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Reduction of Coupling Interface Degrees of Freedom for Mixed-interface Component mode Synthesis

Y. Tang(1), H. Qin(2)

(1) *Dalian University of Technology, Dalian 116024, China, ytang@dlut.edu.cn*
(2) *Dalian University of Technology, Dalian 116024, China, hqin@dlut.edu.cn*

Abstract

Component mode synthesis (CMS) is an efficient method to reduce the size of numerical models used to analyze the dynamic/seismic performance of engineering structures, especially when structures are large and complex. In a CMS, original structures are usually modeled by separate interconnected components or substructures, and then reduced-order modes of each substructure will be obtained for mode synthesis. According to the interface conditions of substructures, CMS can be classified as constrained-interface CMS (CCMS), free-interface CMS (FCMS) and mixed-interface CMS (MCMS). Traditional CMS methods work mainly on the reduction of internal DOFs by retaining a small number of low-frequency modes, however the coupling interface DOFs will rules the mode synthesis when the number of substructures is increasing. In this paper, a new coupling interface degrees of freedom (DOFs) reduction technique for MCMS is proposed. This approach employs a set of interpolation functions based on Finite Element Method (FEM) to realize interface nodal coordinate transformations for each substructure, and then only a small number of interpolation basic nodes will be involved in mode synthesis and the following dynamic analysis. Unlike the majority of available CMS methods that retain the full dimension of coupling interface DOFs, the proposed method allows to reduce the interface DOFs significantly. One typical numerical model, which is a rectangular beam with two ends fixed is presented to demonstrate the computational accuracy and efficiency of the proposed method. The results indicate that, good accuracy with a small number of retained interfacial DOFs can be obtained for both eigenvalue and time-history analysis by the reduction method proposed in this paper. The optimal number and distribution of the interpolation basic nodes are discussed on eigenvalue analysis as well. It is shown that the more the interpolation basic nodes are involved in mode synthesis, the better precision will be received. And when the sub-regions are nearly square, the precision is best. That is, the number and distribution of the interpolation basic nodes should be synthetically considered to get reliable solutions.

Keywords: mixed-interface component mode synthesis, coupling interface degrees of freedom, mode reduction technique, coordinate transformation, interpolation function

1. Introduction

CMS is one of the most efficient ways of model reduction for solving structural dynamic problems. CMS was firstly proposed by Hurty [1], and it's an efficient method for analyzing the dynamics of large, complex structures which are often described by separate interconnected components/substructures. Hurty used three displacement modes (i.e. rigid-body, constraint and normal modes) to form a complete set of modes for each substructure, and he constructed a transformation relating component coordinates to system coordinates [2]. Craig and Bampoton [3] improved the approach by Hurty, and they employed two forms of generalized coordinates: one is interface generalized coordinates that is related to constraint modes, the other is normal modes that is related to the free vibration modes of the substructures. As completely restrained interfaces are imposed on each component at the connections, this approach can be classified as CCMS. Hou [4] provided a new modal synthesis technique, in which the interface conditions used for determining component vibration modes were free-free and both compatibility and equilibrium at boundaries were satisfied for modal coupling. This approach can be classified as FCMS. Benfield and Hruda [5] developed a comprehensive method of component modal substitution that is applicable to redundant complex structures.

Using this method, the natural modes of vibration for each component are determined with free-constrained interface conditions, known as MCMS.

In any type of the CMS methods, the most important and difficult task is choosing an appropriate reduced component modes involving modal reduction and mode synthesis, and this continues to be an area of research. Rubin [6] provided an improved FCMS method based upon an incomplete set of free-boundary normal modes, augmented with a low-frequency account for the contribution of residual modes. Park [7] proposed a new flexibility-based CCMS method and they employ a localized Lagrange multipliers that leads to a linearly independent set of interface forces without any redundancies at multiply connected interface nodes. Kim [8] presented a new CMS method based on the concept of the automated multi-level substructuring (AMLS) method. They enhanced the traditional transformation matrix for the AMLS method by considering the inertia effect of residual modes. So that the original component models can be more precisely reduced and the accuracy of reduced models is dramatically improved. However, most of those improvements work on the reduction of internal DOFs. When the number of substructures is increasing, the coupling interface DOFs will rule the following works, and retaining a full dimension of the coupling interface DOFs will definitely lower the computational efficiency of mode synthesis. Some researchers have worked out several methods to reduce the interface DOFs. Markovic [9] introduced a flexibility-based CMS method that employed partitioning via localized Lagrange multiplier with a particular component mode selection criterion, which is independent of loading conditions. Lou [10,11] developed an interface DOFs reduction technique for both CCMS and FCMS, where a set of nodal displacement shape functions via linear interpolation (LI) based on Finite Element Method (FEM) was applied to establish an extra transformation between the master and slave nodes on the connections.

