SEISMIC DEMANDS ON STEEL BRBF CONSIDERING VARIOUS BRB HYSTERETIC MODELS

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Abstract

Steel frames equipped with buckling restrained braces (BRBF) are widely adopted as lateral load resisting system in recent decades. The buckling restrained braces (BRB) can provide superior load-carrying capacity, as well as excellent energy dissipating capability than the conventional steel braces. For simplicity, the seismic behaviour of BRB has often been modelled as bilinear or trilinear models. Many important research results on seismic demands of BRBF, such as maximum inter-story drift, cumulative plastic ductility, and residual inter-story drift, are based on these simplified models.

However, experimental studies have identified significant isotropic hardening as well as tension-compression asymmetry characteristics of BRB under cyclic loading. The isotropic hardening effect of BRB is due to the strain hardening of steel core plates, while the capacity asymmetry in tension and compression is mainly due to friction between core and sleeve in compression. Specifically, the capacity of BRB in compression may as higher as 15% than that in tension.

In recent decade, some researchers have developed various novel models to represent realistic BRB hysteretic behaviour in numerical studies, such as trilinear force-deformation analytical model, Bouc-Wen model, modified Ramberg-Osgood combined hardening model and Menegotto-Pinto combined hardening model. Zona et al. reviewed the benefits and limitations of those models and developed a new refined hysteretic model of BRB, (hereinafter referred to as Zona model), which accurately considered kinematic and isotropic hardening as well as the tension-compression asymmetry behaviour of BRB. This refined BRB model has been developed into OpenSees 2.5.0, called SteelBRB uniaxialMaterial.

How much deviation will be brought by using a simplified model of BRB to quantify seismic demands on BRBF is still a research gap. Since the refined hysteretic model of BRB is available currently, it is time to systematically quantify the seismic demands on BRBF considering various BRB hysteretic models, thus providing an insight into the performance-based seismic design of BRBF. In this study, three and six-story BRBF are adopted as the prototype structures. Three hysteretic models of BRB, including Zona model and two bilinear models, are considered. The maximum inter-story drifts, cumulative plastic ductility and residual inter-story drifts response are compared under both design-based earthquakes and maximum considered earthquakes. It indicates that the simplified hysteretic models of BBR can predict the maximum inter-story drift response of BRBF with acceptable accuracy. Meanwhile, the simplified models have natural deficiencies in predicting the structural residual drift and cumulative plastic ductility of BRB. This limitation should be fully considered if simplified models are adopted in seismic performance evaluation of the BRBF.

Keywords: Buckling-restrained braces, Seismic demands, hysteretic models, nonlinear time history analysis.

1. Introduction

Steel frames equipped with buckling restrained braces (BRBF) are widely adopted as lateral load resisting system in recent decades. [1–5]. The buckling restrained braces (BRB) can provide superior load-carrying capacity, as well as excellent energy dissipating capability than the conventional steel braces. [6,7]. Various types of BRB have been developed, such as BRB with concrete-infilled steel tube [8], steel assembled BRB [9], self-centring BRB [10–12], and double-stage yield BRB [13,14].

Maximum inter-story drift and cumulative plastic ductility of BRB are among the most important engineering demand parameters (EDPs) for seismic design of BRBF [6]. Maximum inter-story drift is the most intuitive parameter that represents the structural peak inter-story drift response under earthquakes. Cumulative plastic ductility is normally defined as the ratio of cumulative inelastic deformation to the initial yield deformation. The structural seismic resilience is an important aspect of earthquake engineering in recent years, wherein the residual drift of structures after earthquakes is the most critical issue to quantify the resilience of structures [15]. Thus, the residual drift is also an important engineering demand parameter. Usually, the seismic demands on BRBF are determined through nonlinear time history analysis on numerical models of BRBF [6,16].

For simplicity, the seismic behaviour of BRB has often been numerically modelled as bilinear or trilinear models in the nonlinear time history analysis of BRBF [15,17–19]. Many important research results on seismic demands of BRBF, such as maximum inter-story drift [6,18], cumulative plastic ductility [6] and residual inter-story drift [15,20], are based on these simplified models. However, experimental studies have identified significant isotropic hardening as well as tension-compression asymmetry characteristics of BRB under cyclic loading [21]. The isotropic hardening effect of BRB is due to the strain hardening of steel core plates, while the capacity asymmetry in tension and compression is mainly due to friction between core and restaining tube in compression, as shown in Fig. 1. Specifically, the capacity of BRB in compression may as higher as 15% than that in tension.

