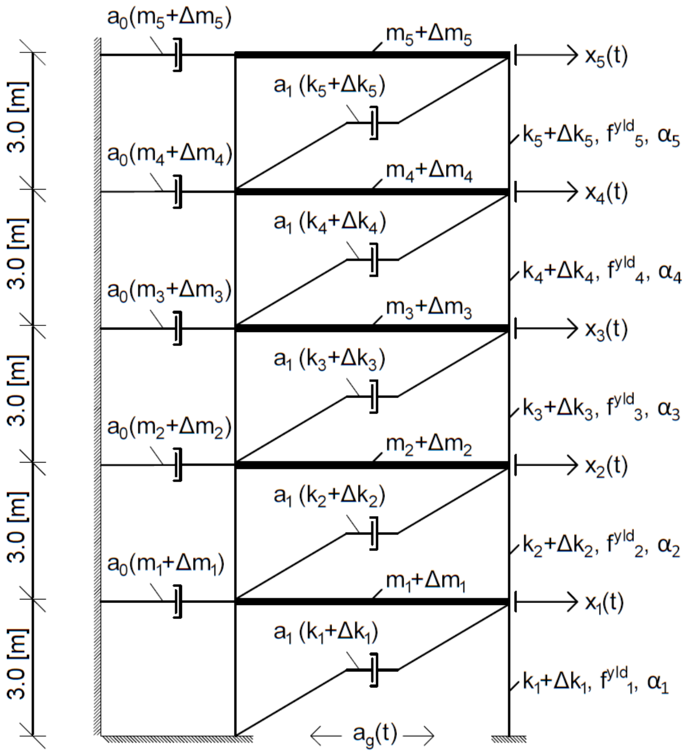


Fig. 3 – Scheme of the yielding five-story shear-type building

Table 1 – Initial five-story yielding shear-type building

| **n** | **$m_n$ $(kN\ sec^2/m)$** | **$k_n$ $(kN/m)$** | **$\Delta c_n$ $(kN\ sec/m)$** | **$f^{yld}_n$ $(kN)$** | **$\alpha_n$** |
|---|---|---|---|---|---|
| 1 | 1.0 | 2,368.7 | 0 | 22.23 | 0.1 |
| 2 | 1.0 | 2,210.8 | 0 | 21.20 | 0.1 |
| 3 | 1.0 | 1,895.0 | 0 | 18.24 | 0.1 |
| 4 | 1.0 | 1,421.2 | 0 | 13.46 | 0.1 |
| 5 | 1.0 | 789.6 | 0 | 7.29 | 0.1 |

Table 2 – Sendai Ground Motion Recordings

| **Name** | **Station** | **Direction** | **Label** | **Date** | **Mag.** | **Sampling (sec)** | **Duration (sec)** | **PGA (g')** |
|---|---|---|---|---|---|---|---|---|
| Sendai01 | IWTH05 | E-W | 2 | 07/04/2011 | 7.4 | 0.01 | 175 | 0.136 |
| Sendai02 | IWTH05 | E-W | 5 | 07/04/2011 | 7.4 | 0.01 | 175 | 0.583 |
| Sendai03 | IWTH05 | N-S | 1 | 07/04/2011 | 7.4 | 0.01 | 175 | 0.161 |
| Sendai04 | IWTH05 | N-S | 4 | 07/04/2011 | 7.4 | 0.01 | 175 | 0.853 |



 

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Sendai05 | MYG004 | E-W | 1 | 07/04/2011 | 7.4 | 0.01 | 191 | 0.903 |
| Sendai06 | MYG004 | N-S | 2 | 07/04/2011 | 7.4 | 0.01 | 191 | 1.266 |

Table 3 – Maximum earthquake response of the initial five-story yielding shear-type building under the April 2011 Miyagi earthquake

| n | $\max_t(\lvert a^{abs}{}_n(t)\rvert)$ (g) | $\max_t\lvert\delta_n(t)\rvert/\delta^{ult}{}_n$ |
|---|---|---|
| 1 | 1.35 | 0.96 |
| 2 | 1.29 | 0.42 |
| 3 | 1.37 | 0.49 |
| 4 | 1.83 | 0.68 |
| 5 | 1.00 | 1.41 |

The first stage of the design methodology utilizes the modified $H_\infty$ synthesis for reducing the absolute accelerations by adjusting the mass and stiffness changes only. Initially, the transfer-function matrices $\mathbf{P}(s)$ and $\mathbf{G}(s)$ of Eq. (6) are defined. In addition, we chose the following constraints on the structural changes:

$$\left.\begin{aligned} \Delta m^{max}{}_n &= 0.2\ m_n \\ \Delta m^{min}{}_n &= -0.05\ m_n \\ \Delta k^{max}{}_n &= 0.5\ k_n \\ \Delta k^{min}{}_n &= -0.5\ k_n \end{aligned}\right\} \forall n = 1, \dots, N$$

After defining the transfer-function matrices and structural constraints, the six-steps $H_\infty$ synthesis algorithms in 3.1 is applied, deriving the structural configurations shown in Table 4. The maximum earthquake response under the April 2011 Miyagi earthquake, in terms of absolute acceleration and ratio of interstory drift to allowable interstory drift, is shown in Table 5. As expected, the absolute acceleration response has been reduced, and the peak absolute acceleration quantity has been reduced by 15%. It is noted that only in the $5^{th}$ story, the allowable interstory drift is violated.