In this work, the interpolation functions based on Finite Element Method (FEM) for MCMS is built, and then the corresponding coordinate transformation matrices for nodal displacements and forces are derived respectively. The efficiency of the approach is demonstrated by two numerical examples. The optimal number and distribution of the interpolation nodes are discussed as well.

2. Review of the classical MCMS

In a global domain, the linear equation of motion of a N-DOFs structure is written as

$$\boldsymbol{M}\,\ddot{\boldsymbol{u}}+\boldsymbol{C}\,\dot{\boldsymbol{u}}+\boldsymbol{k}\,\boldsymbol{u}=\boldsymbol{F} \tag{1}$$

where, $\boldsymbol{M}$, $\boldsymbol{C}$, $\boldsymbol{K}$ are the mass matrix, the damping matrix and the stiffness matrix of the global structure, respectively; $\ddot{\boldsymbol{u}}$, $\dot{\boldsymbol{u}}$, $\boldsymbol{u}$ are the acceleration vector, the velocity vector and the displacement vector, respectively; $\boldsymbol{F}$ is the external force vector.

In the classical MCMS, the original system is divided into two groups: mater substructures and slave substructures [5]. Take one master substructure and one slave substructure as demonstration, and constrain the interconnection of the master substructure while set free that of the slave substructure. The first coordinate transformations that transforming the physical coordinates $\boldsymbol{u}$ to generalized coordinates $\boldsymbol{q}$ are done in each substructure according to the different interconnection conditions. That is,

$$\{\boldsymbol{u}_c\}=\begin{Bmatrix}\boldsymbol{u}_c^I\\ \boldsymbol{u}_c^B\end{Bmatrix}=\begin{bmatrix}\boldsymbol{\Phi}_m & \boldsymbol{\Phi}_c\\ \boldsymbol{0} & I\end{bmatrix}\begin{Bmatrix}\boldsymbol{q}_m\\ \boldsymbol{u}_c^B\end{Bmatrix}=\left[\boldsymbol{T}_c^1\right]\begin{Bmatrix}\boldsymbol{q}_m\\ \boldsymbol{u}_c^B\end{Bmatrix} \tag{2}$$

$$\{\boldsymbol{u}_f\}=\left[\boldsymbol{\Phi}_k \quad \boldsymbol{\Psi}_d\right]\begin{Bmatrix}\boldsymbol{q}_k\\ \boldsymbol{f}_f^B\end{Bmatrix}=\left[\boldsymbol{T}_f^1\right]\begin{Bmatrix}\boldsymbol{q}_k\\ \boldsymbol{f}_f^B\end{Bmatrix} \tag{3}$$

$$\left[\boldsymbol{\Psi}_d\right]=\left[\boldsymbol{\Phi}_d\right]\left[\Lambda_d^{-1}\right]\left[\boldsymbol{\Phi}_d^B\right]^{\mathrm{T}} \tag{4}$$

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where, $\boldsymbol{\Phi}_c$ is the constrained mode; $\boldsymbol{\Phi}_k$ and $\boldsymbol{\Phi}_m$ are the truncated normal mode involving in mode synthesis; $\boldsymbol{\Phi}_d^B$ and Λ_d are the abandoned normal mode corresponding to the coupling interface DOFs and the remaining eigenvalue, respectively; $\boldsymbol{T}^1$ is the first coordinate transformation matrix; subscripts c and f indicate the master substructure with constrained interconnections and the slave substructure with free interconnections, respectively; superscripts I and B indicate the internal DOFs and the interface DOFs. The corresponding generalized stiffness and mass matrix can be written as

$$\left[\boldsymbol{K}_f^{(1)}\right]=\left[\boldsymbol{T}_f^1\right]^{\mathrm{T}}\left[\boldsymbol{K}_f\right]\left[\boldsymbol{T}_f^1\right] \quad , \quad \left[\boldsymbol{M}_f^{(1)}\right]=\left[\boldsymbol{T}_f^1\right]^{\mathrm{T}}\left[\boldsymbol{M}_f\right]\left[\boldsymbol{T}_f^1\right] \tag{5}$$