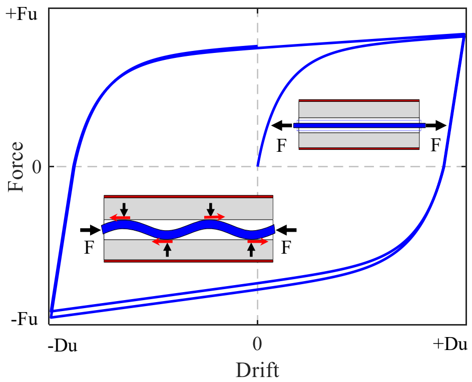


Fig. 1 – Characteristics of the BRB hysteretic behaviour

In recent decade, some researchers have developed various novel models to represent realistic BRB hysteretic behaviour in numerical studies [22], such as trilinear force-deformation analytical model [17], modified Ramberg-Osgood combined hardening model [23] and Menegotto-Pinto combined hardening model [24]. Zona and Dall' Asta [22] critically reviewed the benefits and limitations of these models and developed a new refined hysteretic model of BRB, (hereinafter referred to as Zona model), which accurately considered kinematic and isotropic hardening as well as the tension-compression asymmetry behaviour of BRB. By comparing responses of BRBF adopting the refined Zona model and other simplified models, Rossi gave comprehensive discussion on the limitation of existing simplified models of BRB on estimating maximum inter-story drift and ductility demands.

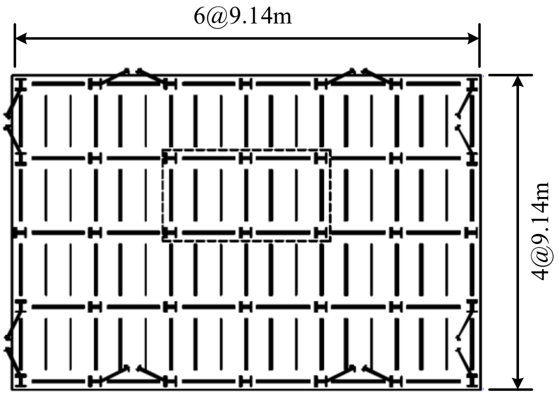
In early years, the simplified models of BRB are generally accepted due to the lack of models, which are able to capture realistic behaviour of BRB with acceptable computational cost. Nevertheless, researchers investigated strain hardening effect of BRB on residual drifts of BRBF, and the results showed that the residual drift demands on BRBF changed significantly only 1% modification of strain hardening ratio [25].

However, the effect of BRB hysteretic models on the seismic demands of BRBF has not been systematically investigated yet.

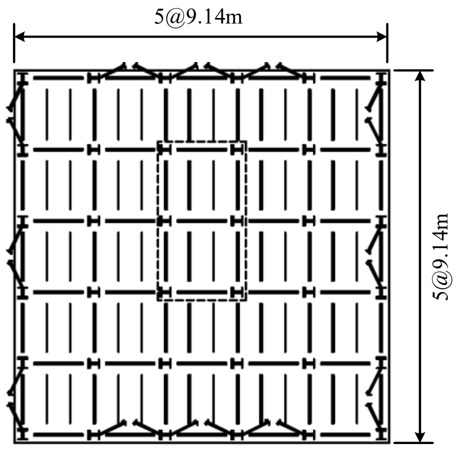
Due to the recent availability of the refined hysteretic model of BRB, opportunity arises to systematically quantify the seismic demands on BRBF considering various BRB hysteretic models to provide an insight into the performance-based seismic design of BRBF. In this study, three- and six-story BRBF are adopted as the prototype structures. Three hysteretic models of BRB, including Zona model, bilinear (EPK model) and the elastic perfectly plastic model (EPP model), are considered. The results of maximum inter-story drift, residual inter-story drift, and cumulative plastic ductility demands obtained by three models are compared under both design-based earthquakes (DBE) and maximum considered earthquakes (MCE).