Table 4 – Five-story yielding shear-type building with mass and stiffness changes

| n | $m_n + \Delta m_n$ $(kN\ sec^2/m)$ | $k_n + \Delta k_n$ $(kN/m)$ | $\Delta c_n$ $(kN\ sec/m)$ | $f^{yld}{}_n$ $(kN)$ | $\alpha_n$ |
|---|---|---|---|---|---|
| 1 | 0.95 | 2380.5 | 0 | 2.03 | 0.1 |
| 2 | 0.95 | 2199.7 | 0 | 2.51 | 0.1 |
| 3 | 0.95 | 1885.5 | 0 | 2.21 | 0.1 |
| 4 | 1.20 | 1414.1 | 0 | 1.22 | 0.1 |
| 5 | 1.20 | 785.7 | 0 | 0.47 | 0.1 |

Table 5 – Maximum earthquake response of the five-story yielding shear-type building with mass and stiffness changes under the April 2011 Miyagi earthquake



 

| n | $\max_t(\lvert a^{abs}_n(t)\rvert)$ *(g)* | $\max_t\lvert\delta_n(t)\rvert/\delta^{ult}_n$ |
|---|---|---|
| 1 | 1.53 | 0.96 |
| 2 | 1.29 | 0.42 |
| 3 | 1.25 | 0.49 |
| 4 | 1.52 | 0.68 |
| 5 | 0.81 | 1.41 |

The second stage utilizes the analysis-redesign in 3.2 so that supplemental damping is optimally added for regulating the interstory drift violation. The first ground motion recording in the active set is "Sendai06" whose allowable interstory violation is 41% at the $5^{th}$ story. The iterative term of Eq. (7) is assigned with convergence parameter of $q = 0.5$ and initial supplemental damping of $\Delta c_1{}^0 = \Delta c_2{}^0 = \Delta c_3{}^0 = \Delta c_4{}^0 = \Delta c_5{}^0 = 10\ kN\ sec/m$. the analysis-redesign procedure needed 13 iteration to converge into the optimal damping distribution shown in Table 6 (i.e., a single damper on the $5^{th}$ floor). After the solution of "Sendai06" has been obtained, the other ground motion recordings did not violate the allowable interstory drift quantity. As shown in Table 7, the analysis-redesign procedure is successful in regulating the interstory drift violation.

Table 6 – Initial five-story yielding shear-type building

| n | $m_n + \Delta m_n$ *($kN\ sec^2/m$)* | $k_n + \Delta k_n$ *($kN/m$)* | $\Delta c_n$ *($kN\ sec/m$)* | $f^{yld}_n$ *($kN$)* | $\alpha_n$ |
|---|---|---|---|---|---|
| 1 | 0.57 | 216.4 | 0 | 2.03 | 0.1 |
| 2 | 1.03 | 261.8 | 0 | 2.51 | 0.1 |
| 3 | 0.96 | 230.0 | 0 | 2.21 | 0.1 |
| 4 | 1.73 | 128.6 | 0 | 1.22 | 0.1 |
| 5 | 2.00 | 50.8 | 6.0 | 0.47 | 0.1 |

Table 7 – Maximum earthquake response of the five-story yielding shear-type building with mass and stiffness changes under the April 2011 Miyagi earthquake

| n | $\max_t(\lvert a^{abs}_n(t)\rvert)$ *(g)* | $\max_t\lvert\delta_n(t)\rvert/\delta^{ult}_n$ |
|---|---|---|
| 1 | 1.31 | 0.71 |
| 2 | 1.17 | 0.46 |
| 3 | 1.51 | 0.43 |
| 4 | 1.26 | 0.90 |
| 5 | 0.90 | 1.00 |

## 5. Conclusions

A two-stage seismic design methodology that combines the modified $H_\infty$ synthesis and the analysis-redesign procedure for the fully stressed design of supplemental damping is proposed. The methodology adopts the relation between the $H_\infty$ norm and the SRSS of the peak absolute accelerations. The general system interconnection, which represents the linearity between the absolute accelerations and the ground acceleration, is assigned with a passive controller whose free-parameters are the changes in mass and changes in stiffness only. In the first stage, The $H_\infty$ synthesis optimizes the general system interconnection and reduces the $H_\infty$ norm while adhering to prescribed maximum/minimum limitations in stiffness changes and mass changes.

The analysis-redesign approach is introduced in the second stage of the methodology and is utilized for limiting the maximum interstory drifts to their prescribed allowable quantities by adding supplemental damping. While the $H_\infty$ synthesis addresses the SRSS of the peak absolute acceleration gains in the frequency domain, the analysis-redesign approach employs an ensemble of ground motion records and addresses the maximum interstory drifts in the time domain. Due to performing analysis in different domains and optimizing different structural parameters in each stage, the methodology successfully handles the absolute accelerations and the interstory drifts, simultaneously, without the need of assigning and adjusting weighting factors to define the relative importance of each response.

The numerical example employs a five-story shear-type building which is analyzed for the April 2011 Miyagi earthquake. The example demonstrates the effectiveness of infusion between the field of optimal control and seismic design in terms of earthquake response and feasible structural changes.

## 6. References

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