and

$$\left[\boldsymbol{K}_c^{(1)}\right]=\left[\boldsymbol{T}_c^1\right]^{\mathrm{T}}\left[\boldsymbol{K}_c\right]\left[\boldsymbol{T}_c^1\right] \quad , \quad \left[\boldsymbol{M}_c^{(1)}\right]=\left[\boldsymbol{T}_c^1\right]^{\mathrm{T}}\left[\boldsymbol{M}_c\right]\left[\boldsymbol{T}_c^1\right] \tag{6}$$

Dual assembly with displacement-coordination and force-equilibrium between adjacent substructures are involved in mode synthesis. That is,

$$\left\{\boldsymbol{f}_f^B\right\}+\left\{\boldsymbol{f}_c^B\right\}=0 \tag{7}$$

$$\left\{\boldsymbol{u}_f^B\right\}=\left[\boldsymbol{\Phi}_k^B\right]\left\{\boldsymbol{q}_k\right\}+\left[\boldsymbol{\Psi}_d^B\right]\left\{\boldsymbol{f}_f^B\right\}=\left\{\mathbf{u}_c^B\right\} \tag{8}$$

where, $\boldsymbol{\Phi}_k^B$ and $\boldsymbol{\Psi}_d^B$ are the truncated modal matrix and the residual attachment mode relating to all the interface DOFs for the master substructure. Multiplying the both sides of Eq. (8) by $[\boldsymbol{\Psi}_d^B]^{-1}$, then $\boldsymbol{f}_f^B$ can be written as

$$\left\{\boldsymbol{f}_f^B\right\}=\left[\boldsymbol{\Psi}_d^B\right]^{-1}\left\{\boldsymbol{u}_c^B\right\}-\left[\boldsymbol{\Psi}_d^B\right]^{-1}\left[\boldsymbol{\Phi}_k^B\right]\left\{\boldsymbol{q}_k\right\} \tag{9}$$

At the j-th vibration mode of the system, the equation of motion of the slave substructure is,

$$\left(\begin{bmatrix}\boldsymbol{K}_c^{II} & \boldsymbol{K}_c^{IB}\\ \boldsymbol{K}_c^{BI} & \boldsymbol{K}_c^{BB}\end{bmatrix}-\omega_j^2\begin{bmatrix}\boldsymbol{M}_c^{II} & \boldsymbol{M}_c^{IB}\\ \boldsymbol{M}_c^{BI} & \boldsymbol{M}_c^{BB}\end{bmatrix}\right)\begin{Bmatrix}\boldsymbol{u}_c^I\\ \boldsymbol{u}_c^B\end{Bmatrix}=\begin{Bmatrix}\mathbf{0}\\ \boldsymbol{f}_c^B\end{Bmatrix} \tag{10}$$

Ignoring the inertia items in Eq. (10), it has

$$\left\{\boldsymbol{f}_c^B\right\}=\left[\boldsymbol{K}_c^{BB}+\boldsymbol{K}_c^{BI}\,\boldsymbol{\Phi}_c\right]\left\{\boldsymbol{u}_c^B\right\}+\left[\boldsymbol{K}_c^{BI}\right]\left[\boldsymbol{\Phi}_m\right]\left[\boldsymbol{q}_m\right] \tag{11}$$

Substituting Eq. (9) and Eq. (11) into Eq. (7) leads to

$$\left\{\boldsymbol{u}_c^B\right\}=\left[\boldsymbol{P}_1 \quad \boldsymbol{P}_2\right]\begin{Bmatrix}\boldsymbol{q}_k\\ \boldsymbol{q}_m\end{Bmatrix} \tag{12}$$

where,

$$\boldsymbol{P}_1=\left(\boldsymbol{I}+\boldsymbol{\Psi}_d^B\,\boldsymbol{K}_c^{BB}+\boldsymbol{\Psi}_d^B\,\boldsymbol{K}_c^{BI}\,\boldsymbol{\Phi}_c\right)^{-1}\boldsymbol{\Phi}_k^B \tag{13}$$

$$\boldsymbol{P}_2=-\left(\left[\boldsymbol{\Psi}_d^B\right]^{-1}+\boldsymbol{K}_c^{BB}+\boldsymbol{K}_c^{BI}\,\boldsymbol{\Phi}_c\right)^{-1}\boldsymbol{K}_c^{BI}\,\boldsymbol{\Phi}_m \tag{14}$$