2. Prototype BRBF

In this study, three and six-story steel frames with BRB, designed by Sabelli et al. [30], were adopted as the prototype buildings to investigate the effect of BRB hysteretic models on quantifying seismic demands on BRBF. These two buildings were assumed to be located in Downtown Los Angeles where in the seismic design category D [6]. As shown in Fig. 2 and Fig. 3, the 3-story building designed with four chevron BRBF in each direction to resist lateral load, while the 6-story building designed with six chevron BRBF in each direction to resist lateral load. For clarity, the 3-story chevron BRBF and 6-story chevron BRBF are represented as BRBF3 and BRBF6. The BRBF3 was designed with a bay width of 9.14 m and constant story height of 3.94 m. For the BRBF6, bay width was 9.14 m which is the same as BRBF3, while story heights of 5.49 m and 3.96 m were designed for ground story and upper stories, respectively. Hinge connection of beam-to-column joint is considered in this study. All the members were sized according to AISC standard [6]. Seismic hazard with a 10% probability of exceedance in 50 years was adopted to determine the lateral earthquake action. Based on the information of seismic action, the stiffness and capacity of BRB in each storey can be determined. The detailed design information of BRB can be found in [6]. Wide flange section W14×48 was adopted for frame beams in both BRBF3 and BRBF6. Section W12×96 was adopted as column section for BRBF3. For BRBF6, section W14×211 was adopted for columns in first to third story, while columns in fourth to sixth story, section W14×132 was adopted. It is worth noting that all the design information of the selected BRBF are exactly the same with that reported in the literature [6].



(a) 3-storey



(b) 6-storey

Fig. 2 – Prototype buildings in plane-view

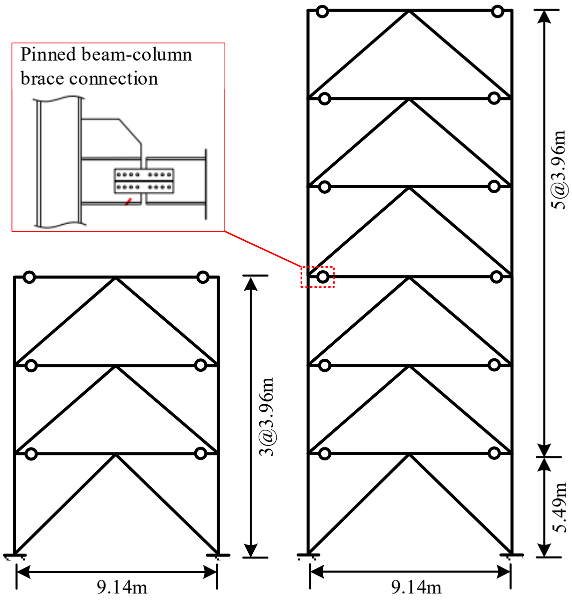


Fig. 3 – Elevation view of bare BRBF

3. Development of numerical models

The Zona model, elastic-plastic with kinematic hardening model (EPK model) and elastic perfectly plastic model (EPP model) are considered to capture the hysteretic behaviour of BRB in the BRBF. Key parameters for these three hysteretic models, such as the initial yield force, the post-yield stiffness, and the maximum yield force, are listed in Table 1. The initial yield force for Zona model and EPK model is the initial tension yield force of BRB, F_{y0}. The yield force in compression of Zona model is considered as 1.08 times of the initial yield force. For the EPP model, the yield force is set to the maximum yield force, $F_{y\max}$. The post-yield stiffness is intended to simulate the average behaviour of BRB from yielding to achievement of the reference state, wherein the reference state means that the fully saturated isotropic hardening condition is fulfilled. The post-yield stiffness for Zona model and EPP model is set to $0.01k_0$, wherein k_0 is the elastic stiffness of BRB. For the EPK model, additional parameters, including a force F^R equal to $1.5F_{y0}$ and a kinematic ductility μ_k^R equal to 15, are defined based on a series of experimental results [22, 26]. It is noteworthy that the same reference state is assumed for all three hysteretic models. Compared with simplified models, two isotropic hardening parameters, δ_r and α, need to be defined in the Zona model. Specifically, δ_r, α are non-dimensional constants that control the rate of isotropic hardening and the trend of transition from elastic range to plastic range, respectively. The Zona model has been implemented in OpenSees as an uniaxialMaterial model SteelBRB [27]. For the EPK model and the EPP model, they can be simulated with uniaxialMaterial Steel01 which is a bilinear steel material model.