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Substituting Eq. (12) into Eq. (9) becomes

$$\left\{ \boldsymbol{f}_f^B \right\} = [\boldsymbol{Q}_1 \quad \boldsymbol{Q}_2] \begin{Bmatrix} \boldsymbol{q}_k \\ \boldsymbol{q}_m \end{Bmatrix} \tag{15}$$

where,

$$\boldsymbol{Q}_1 = \left[\boldsymbol{\Psi}_d^B \right]^{-1} \left(\boldsymbol{P}_1 - \boldsymbol{\Phi}_k^B \right) \tag{16}$$

$$\boldsymbol{Q}_2 = \left[\boldsymbol{\Psi}_d^B \right]^{-1} P_2 \tag{17}$$

The second coordinate transformation for the two substructures and the corresponding generalized stiffness and mass matrix can be written as

$$\begin{Bmatrix} \boldsymbol{q}_k \\ \boldsymbol{f}_f^B \end{Bmatrix} = \begin{bmatrix} \boldsymbol{I}_k & \boldsymbol{0} \\ \boldsymbol{Q}_1 & \boldsymbol{Q}_2 \end{bmatrix} \begin{Bmatrix} \boldsymbol{q}_k \\ \boldsymbol{q}_m \end{Bmatrix} = \left[\boldsymbol{T}_f^2 \right] \begin{Bmatrix} \boldsymbol{q}_k \\ \boldsymbol{q}_m \end{Bmatrix} \tag{18}$$

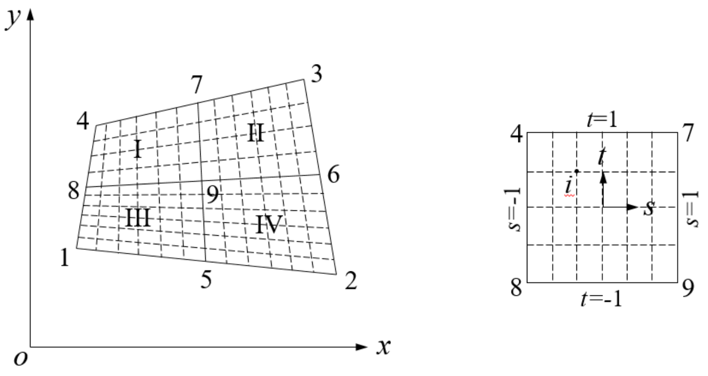
$$\begin{Bmatrix} \boldsymbol{q}_m \\ \boldsymbol{u}_c^B \end{Bmatrix} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{I}_m \\ \boldsymbol{P}_1 & \boldsymbol{P}_2 \end{bmatrix} \begin{Bmatrix} \boldsymbol{q}_k \\ \boldsymbol{q}_m \end{Bmatrix} = \left[\boldsymbol{T}_c^2 \right] \begin{Bmatrix} \boldsymbol{q}_k \\ \boldsymbol{q}_m \end{Bmatrix} \tag{19}$$

Assembling the generalized stiffness and mass matrices of all substructures after the second transformation, the generalized stiffness and mass matrices of the structural system can be formed to solve the eigenvalue problem and dynamic responses of whole structure system.

3. Reduction of the coupling interface DOFs

3.1 Formulations

In this section, we present a new reduction strategy to minimize the interface DOFs that involve in the following mode synthesis. Assuming that the coupling interface is quadrilateral and discretized by 4-node arbitrary quadrilateral elements based on FEM, then we decompose it into several subdomains (I~IV) as shown in Fig.1 (a).



(a) (b)

Fig. 1 – FEM model of the coupling interface and the local coordinate of a subdomain. (a) Arbitrary quadrilateral element; (b) 4-node quadrilateral isoperimetric element.

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Choosing a small number of nodes (1~9) on the interface as interpolation basic nodes (IBNs), and the displacement or force of node i can be described approximately by that of the IBNs via a set of shape functions $N_j(s_i,t_i)$ based on FEM [13]:

$$u_e^i = \sum_{j=1}^{n} N_j(s_i,t_i) u_e^j \quad or \quad f_e^i = \sum_{j=1}^{n} N_j(s_i,t_i) f_e^j \tag{20}$$

where, subscript e=x, y, z is the three directions in the global coordinate system; n is the number of IBNs in the subdomain that node i is in. Introducing LIP to set up the shape function as,

$$N_j(s_i,t_i) = \frac{1}{4}(1+ss_i)(1+tt_i) \tag{21}$$

The nodal displacement or force vector relative to the coupling interface DOFs of the constrained-interface or the free-interface substructure can be obtained from assembling the interpolation transformation of all re-regions. There are

$$\{\boldsymbol{u}_c^B\} = [\boldsymbol{N}_c]_{n_0 \times n} \{\boldsymbol{u}_c^b\} \quad or \quad \{\boldsymbol{f}_f^B\} = [\boldsymbol{N}_f]_{n_0 \times n} \{\boldsymbol{f}_f^b\} \tag{22}$$

in which, N_c is the shape function of the slave substructure; N_f is the shape function of the master substructure; n_0 is the total number of DOFs on the coupling interface; $\boldsymbol{u}_c^b$ is the displacement vector with IBNs, $n\times1$; $\boldsymbol{f}_f^b$ is the force vector with IBNs, $n\times1$. Substituting Eq. (22) into Eq. (2) and Eq. (3), there are

$$\begin{Bmatrix} \boldsymbol{q}_m \\ \boldsymbol{u}_c^B \end{Bmatrix} = \begin{bmatrix} \boldsymbol{I}_m & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{N}_c \end{bmatrix} \begin{Bmatrix} \boldsymbol{q}_m \\ \boldsymbol{u}_c^b \end{Bmatrix} = [\boldsymbol{T}_c^R] \begin{Bmatrix} \boldsymbol{q}_m \\ \boldsymbol{u}_c^b \end{Bmatrix} \quad or \quad \begin{Bmatrix} \boldsymbol{q}_k \\ \boldsymbol{f}_f^B \end{Bmatrix} = \begin{bmatrix} \boldsymbol{I}_k & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{N}_f \end{bmatrix} \begin{Bmatrix} \boldsymbol{q}_k \\ \boldsymbol{f}_f^b \end{Bmatrix} = [\boldsymbol{T}_f^R] \begin{Bmatrix} \boldsymbol{q}_k \\ \boldsymbol{f}_f^b \end{Bmatrix} \tag{23}$$

3.2 Modal synthesis with reduced coupling interface DOFs

The process of mode synthesis with reduced coupling interface DOFs is basically as same as mentioned in Section 2, except that the coordinates involving in docking are $\boldsymbol{u}_c^b$ and $\boldsymbol{f}_f^b$ instead of $\boldsymbol{u}_c^B$ and $\boldsymbol{f}_f^B$. Assuming that the shape function and the number of interpolation basic nodes chosen for all the substructures are same, so that $\boldsymbol{N}_c$=$\boldsymbol{N}_f$=$\boldsymbol{N}$. Substituting Eq. (22) into Eq. (7) and Eq. (8) leads to

$$[\mathbf{N}]\{\boldsymbol{f}_f^b\} + [\mathbf{N}]\{\boldsymbol{f}_c^b\} = 0 \tag{24}$$

$$[\boldsymbol{N}]\{\boldsymbol{u}_f^b\} = [\boldsymbol{N}]\{\mathbf{u}_c^b\} \tag{25}$$

Then,

$$\begin{Bmatrix} \boldsymbol{q}_k \\ \boldsymbol{f}_f^B \end{Bmatrix} = \begin{bmatrix} \boldsymbol{I}_k & \boldsymbol{0} \\ \boldsymbol{Q}_1^R & \boldsymbol{Q}_2^R \end{bmatrix} \begin{Bmatrix} \boldsymbol{q}_k \\ \boldsymbol{q}_m \end{Bmatrix} = [\boldsymbol{T}_f^3] \begin{Bmatrix} \boldsymbol{q}_k \\ \boldsymbol{q}_m \end{Bmatrix} \tag{26}$$

$$\begin{Bmatrix} \boldsymbol{q}_m \\ \boldsymbol{u}_c^B \end{Bmatrix} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{I}_m \\ \boldsymbol{P}_1^R & \boldsymbol{P}_2^R \end{bmatrix} \begin{Bmatrix} \boldsymbol{q}_k \\ \boldsymbol{q}_m \end{Bmatrix} = [\boldsymbol{T}_c^3] \begin{Bmatrix} \boldsymbol{q}_k \\ \boldsymbol{q}_m \end{Bmatrix} \tag{27}$$