To demonstrate the characteristics of the three models, numerical simulation on hysteretic behaviour of a BRB specimen under static cyclic loading was conducted by adopting different BRB hysteretic models. This BRB experimental test result is reported by Chou and Chen [26], wherein the data is from specimen 1. As shown in Fig. 4, Zona model can capture the kinematic and isotropic hardening behaviour of the BRB specimen very well. However, the EPK model will overestimate the post-yield stiffness of the specimen, meanwhile, the EPP model will overestimate the yield capacity of the specimen. Since the accuracy of Zonal model has also been validated in previous, hereinafter we take the responses of BRBF adopting Zonal model as the exact seismic responses.

Table 1 – Parameters for the three hysteretic models

Hysteretic models	Zona model	EPK model	EPP model
Initial yield force	$F_y = F_{y0}$	$F_y = F_{y0}$	$F_y = F_{y\max}$
Post-yield stiffness	$k_1 = 0.01k_0$	$k_1 = \dfrac{F_R - F_{y0}}{\Delta l_{y0}(\mu_k^R - 1)}$	$k_1 = 0.01k_0$
Additional parameters	$F_{y\max} = \dfrac{F^R - \frac{k_1}{k_0}F_{y0}\mu_k^R}{1-(k_1/k_0)} \cong 1.364F_{y0}$ $\alpha = 0.6, \delta_r = 0.2$	$F_R = 1.5F_{y0}, \mu_k^R = 15$ Δl_{y0} is elongation at yield	$F_{y\max} = 1.364F_{y0}$