where,

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$$\boldsymbol{P}_1^R = \left(\boldsymbol{K}_c^{BB} \cdot \boldsymbol{N} \cdot \boldsymbol{N}^{\mathrm{T}} \cdot \boldsymbol{\Phi}_c \cdot \boldsymbol{N} + \boldsymbol{N} \cdot \left[\boldsymbol{\Psi}_d^n \right]^{-1} \boldsymbol{N}^{\mathrm{T}} \cdot \boldsymbol{N} \right)^{-1} \cdot \boldsymbol{N} \cdot \left[\boldsymbol{\Psi}_d^n \right]^{-1} \cdot \boldsymbol{\Phi}_k^n \tag{28}$$

$$\boldsymbol{P}_2^R = -\left(\boldsymbol{K}_c^{BB} \cdot \boldsymbol{N} \cdot \boldsymbol{N}^{\mathrm{T}} \cdot \boldsymbol{K}_c^{BI} \cdot \boldsymbol{\Phi}_c \cdot \boldsymbol{N} + \boldsymbol{N} \cdot \left[\boldsymbol{\Psi}_d^n \right]^{-1} \cdot \boldsymbol{N}^{\mathrm{T}} \cdot \boldsymbol{N} \right)^{-1} \cdot \boldsymbol{K}_c^{BI} \boldsymbol{\Phi}_m \tag{29}$$

$$\boldsymbol{Q}_1^R = \left[\boldsymbol{\Psi}_d^n \right]^{-1} \cdot \boldsymbol{N}^{\mathrm{T}} \cdot \boldsymbol{N} \cdot \boldsymbol{P}_1^R \tag{30}$$

$$\boldsymbol{Q}_2^R = \left[\boldsymbol{\Psi}_d^n \right]^{-1} \cdot \boldsymbol{N}^{\mathrm{T}} \cdot \boldsymbol{N} \cdot \boldsymbol{P}_2^R - \left[\boldsymbol{\Psi}_d^n \right]^{-1} \cdot \boldsymbol{\Phi}_k^n \tag{31}$$

The generalized stiffness and mass matrices of the system with reduced interfacial DOFs can then be formed to solve the eigenvalue problem and dynamic responses of whole structure system.

4. Numerical example

A beam with two fixed ends is taken as an example to investigate the performance of the proposed interfacial induction method, and the structural properties are: l=6m, w=0.6m, h=0.4m, ρ=2500kg/m^3, E=30000Mpa, v=0.3. Dividing the whole system into two substructures (numbered 1 and 2 in Fig. 2), and the coupling interface of the substructure 1 is constrained while that of the substructure 2 is set free. The coupling interface is rectangle in this case with 25 nodes and 75 DOFs as shown in Fig. 3 (a). Fig. 3 (b)~(d) indicate three schemes of the chosen for IBNs with the number of 4, 9 and 10, respectively.

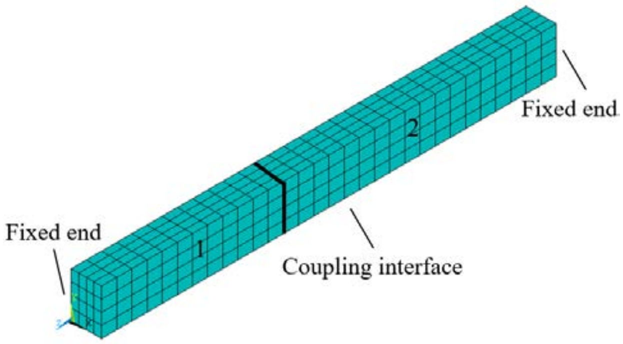

Fig. 2 – Finite element model of the beam

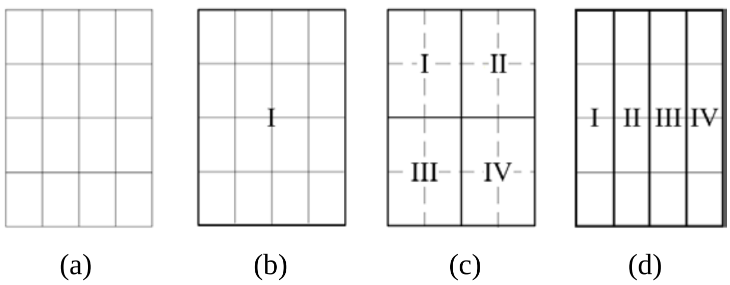

Fig. 3 – Schemes of the chosen for IBNs. (a) Full interfacial DOFs; (b) one subdomain (I) with 4 IBNs and 12 DOFs; (c) four subdomains (I~IV) with 9 IBNs and 27 DOFs; (d) four subdomains (I~IV) with 10 IBNs and 30 DOFs.