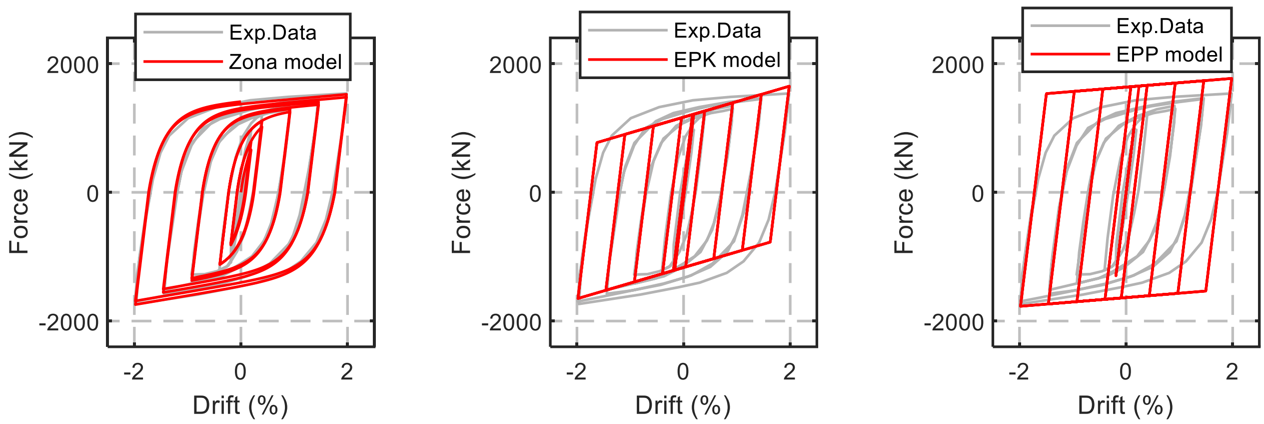


Fig. 4 – Simulation results of different hysteretic models

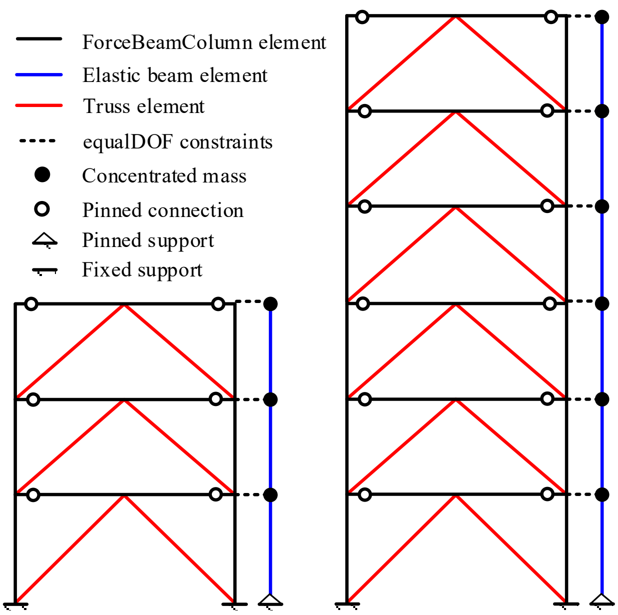


Fig. 5 – Numerical models of BRBF

Fig. 5 shows the numerical models of BRBF3 and BRBF6 in OpenSees [27]. The frame beams and columns were modelled with ForceBeamColumn elements, wherein a plastic hinge was specified at each end of an element. Nonlinear behaviour of a structural member can be fully captured with only one force-based element. Material nonlinearity of steel beams and columns was simulated with uniaxialMaterial Steel02, wherein elastic modulus, yield strength and strain hardening ratio are adopted as 200Gpa, 345 MPa and 0.03, respectively. Bending moment was released at the beam end of each beam-column-brace joint to simulate the pinned connection of beam end. The truss elements and nonlinear uniaxialMaterial modelling scheme, which has been validated against the experimental test results, were adopted to simulate the BRB. A leaning column with concentrated masses was adopted to simulate the gravity frame. The concentrated masses were distributed at all the floor levels, and an 'equalDOF' constraint was used to coupling horizontal displacement of BRBF and leaning columns at each floor. Concentrated mass at each floor, stiffness and strength of leaning column in each story are obtained from the design information [6]. The fundamental periods of the numerical models are 0.54 and 1.02 seconds for BRBF3 and BRBF6, respectively.

4. Statistical results of the seismic demands

Two sets of ground motions, considering design level earthquakes as well as maximum considered level earthquakes, were selected as input for nonlinear time history analysis in this study. Each set includes 20 ground motion records and was originally developed in the SAC Joint Venture steel project [28]. The first set ground motions were considered to represent the designed based earthquake level (DBE) in ASCE 7-10 [28] with probability of exceedance of 10% in 50 years. The second set ground motions were considered to represent the maximum considered earthquake level (MCE) in ASCE 7-10 with probability of exceedance of 2% in 50 years. The mean values of seismic demands, including maximum inter-story drift (Max drift), residual inter-story drift (Res drift) and cumulative plastic ductility (CPD) were obtained and shown in Fig. 6 and Fig. 7. As shown in Fig. 6, the mean values of maximum inter-story drifts estimated by all the three models are almost identical under DBE earthquakes. The mean residual drift estimated by the EPK model is very close to that of the refined Zona model, meanwhile, the EPP model greatly overestimated the residual inter-story drift under the DBE earthquakes. For the cumulative plastic ductility, the simplified BRB hysteretic models (i.e. EPK and EPP models) always underestimated the seismic demand significantly than that by the refined Zona model. As shown in Fig. 7, under the MCE earthquakes, the simplified BRB models can capture the maximum inter-story drift response of BRBF very close to that of the refined Zona model. However, with respect to the residual inter-story drift response, the results of EPK model and EPP model are quite different. Specifically, compared with the refined Zona model, the EPK model tends to underestimate the residual inter-story drift and the EPP model tends to overestimate the residual inter-story drift response of BRBF. This result is due to the difference in post-yield stiffness and yield capacity of the three hysteretic models. The post-yield stiffness of the Zona model is reduced with the increment of axial deformation, which is close to the actual hysteretic behaviour of BRB. While the post-yield stiffness of EPK model is a constant value larger than that of the Zona model under relative strong earthquakes. The post-yield stiffness of EPP model is a constant value of $0.01k_0$, which is the lowest of all three models. Since the structural residual drift response always increased with the reducing post-yield stiffness of BRB, the EPK model and EPP model tend to underestimate and overestimate the residual inter-story drift, respectively. As for the cumulative plastic ductility, the EPK model can capture the result very close to that of the refined Zona model under the MCE earthquakes. Moreover, the EPP model always underestimates the cumulative plastic ductility under either DBE or MCE earthquakes. This phenomenon is mainly due to the yield capacity given in the EPP model which is much higher than the nominal yield capacity of BRB.

Table 2 lists the mean and mean plus one standard deviation values of the seismic demand indices obtained from the nonlinear time history analysis. Statistically, the probabilities of exceedance of the mean and the mean plus one standard deviation values are 50% and 15.8%, respectively. The maximum value of each seismic demand index is bolded in Table 2. These values can be adopted as a reference for determining the engineering demand parameters in seismic design. For instance, the cumulative plastic ductility demand on

BRB can be adpted as the mean plus one standard deviation value of CPD under MCE earthquakes, which is 252.

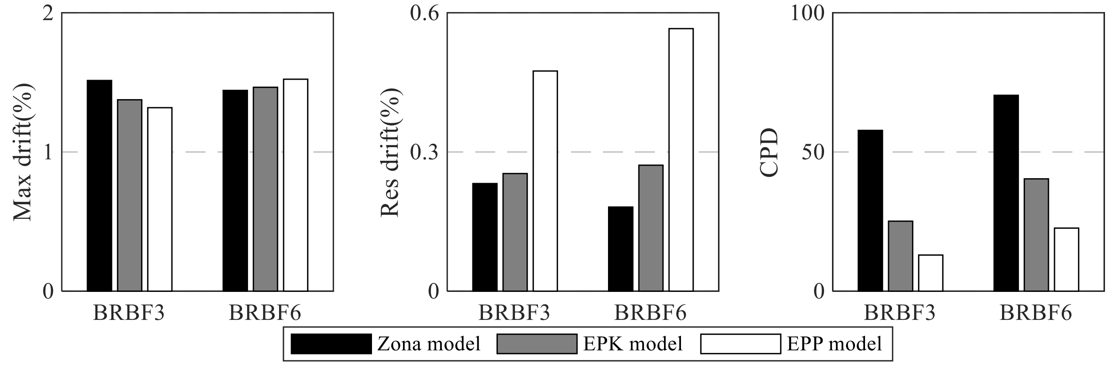


Fig. 6 – Comparison of seismic demands on BRBF subject to DBE earthquakes

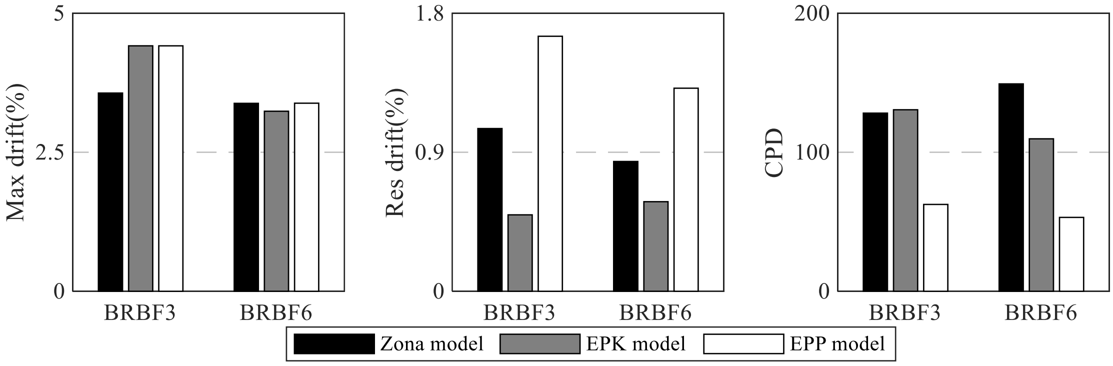


Fig. 7 – Comparison of seismic demands on BRBF subject to MCE earthquakes

Table 3 lists the errors of seismic demands obtained by the simplified BRB hysteretic models (i.e. EPK and EPP models) comparing to those obtained by the refined Zona model. For the simplified models, the relative errors of mean values of maximum inter-story drifts are within ±24%. For the residual inter-story drift, the relative estimation error of EPK model is in the range of -54% to +50%, while the relative estimation error of EPP model may as large as 56% to 212%. As for the cumulative plastic ductility (CPD), the relative estimation error of EPK model is in the range of -57% to +2%, meanwhile, the relative estimation error of EPP model is in the range of -78% to -51%. These significant relative errors are mainly due to the difference in yield capacity and post-yield stiffness of the three BRB hysteretic models. Specifically, the excessively high yield capacity of EPP model always results in underestimated cumulative plastic ductility, and the low post-yield stiffness of EPP models always results in overestimated residual drift. Meanwhile, the accuracy of results estimated by the EPK model depends on the assumed reference state (i.e. a value of the kinematic ductility), thus it is not suitable for prediction of residual drift and cumulative plastic ductility under various seismic intensities. Based on the above statistical results, we can find that the simplified models can capture the maximum inter-story drift response with acceptable accuracy, however, the simplified models have natural deficiencies in predicting the structural residual drift and cumulative plastic ductility of BRB.