Three methods are introduced for accuracy analysis, including: (1) FEM with full DOFs of the whole system, which is referred as referential solution; (2) MCMS method with full interfacial DOFs [12]; and (3)

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MCMS method with reduced interface DOFs (MCRD). The substructural modes selected for substructure 1 and 2 are 5 and 10, respectively. The first five modal frequencies of the beam derived from each method are listed in Table 1, where the precisions measured through the relative error with respect to FEM are shown in parenthesis in the table. The displacement responses of the beam derived from the proposed interfacial DOFs reduction method with three reduction schemes are illustrated in Fig. 4, where the seismic excitation is El Centro (NS, 1940) with a scaled peak ground acceleration (PGA) of 0.1g.

The results shown both in Table 1 and Fig. 4 indicate that the proposed interfacial DOFs reduction method can achieve favorable precision when compared with the referential solutions. It is also seen that the number and distribution of the IBNs has important influence on the numerical precision. Although the number of the interpolation basic nodes in Scheme d is 10 and the number of the interpolation basic nodes in Scheme c is 9, the numerical precision of Scheme c is better than that of Scheme d. It means that it is better the shape of the sub-region is toward to square.

Table 1 – The first five modal frequencies of the beam (Hz) and relative error (%)

Method	**Mode**					**DOFs**
FEM	39.10	56.62	104.63	146.16	157.97	2175
MCMS	39.47(0.94)	56.36(-0.46)	95.67(-8.56)	134.88(-7.72)	159.02(0.66)	165
MCRD-4	41.21(5.39)	58.22(2.83)	88.77(-15.16)	132.56(-9.51)	155.57(-1.52)	39
MCRD-9	39.76(1.68)	56.32(-0.53)	95.24(-8.97)	133.63(-8.57)	159.97(1.26)	69
MCRD-10	40.09(2.53)	56.09(-0.93)	93.81(-10.34)	132.77(-9.16)	160.09(1.34)	75

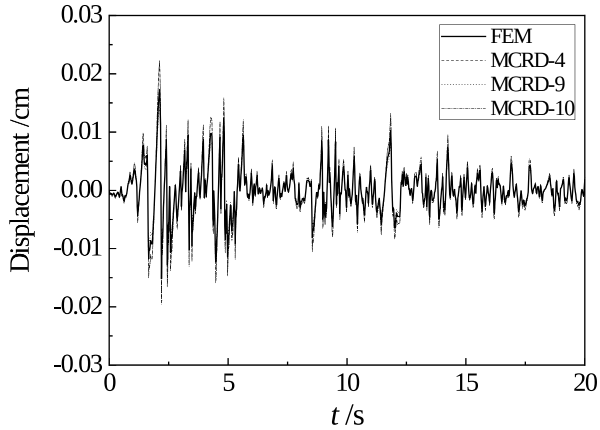

Fig. 4 – Seismic responses of the beam

5. Conclusions

CMS method is one of the well-developed model order reduction technique that describes the whole structure with several independent components/substructures. In this paper, we focus on reducing the coupling interface DOFs to solve the problems of docking difficulties and incomplete boundary conditions. Displacement and force interpolation functions of the coupling interface DOFs based on FEM are set up for MCMS method. The corresponding interpolation transformations are done in each substructure, and only a small number of IBNs are kept for mode synthesis so that the computational efficiency is greatly enhanced. The numerical performance of the MCMS method with reduced interface DOFs is demonstrated by a simple example. The results indicate that favorable accuracy with a least number of retained DOFs involved in

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mode synthesis can be obtained when compared with the referential solutions. The number and distribution of the IBNs chosen for modal synthesis have significant influence on the numerical precision. It is noticed that the more the interpolation nodes involved, the higher the calculation accuracy will be. When the IBNs uniformly distribute on the coupling interface, the accuracy is the best.

6. Acknowledgements

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