Table 2 Comparison of seismic demands on BRBF

Case	**Max. drifts (%) Mean (Mean+1σ)**		**Res. Drifts (%) Mean (Mean+1σ)**		**CPD Mean (Mean+1σ)**	
	DBE	**MCE**	**DBE**	**MCE**	**DBE**	**MCE**

BRBF3+Zona	1.51 (1.85)	3.56 (4.18)	0.23 (0.27)	1.05 (1.20)	58 (86)	128 (170)
BRBF3+EPK	1.38 (1.77)	4.41 (5.27)	0.25 (0.27)	0.49 (0.63)	25 (41)	**131 (252)**
BRBF3+EPP	1.32 (1.61)	**4.41 (5.75)**	0.47 (0.54)	**1.65 (1.99)**	13 (22)	62 (118)
BRBF6+Zona	1.44 (1.76)	3.38 (3.98)	0.18 (0.21)	0.84 (0.92)	**70 (103)**	149 (191)
BRBF6+EPK	1.46 (1.78)	3.24 (3.82)	0.27 (0.31)	0.58 (0.69)	40 (62)	110 (158)
BRBF6+EPP	**1.52 (1.80)**	3.38 (4.08)	**0.57 (0.66)**	1.31 (1.50)	23 (37)	53 (75)

Table 3 Relative errors of seismic demands on BRBF with simplified BRB hysteretic models (%)

Case	**Max. drifts (%) Mean (Mean+1σ)**		**Res. Drifts (%) Mean (Mean+1σ)**		**CPD Mean (Mean+1σ)**	
	DBE	**MCE**	**DBE**	**MCE**	**DBE**	**MCE**
BRBF3+EPK	-9.15(-4.59)	23.82(25.97)	9.30(2.46)	-53.03(-47.44)	-56.45(-51.99)	1.88(48.61)
BRBF3+EPP	-12.90(-13.09)	23.81(37.54)	104.57(101.32)	56.70(65.41)	-77.52(-74.54)	-51.26(-30.57)
BRBF6+EPK	1.58(1.06)	-4.21(-3.98)	49.76(46.97)	-30.97(-24.69)	-42.65(39.61)	-26.49(-17.15)
BRBF6+EPP	5.59(2.56)	0.08(2.47)	212.08(208.34)	56.47(62.89)	-67.80(-63.83)	-64.38(-60.48)

5. Conclusions

The seismic demand indices, including maximum drift, residual drift and cumulative plastic ductility were statistically analyzed. The following conclusions can be drawn,

1. The refined (Zona) hysteretic model of BRB is recommended for quantifying the seismic demands on BRBF.

2. The simplified (EPK and EPP) hysteretic models of BBRs can predict the maximum inter-story drift response of BRBF with acceptable accuracy.

3. the simplified models have natural deficiencies in predicting the structural residual drift and cumulative plastic ductility of BRB. This limitation should be fully considered if simplified models are adopted.

6. Acknowledgements

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